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FOREIGN JOURNALS.

NATURAL PHILOSOPHY.

EDITED BY

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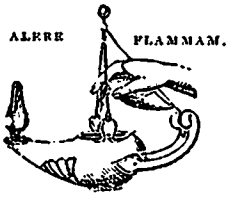
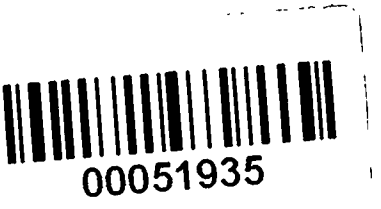


"Every translator ought to regard himself as a broker in the great intellectual traffic of the world, and to consider it his business to promote the barter of the produce of mind. For, whatever people may say of the inadequacy of translation, it is, and must ever be, one of the most important and meritorious occupations in the great commerce of the human race."—Goethe, *Kunst und Alterthum.*



CONTENTS.

PART I.

Page

ART. I.—On the Mechanical Equivalent of an Electric Discharge, and the Heating of the Conducting Wire which accompanies it. By R. CLAUSIUS 1

ART. II.—On the Processes which have taken place during the formation of the Volcanic Rocks of Iceland. By R. BUNSEN 33

ART. III.—On the Dependence of Radiant Heat in its passage through Crystals upon the direction of transmission. By H. KNOBLAUCH, Professor of Natural Philosophy in the University of Marburg 99

PART II.

ART. III.—On the Dependence of Radiant Heat in its passage through Crystals upon the direction of transmission. By H. KNOBLAUCH, Professor of Natural Philosophy in the University of Marburg (*continued*) 101

ART. IV.—On the Conservation of Force. By Dr. H. HELMHOLTZ 114

ART. V.—On the Connexion of Diamagnetism with Magnetism and Electricity. By M. W. WEBER 163

ART. VI.—On the Work performed and the Heat generated in a Conductor by a Stationary Electric Current. By R. CLAUSIUS 200

PART III.

ART. VI.—On the Work performed and the Heat generated in a Conductor by a Stationary Electric Current. By R. CLAUSIUS (*continued*) 201

Page

ART. VII.—On the Deviation of Projectiles; and on a remarkable Phænomenon of Rotating Bodies. By G. MAGNUS 210

ART. VIII.—On the Motion of Liquids in a closed Galvanic Circuit. By G. WIEDEMANN 232

ART. IX.—Researches upon the Optical Characters of Double Refraction in Isomorphous Substances. By M. H. DE SENARMONT 257

PART IV.

ART. IX.—Researches upon the Optical Characters of Double Refraction in Isomorphous Substances. By M. H. DE SENARMONT (*continued*) 281

ART. X.—On the Phænomena of closed Electro-Magnets. By J. C. POGGENDORFF 294

ART. XI.—On the Nature of those Constituents of the Atmosphere by which the Reflexion of the Light within it is effected. By R. CLAUSIUS 303

ART. XII.—On the Blue Colour of the Sky and the Morning and Evening Red. By R. CLAUSIUS 326

ART. XIII.—On the Theory of Diamagnetism, the Explanation of the transition from the Magnetic to the Diamagnetic deportment, and the Mathematical treatment of the Phænomena observed in Crystals. By M. PLÜCKER, Professor of Natural Philosophy in the University of Bonn 332

FIVE PLATES.

Fig. 1.

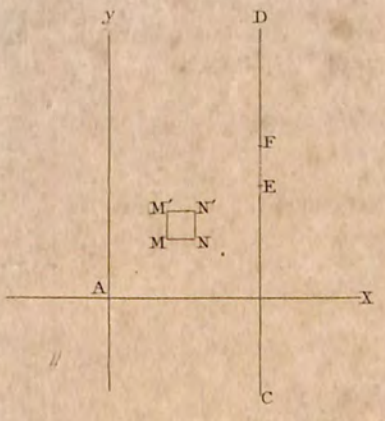
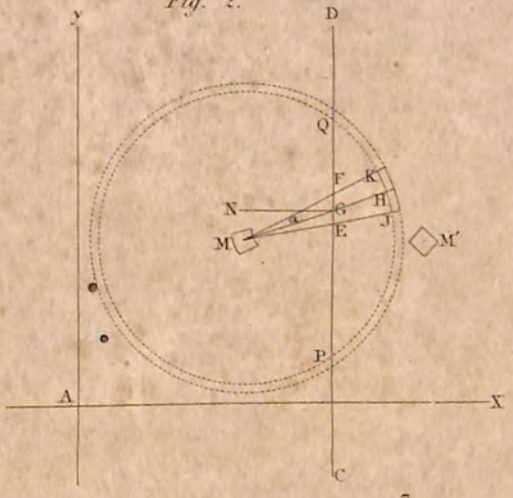
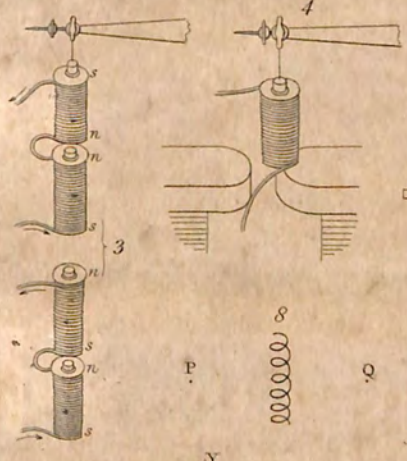
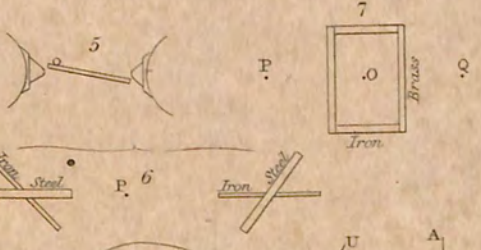
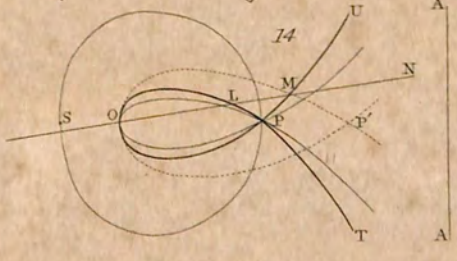


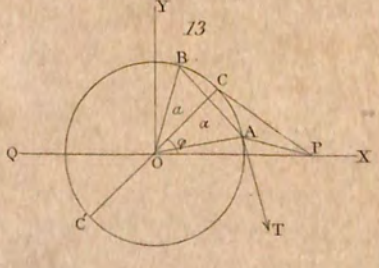
Fig. 2.

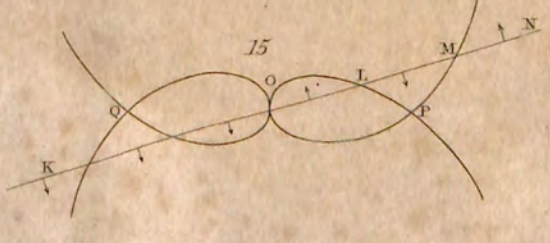












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SCIENTIFIC MEMOIRS.

NATURAL PHILOSOPHY.

Article I.

On the Mechanical Equivalent of an Electric Discharge, and the Heating of the Conducting Wire which accompanies it. By R. Clausius.

[From Poggendorff's *Annalen*, No. 7, 1852.]

By the application of heat we are able to obtain mechanical work, and it is known that electric currents are also capable of producing various mechanical effects, and also of producing heat. All those phænomena possess in themselves great interest, and this interest is considerably increased by the practical applications which have been made of these forces, and which may possibly yet be made of them. To this is added the circumstance that these actions are capable of strict mathematical treatment, and are therefore peculiarly suited to the investigation of the relation which subsists between them and their causes, and also of the varied influences which they exert upon each other.

They have indeed already been made the subject of various investigations. Particular attention has been heretofore devoted to galvanic electricity and to electro-magnetism, inasmuch as

these have been most frequently employed and upon the largest scale. In such an investigation, however, it appears to me more suitable to commence with *machine electricity*; although this subject presents greater difficulties to *mathematical treatment*, still in *principle* it is the most simple; for here we have to do with electricity alone, unaccompanied by the incidental actions of chemistry and magnetism.

In the following pages I have endeavoured *to reduce the effect produced by an electric discharge to a definite unit derived from the principles of mechanics*, and have compared in certain simple cases the result arrived at with those of experiment. The coincidence has, as will be observed, been so satisfactory, that in my opinion the deduced result is not only an undoubted law of electric action, but also a new corroboration of the mechanical theory of heat.

Let us imagine a system of material points possessing the masses m, m', m'', &c., and having, in a rectangular system of coordinates, at a certain point of time t, the coordinates x, y, z; x', y', z'; x'', y'', z'', &c. Let these masses be acted upon by a system of given forces, and let the components of the total force which acts upon m be X, Y, Z, and for m', X′, Y′, Z′, &c. Let the points be either quite free to move, or else limited in their motions, which last will of course be the case when the points are in any way connected together, as also when certain exterior conditions of motion exist; for example, if one of the points should be compelled to move in a certain surface, or along a certain line. The conditions, however, must not be such, that by them alone, and without the exercise of the given forces, motion can take place; which, for instance, would be the case if the surface or line just supposed, and which the point cannot forsake, were itself in a state of motion, or such that the motion of the given masses should be able to set other masses in motion which are not embraced in the system. That is, in other words, *the moving forces and the masses moved by them must be given explicitly*. Finally, let the velocities of the masses m, m', m'', &c. at the time t, be denoted by v, v', v'', &c., we then obtain the following general expression,

$$\tfrac{1}{2}\Sigma m d(v^2) = \Sigma(X dx + Y dy + Z dz), \quad . \quad . \quad . \quad . \quad (1)$$

in which the signs of summation Σ refer to all the masses of the system, and to their coordinates, forces, and velocities.

In this equation the left-hand side is immediately integrable, and so is the right side, if we regard the quantities x, y, z; x', y' z', &c., not as independent variables, but as what they really are, all of them *functions of one and the same variable, namely, of the time t.* We thus obtain

$$\frac{1}{2}\Sigma mv^2 = \int \Sigma(\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz) + \text{const.};$$

or, if we call the velocities at any time of commencement whatever v_0, v_0', v_0'', &c., and take this time as the lower limit of the integral,

$$\frac{1}{2}\Sigma mv^2 - \frac{1}{2}\Sigma mv^2{}_0 = \int_{t_0}^{t} \Sigma(\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz). \quad . \quad . \quad (2)$$

The quantities which appear in these equations are of very common occurrence in mechanics, and have on this account received particular names. With regard to that on the left-hand side, it is known that the product mv^2 is called the *vis viva* of the mass m; and according to this Σmv^2 would express the *vis viva* of the entire system. Here, however, only the half of this quantity makes its appearance; and as this will always be the case in the following pages, and likewise occurs so in most mechanical investigations, we propose to call, as other authors also have done, *the quantity* $\frac{1}{2}\Sigma mv^2$ *the vis viva of the system.*

The quantity on the right-hand side can, in the first place, be brought to a more simple form. Let ds represent the element of its path which the mass m describes during the time dt, and of which dx, dy, dz are the three projections on the three axes of coordinates; further, let S be the component of the total force acting upon m in this direction, we then have

$$\mathrm{S}ds = \mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz;$$

the quantity Sds is that which in treatises on mechanics is called *the quantity of work produced by the force acting upon* m *during the time* dt, bearing in mind that this work must be regarded as positive or negative, according as the element ds is in the same direction as S or opposed to it.

Applying these names, the meaning of the above equation may

be thus expressed: *the increase of vis viva in the system during any given time is equal to the quantity of mechanical work produced during the same time in the system.*

The determination of the work may be much simplified in particular cases which very often present themselves.

For instance, suppose a portion of the given forces to consist of attractions and repulsions which remain equal throughout the whole time, these forces being either such as might be exerted by other points upon the given ones, in which case the former must remain motionless, or such as the given points may exercise upon each other; let the intensity of each of these forces depend solely on the distance, and not upon the position which the attracting or repelling points may occupy; they may be any function whatever of the distance. The portion of the total sum due to these forces will then be expressed by

$$\Sigma_1(X dx + Y dy + Z dz);$$

and not only is this a complete differential, because all the quantities which appear in it are functions of one and the same variable t, but it also remains so if the single quantities x, y, z; x', y', z', &c., as far as is permitted by the express conditions to which the general motion of the system is subject, be regarded as *independent* variables. From this it follows that the value of the integral

$$\int \Sigma_1(X dx + Y dy + Z dz)$$

will be completely determined when the first and last values of x, y, z; x', y', z' are known, without its being necessary to know anything more regarding the nature of the motions by which the masses m, m', &c. pass from one position to another.

Further, to take a more special case; let the *exterior* forces proceed partly from a system of immoveable masses μ, μ', μ'', &c., and let these forces be *inversely proportional to the squares of the distances*, so that if ρ denote the distance of the two masses m and μ from each other, the force acting between them is $\pm \frac{m\mu}{\rho^2}$, which is positive or negative according as the force is one of attraction or of repulsion. Then for this portion Σ_2 of the total sum we have

$$\int \Sigma_2(X dx + Y dy + Z dz) = \Sigma \pm \frac{m\mu}{\rho} + \text{const.}, \quad . \quad . \quad . \quad (3)$$

where the sign of summation at the right-hand side refers to all possible combinations of every single mass of the given system with every one of the exterior systems.

In like manner let a portion of the *mutual* action of the masses of the given system itself *be assumed to be inversely proportional to the square of the distance*, so that if r denote the distance of two masses m and m' from each other, $\pm\frac{m\,m'}{r^2}$ will denote the attraction or repulsion exerted between them. We obtain for this portion Σ_3 of the entire sum

$$\int\Sigma_3(X\,dx+Y\,dy+Z\,dz)=\Sigma\pm\frac{m\,m'}{r}+\text{const.}, \quad . \quad . \quad (4)$$

where the sign of summation to the right refers to all possible combinations of every two of the given masses.

Now the quantity $\Sigma_3\pm\frac{m\mu}{\rho}$ is *the potential of the exterior system of masses upon the given system**, and $\Sigma\pm\frac{m\,m'}{r}$ *the potential of the given system of masses upon itself*; and hence the two last-mentioned portions of the work consist simply in the increasing of these potentials.

To these propositions, which indeed are those of ordinary mechanics, but which it was necessary to premise here, let us now add, *that heat consists in a motion of the ultimate particles of bodies, and is the measure of the vis viva of this motion*; so that if in any portion of the given system heat be excited, this

* The quantity $\pm\frac{\mu}{\rho}$, where ρ denotes the distance of the mass μ from the point o, is usually called, after the manner of Gauss, the potential of the said mass at this point; the potential is, in this case, not referred to the actual mass m which occupies the position o, but to an assumed *unit* of mass. This quantity $\pm\frac{\mu}{\rho}$ is certainly of such importance in mechanics as to need a special name; in like manner, however, the quantity $\pm\frac{m\mu}{\rho}$ is of great import; so that a short name is necessary here also, and it also is sometimes named potential. This double use of the word can, however, easily lead to error, and to distinguish one from the other I have applied to $\pm\frac{\mu}{\rho}$ the old name *potential function*, used by Green, and applied the word *potential* exclusively to the quantity $+\frac{m\mu}{\rho}$.

must be introduced into the calculation as an increase of the *vis viva*; and now let us turn to the consideration of electricity.

There can exist no doubt that the equation (2), which referred originally to the motions of *material masses*, is also applicable to the *electricity* within these masses; a statement already made in a very general manner by Helmholtz, and partly established*. To give, however, the quantity $\frac{1}{2}\Sigma mv^2$, which appears in this equation, a meaning so general as to embrace the motion both of the material masses and of the electricity, we must, in the first place, decide whether the electricity is subject to the laws of inertia, so as to permit of *vis viva* being ascribed to it. This question is manifestly of the greatest importance to the whole science of electricity; notwithstanding this, however, we will not enter upon its discussion here, inasmuch as its solution is without influence upon the present investigation.

If the work produced by the electricity in any change of arrangement whatever be compared with the cases already cited, we readily observe that it is included in the last of these cases, inasmuch as in the production of this work the forces in action consist only in the mutual attraction or repulsion of the particles of electricity, this being inversely proportional to the square of the distance. From this it immediately follows, *that the said work is independent of the manner in which the alteration is effected, and depends only on the primitive and final conditions, and that the work is measured by the increase of the potential of the entire electricity upon itself*†.

* *Ueber die Erhaltung der Kraft* (On the Conservation of Force), a physical memoir by Dr. H. Helmholtz. Berlin bei G. Reimer, 1847. It is to be regretted that the author of this ingenious essay has not entered more fully into the details of his subject. From this cause certain portions appear to me to be inexact, and others difficult to be understood; such a mass of thought it would, however, be difficult to press into a narrow space. [The memoir here referred to will appear in the next Part of this work.—Eds.]

† Helmholtz gives in a case cited by him as an example (see p. 40) the following expression for the work produced:

$$-\left(V+\frac{W_a+W_b}{2}\right),$$

while, with the same meaning attached to the letters, the increase of the potential is

$$-(V+W_a+W_b).$$

This divergence from the above proposition is simply due to the fact that he

According to this it will not be necessary for us to consider the varying condition which exists *during* a discharge, but only to consider the constant conditions existing before and after the discharge; and as in these two states the electricity is at rest, the question whether the electricity possess *vis viva* or not may remain unnoticed.

In like manner it is unnecessary to decide whether there are two kinds or only a single kind of electricity; for, that the attractions and repulsions take place *as if* there were two electricities is sufficiently proved; and we can therefore, in the present instance, where we have only to do with the *forces*, always speak of two electricities as the carriers of these forces, without therefore attributing to them an actual existence.

Besides the work already mentioned as produced by the attractive and repulsive forces of the electricity, and which is measured by the increase of the potential, we find in the case of discharge many additional effects in which other forces come into play, and of which a few of the most common may be introduced here.

At one or more places electric sparks spring over, by which a stratum of air or other non-conducting body is broken through. When the electric current passes in one place through a thin wire, the latter undergoes mechanical changes which may vary between the limits of small indentations impressed on the wire and the complete reduction of the latter to powder. When the current passes through electrolytic bodies, chemical decompositions take place. In bodies which lie in the immediate neighbourhood of the current, induced currents or magnetic effects may be excited, &c.

These actions must, in reference to the forces under consideration, be regarded as *negative* work, inasmuch as the nature of the actions is, that the corresponding forces, as for example in the first case, the resistance of the layer of air or other body penetrated by the spark, in the second case, the solidity of the wire, &c., are *overcome*, and therefore motions take place which are *opposed* to the forces. If all these quan-

reckons the potential of a mass upon itself double what it really is. In other cases, where, instead of the potential, he introduces quantities corresponding to the potential function, his formulas and mine are coincident.

tities of work be united in an algebraic sum with the increase of the potential, the said sum forms the right-hand side of the equation (2), that is, of the equation

$$\frac{1}{2}\Sigma m v^2 - \frac{1}{2}\Sigma m v_0^2 = \int_{t_0}^{t} \Sigma\,(X dx + Y dy + Z dz),$$

while the member to the left expresses the increase of the *vis viva* of the entire system due to the discharge. This last can also be of a twofold nature. In the first place, under favourable circumstances, actual visible motions may be produced in the system by the electric attractions and repulsions; secondly, and more especially, heat will be excited by the current in the conductors.

Let us suppose all the above-mentioned quantities of negative work to be positive and brought over to the left-hand side of the last equation, so that only the increase of the potential remains to the right; the proposition contained in the equation can be expressed in a very complete manner as follows:—

The Sum of all the effects produced by an electric discharge is equal to the increase of the potential of the entire electricity upon itself,

in which place under *electric discharge* is meant every alteration in the arrangement of the electricity, by which the electric states of the various portions of a system of conducting bodies, in which the earth itself may be included, entirely or partially neutralize each other.

We will now apply this general proposition, which for the sake of brevity will be hereafter called the principal proposition, to the special case of a Leyden jar, or to a battery composed of a number of such jars; this case is of peculiar interest on account of its frequent occurrence, and presents the best opportunity of comparing the results of theory with those of experience. With regard to the latter, the investigations of Riess, which are carried out with the greatest care and ability, furnish us with a fund of materials in which the greatest confidence may be placed; the comparison of the same with our principal proposition is rendered the more easy by the circumstance that Riess himself has deduced from the facts which he has observed determinate laws, which he has expressed in strict mathematical formulas.

In the first place, let it be required to determine the *value of the potential* in the case of a charged Leyden jar or battery.

In order to present the matter more simply to the mind, we will commence with the special case of a jar of particular form, and apply the expression thus found to the deduction of the general expression. We will first choose a form without indeed an actual existence, but which, however, in all essential points must be subject to the same laws as the common Leyden jar, and which leads to results of extraordinary simplicity. The glass vessel shall form a *closed hollow sphere*, possessing at all points the same thickness, and completely covered with tinfoil both inside and out. Let us suppose a quantity of electricity Q to be imparted by some means or other to the interior surface, where we assume as unit such a quantity of positive electricity as exerts upon an equal quantity of the same electricity the unit of force at the unit of distance. Let the outer surface stand in connexion with the earth, and let the quantity of electricity which it receives from the latter be denoted by Q'.

In this case it is evident that Q and Q' must spread themselves *uniformly* over both the respective surfaces, and in consequence of this the determination of the potential function and of the potential is greatly facilitated.

The potential function V of any quantity of electricity Q at any point O, will, in general, be determined by the equation

$$V = -\int \frac{dq}{r},$$

where dq denotes an element of electricity, and r its distance from the point O, the integral extending over the entire quantity. For the particular case however where Q is spread uniformly over a spherical surface we do not need this general equation, but can apply the following two propositions:—

1. Within the sphere the potential function is everywhere the same, that is, if r be the radius of the sphere,

$$V = -\frac{Q}{r}.$$

2. Without the sphere, and for a distance R from its centre, the potential function is

$$V = -\frac{Q}{R}.$$

On the surface both expressions give the same value, and hence both propositions in this case hold good simultaneously. In the instance before us the tinfoil coatings form two concentric spherical surfaces, whose radii may be called a and $a+c$, c being the thickness of the glass. Let us first consider a point on the interior coating; in this case we can apply the first proposition to both the spherical surfaces; connecting the potential functions of both quantities of electricity Q and Q′ together, we obtain

$$V=-\frac{Q}{a}-\frac{Q'}{a+c}. \quad \ldots\ldots\ldots \quad (5)$$

For a point on the outer coating, on the contrary, the second proposition is to be applied, and if in this case we call the potential function V′, we have

$$V'=-\frac{Q}{a+c}-\frac{Q'}{a+c}. \quad \ldots\ldots\ldots \quad (6)$$

Through the condition that the exterior surface is in communication with the earth, we possess a means of estimating the quantity of electricity Q′. It is known that when several conducting bodies are connected together, that the equilibrium of the electricity is so established that the potential function in the interior of the entire system possesses the same value. Hence, as in the earth, where in general equal quantities of positive and negative electricity are present, the potential function is zero; this must also be the case for the exterior covering. We have therefore

$$V'=0;$$

and hence, according to (6),

$$Q'=-Q,$$

and thus (5) becomes

$$V=-Q\frac{c}{a(a+c)}; \quad \ldots\ldots\ldots \quad (7)$$

when the fraction is developed according to C, the superficial content of the inner coating, that is $4a^2\pi$, being denoted by S, we have

$$V=-\frac{Q}{S}4\pi c\left(1-\frac{c}{a}+\frac{c^2}{a^2}-\text{\&c.}\right). \quad \ldots \quad (7a)$$

We are now in a condition easily to determine the potential

of the entire electricity upon itself. The potential W of a given quantity of electricity upon itself is, in general,

$$W=-\frac{1}{2}\iint\frac{dq.dq'}{r},$$

where dq and dq' represent any two elements of the electricity, r their distance asunder, and both the integrals extend over the entire quantity, the factor $\frac{1}{2}$ must be introduced, because in the double integral every combination of every two elements dq and dq' appears twice. Now as

$$-\int\frac{dq}{r}=V,$$

instead of the foregoing expression we can write

$$W=\frac{1}{2}\int V dq. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Now, as before mentioned, in every connected conducting body the potential function is constant, and may therefore be taken from under the sign of integration; the integral that remains represents simply the quantity of electricity distributed over the body. If we apply this to the two coatings of a Leyden jar with the potential functions V and V', we obtain as the total potential of both the quantities Q and Q' upon themselves,

$$W=\frac{1}{2}(Q.V+Q'.V'); \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and if for our special case we set $V'=0$, and in the place of V set the value of it, as found by (7) or (7*a*), we obtain the required potential in the case of a charged *spherical* jar,

$$W=-Q^2.\frac{c}{2a(a+c)}, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

or

$$W=-\frac{Q^2}{S}.2\pi c\left(1-\frac{c}{a}+\frac{c^2}{a^2}-\&c.\right). \quad . \quad . \quad (10a)$$

The *plate of Franklin with circular coatings* may be regarded as the next simplest form of the Leyden jar. In a former memoir I have devoted especial attention to this form, and from the results there obtained I will introduce but one here, which corresponds to the foregoing example. This is the case in which one of the metallic coverings is supposed to be connected with

the earth, while a quantity of electricity Q is communicated to the other. On the former the value of the potential function is of course zero, while on the latter, when c denotes the thickness of the glass, and a the radius of the circle, the value is

$$V=-Q\frac{4c}{a^2}\left[1-\frac{c}{a\pi}\left(\log\frac{17{\cdot}68a}{c}+2\right)\right],$$

for which, when the superficial content of one of the coatings, that is $a^2\pi$, is denoted by S, we can write

$$V=-\frac{Q}{S}4\pi c\left[1-\frac{c}{a\pi}\left(\log\frac{17{\cdot}68a}{c}+2\right)\right]. \quad . \quad (11)$$

To obtain from this the potential of the total electricity upon itself, it is only necessary to multiply by $\frac{1}{2}Q$, hence

$$W=-\frac{Q^2}{S}2\pi c\left[1-\frac{c}{a\pi}\left(\log\frac{17{\cdot}68a}{c}+2\right)\right]. \quad . \quad . \quad (12)$$

Comparing the expressions (11) and (12) with ($7a$) and ($10a$), we observe that the principal members $-\frac{Q}{S}4\pi c$ and $-\frac{Q^2}{S}2\pi c$ coincide in both cases, and the divergence first appears in those members which, in regard to c, are of a higher order than the first, and hence if c is small compared with the dimensions of the coatings, they may be neglected. The same may be proved in general for every other Leyden jar in which the glass is of a uniform thickness, that is, where c is constant. Neglecting therefore the members of a higher order, we obtain the equations

$$V=-\frac{Q}{S}4\pi c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$W=-\frac{Q^2}{S}2\pi c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

by means of which jars of various forms, magnitude, and thickness of glass may be compared with each other, with this condition only, that the thickness c is constant in each, which it must be confessed is only partially the case with jars actually applied.

When instead of different jars, a battery of jars, all of the same size, is applied—a case of frequent occurrence in practice—and that the superficial content is changed solely by increasing or decreasing the number of jars, the various cases that then pre-

sent themselves can be compared with each other, without neglecting the members of a higher order, and without the limiting assumption made with respect to the thickness of the glass.

Whatever the nature of the jar may be, when s is the surface of its interior coating, and q the quantity of electricity upon it, we can set

$$V = -\frac{q}{s}k, \quad \ldots \ldots \ldots \quad (15)$$

$$W = -\frac{q^2}{s}\cdot\frac{k}{2}, \quad \ldots \ldots \ldots \quad (16)$$

where k is a constant dependent on the nature of the jar, which, though not known, is the same for equal jars, and besides this is of the first order in respect to the mean thickness of the glass.

Now let n such jars be taken, and having been all charged to the same degree, let all the inner and all the outer coatings be connected together. Then if we do not take into account the influence exerted by the jars upon each other, in case they stand too near, nor the influence of the electricity distributed over the connecting pieces, then as regards the *potential function* no change will be effected by the combination, but the *potential*, on the contrary, in the entire battery will have an n times greater value than in every single jar. Hence calling the surface of the common interior coating S, and the total electricity distributed over it Q, to obtain the quantities V and W for the entire battery it is only necessary to set in the foregoing equations in the place of q its value $\frac{Q}{n}$, and for s its value $\frac{S}{n}$, and to multiply the second equation by n. In this case n disappears in both equations, and we again obtain

$$V = -\frac{Q}{S}k \quad \ldots \ldots \ldots \quad (17)$$

$$W = -\frac{Q^2}{S}\cdot\frac{k}{2} \quad \ldots \ldots \ldots \quad (18)$$

Regarding then the potential of a charged Leyden jar or battery as known, the increase of the potential by the act of discharge, and hence the work produced by the electricity are given. If the

discharge take place only partially, and the potential of the residue be denoted by W_1, then the work produced will be

$$=W_1-W, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which, as W and W_1 are always negative, and W_1 has an absolutely smaller value than W, is a positive quantity. If, on the contrary, a complete discharge takes place, we must set $W_1=0$, and in this case the work produced is

$$=-W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

We will now consider the effects which are due to the discharge.

Let us suppose the discharge to be occasioned by connecting one coating with the other by conducting bodies, whose ends are brought either so near that a spark springs from one to the other, or else brought into actual contact. In this case, during the act of approximation, an electric action takes place, inasmuch as the approximated ends of the connecting arc attract each other, and thus render their approximation easier. In our case, however, where the greatest portion of the electricity is bound to the coatings, and hence cannot contribute to this attraction, the latter must be so trifling that it may be neglected altogether.

Further, to simplify the matter, we will for the present not take into account the excitation of induced currents, nor any permanent changes which are due to magnetical or chemical action; we will assume that *the work expended at the places where the connexion is interrupted, and where a spark must spring across, together with the heat generated in the system*, are the only actions which present themselves. Then, in accordance with our principal proposition, *the sum of these both must be equal to the increase of the potential.*

Let it be next assumed that in a series of experiments *the strength of the discharge, that is, the increase of the potential, remains the same, but that the connexion is altered,* in this case *the sum of the actions must be constant.*

With regard to the development of heat, we possess, in reference to its dependence on the nature of the connexion, the following two important laws established by Riess*:—

* Pogg. *Ann.* vol. xliii. and xlv.

1. *The quantity of heat developed in two different wires, of which the circuit is composed, is directly proportional to their reduced lengths*; when under the term *reduced length* the quantity $\frac{\lambda}{\rho^2}x$ is understood, in which λ signifies the actual length, ρ the radius, and x a quantity depending on the nature of the wire, which Riess has named its retarding force, and which corresponds to the inverse value of the conductive power.

2. *When, other circumstances remaining the same, the circuit is lengthened, by introducing into it a wire of the reduced length* l, *the effect is that the heating of another wire which forms a portion of the circuit is lessened in the proportion of* $1+bl:1$, *where* b *expresses a constant to be determined by experiment.*

Both propositions may be expressed by the following equation*—

$$C=\frac{l'}{1+bl}\cdot A, \quad \ldots\ldots\ldots \quad (21)$$

where l' denotes the length of the portion of wire under consideration, and C the quantity of heat excited in it, while b and l retain the signification already given to them, and A is a quantity dependent on the strength of the charge, which in our present case, inasmuch as we have to deal with equal discharges only, is constant.

This equation contains a corroboration of the conclusion already drawn. The wire l will of course be likewise warmed by the discharge; according to the foregoing equation the quantity of heat developed in it will be $\frac{l}{1+bl}A$. The consequence of this will be, if the total sum of the actions remains constant, a decrease of the remaining actions, which is indeed proved by Riess's second proposition, and by the equation. With this general coincidence we must rest satisfied for the present. An exact quantitative investigation whether the decrease of all the remaining actions taken together is actually equal to the quantity of heat expressed by $\frac{l}{1+bl}A$, seems to me incapable of accomplishment without new data of observation.

Vorsselman de Heer has, it is true, deduced a general propo-

* Pogg. *Ann.* vol. xlv. p. 23.

sition from the equation (21), which might at first sight be taken for a complete corroboration of our conclusion. According to this proposition, *the total heat which is excited in the entire circuit by an electric discharge is independent of the nature of the circuit**. This proposition is indeed brought forward by Helmholtz as coincident with the theory†; I do not think, however, that it is quite applicable, inasmuch as it contains many inaccuracies.

In the first place, Vorsselman de Heer limited the consideration expressly to "the wire which unites both the coatings of the battery‡." The development of heat extends however to the other bodies of the system, a portion of it indeed is excited within the battery itself; and another portion, in case the battery and connecting wire are not insulated but are connected with the earth, within the conducting wire and the earth. The latter portion will in general be trifling, as the excess alone of the one or the other electricity escapes to the earth, and this, in comparison with the total quantity of electricity, is very small; the same may perhaps be assumed with regard to the former portion, under the condition that the connecting wire possesses a considerable reduced length. With very short connecting wires, on the contrary, such an assumption would be unwarrantable, and this portion must, at all events for the present, be regarded as unknown.

Further, he treats the connecting wire so as if it were one continuous piece. In this respect Riess§ has already drawn attention to the fact that his connexion was established by several pieces of wire attached together; and as his experiments refer only to the continuous wires, and not to the heat developed at the points of junction, so his proposition, so far as it is a conclusion from his experiments, was refuted. From a theoretic point of view, indeed, those places of junction where actual metallic contact takes place, and where the electricity in its passage gives rise to no mechanical alterations, would be embraced in the general proposition, without its being necessary to know the quantity of heat developed in each singly. It is otherwise, however, with the places where an *interruption* occurs, where a

* Pogg. *Ann.* vol. xlviii. p. 298.
† His memoir, p. 44.
‡ Pogg. *Ann.* vol. xlviii. p. 297.
§ Pogg. *Ann.* vol. xlviii. p. 320.

spark springs over. An exterior mechanical action takes place here, which, as so much work consumed, must be deducted from the total action, in order to obtain that portion which is changed into heat within the system of bodies under consideration.

With respect to the magnitude of the work here expended, and its influence on the development of heat, I am enabled to adduce an experimental corroboration of the theory. It is, in the first place, clear that the consumption of work must depend on the resistance presented to the passage of the electricity by the non-conducting layer, and hence that it will be more considerable when the ends of the connecting wires are separated by a non-conducting *solid* body than when merely *air* intervenes between them. From this it follows, that in the first case an air-thermometer placed in some other portion of the circuit must be less warmed than in the latter case; this indeed has been established by a series of experiments carried out by Riess.

At the place of interruption were placed either two small discs, or two small balls, or two points opposite to each other, at a distance of 0·2 of a line apart in each case. Between these the bodies mentioned in the first column of the following table were introduced one after the other; and the quantities of heat developed under precisely the same conditions are contained in the following columns. Where Riess has given several numbers I have given the mean:—

Body introduced.	Heat developed in Air-thermometer on the passage of the spark.		
	Between the discs.	Between the balls.	Between the points.
Layer of air	15·9	15·4	15·1
A card	11·7	12·0	11·6
Two cards with tinfoil between them	9·7	9·3	
Two cards.....	8·0	8·8	10·4
A plate of mica.....	6·8	4·7	4·8

The influence of the solidity of the bodies penetrated by the spark is very plainly visible here*; and it is at the same time

* Only in the case where two cards with tinfoil between them were applied do we find an exception, inasmuch as those three bodies exercised a less influence than the two cards alone. According to this we must assume that the plate of tinfoil, although it also was penetrated, caused, not an increased con-

observable, from the great difference in the numbers, how considerable the quantity of work expended by the spark, in cases when difficulties are presented to its passage, may be. An exact measure of this work cannot be deduced from the experiments; and such a measure, in my opinion, we do not yet possess, even for the simplest and most important case where the spark passes through air.

At first sight it might be imagined that *the quantity of work expended must, when the density of the air is uniform, be proportional to the thickness of the layer.* When, however, the nature of the circuit or of the charge is changed, the distance across which the spark passes remaining unaltered, such differences are observed, even in the exterior manifestations of the spark, its noise and magnitude, that such sparks cannot be regarded as alike, in respect to the quantity of work which they have consumed.

Further, it might be perhaps concluded from some of the observations communicated by Riess*, *that the work consumed by a spark in passing through the air is so small that it might be neglected.* Riess has conducted the experiments with the little discs and balls before mentioned, bringing them first into contact, and then within different distances of each other, so that in the first case the electricity passed without a spark, and in the latter case with a spark; for each of these cases he observed the quantity of heat developed in the connecting wire under the same conditions of experiment. In these instances the quantity of heat developed when the discs were separated by an interval of air, was only a little less than by actual contact; and in some cases it was actually greater, which was the more extraordinary, inasmuch as a residue here remained in the battery, while in those cases where contact was established a complete discharge took place. I believe, however, that these observations would not justify the foregoing conclusion.

sumption of work, but the contrary; and this seems to involve a contradiction. I believe, however, that this assumption, even although it is as yet incapable of being firmly established, is not to be regarded as contrary to common sense. For with regard to the quantity of work expended, we have not only to take into account the bodies which were penetrated, but also the manner in which the penetration is effected, and this latter will certainly be affected by the conducting body introduced between the cards.

* Pogg. *Ann.* vol. xliii. p. 78.

With regard, in the first place, to the fact that where a spark occurred a residue remained in the battery, the influence of this residue must not be rated too high. This residue, even in the extreme case that the distance between the discs was equal to the length of the stroke of the battery, could only be about $\frac{2}{13}$ of the entire charge*. By this, however, the total action of the discharge would not be diminished by $\frac{2}{13}$ of the whole, but only by $\left(\frac{2}{13}\right)^2$, or $\frac{1}{42}$. The total action is, according to our principal proposition, equal to the increase of the potential, and the potential in the case of any given battery is proportional to the square of the quantity of electricity. Hence, if we denote the quantity of the entire charge by Q, and that of the residue by Q_1, the increase of the potential is

$$W_1 - W = A(Q^2 - Q_1^2),$$

where A is a number independent of the quantity of electricity.

This gives, according as we set,

$$Q_1 = 0, \text{ or } = \frac{2}{13}Q,$$

$$W_1 - W = AQ^2, \text{ or } = AQ^2\left[1 - \left(\frac{2^2}{13}\right)\right].$$

Further, besides those sparks which were intentionally called forth by placing the discs or balls at a distance from each other, we must also take those into account which were the necessary accompaniment of the mode of experiment. Riess, in order to make the discharges as regular as possible, obtained them by a peculiar apparatus constructed by himself†, which was so arranged that at every discharge *two* sparks sprang over. Now it may be deduced from other experiments by Riess‡, that by making an interruption anywhere in the circuit, the distance across which the spark will spring in any other portion of the circuit will be diminished, and hence in the case before us, at the same time that they create the new spark between the balls or discs, the two other sparks must themselves become shortened, from which we can infer a partial compensation as regards the work con-

* See Riess, Pogg. *Ann.* vol. liii. p. 11.

† Pogg. *Ann.* vol. xl. p. 339. ‡ Pogg. *Ann.* vol. liii. p. 11.

sumed. In many cases the two latter sparks indeed disappeared totally, inasmuch as the discharge did not take place until the two balls of the apparatus were in contact*. Hence *one* new spark had made its appearance, and for this *two* old ones had disappeared, which justifies the expectation that a lessening of the quantity of work consumed, and a corresponding increase of heat would be the result. It was, in point of fact, in *these cases* that Riess observed the increased temperature of the connecting wire. We thus see that in explaining these phænomena it is not necessary to assume that the work consumed by a spark is very small; and to me it appears that the experiments do not furnish ground for a safe conclusion as to the magnitude of the same quantity of work.

Although it thus appears that, on account of the unknown quantities which enter into the production of the total action, it is impossible to demonstrate an *exact quantitative* coincidence between the equation (21) and our principal proposition, we could perhaps try the inverse method, and assuming both, and their connexion with each other, to determine the said unknown quantities, or at least their sum; indeed the form of the equation seems to invite such an effort. It must however be remembered that the equation itself, being an empirical one, must not have absolute accuracy ascribed to it, as is also shown by the numbers of Riess. In two series of experiments in which he introduced wires of different lengths and thicknesses, thus changing only the quantity l in the expression on the right side of equation (21), he determined from each observation the constant b. The values so found diverge from each other in the first series between 0·01358 and 0·01101, and in the second series between 0·00000926 and 0·00000840†; and although these differences, considering the great variety of the wires introduced, cannot be regarded as considerable, still they are worthy of regard, inasmuch as a certain regularity is observed in them. In both series, when the reduced length l of the wire is increased, the corresponding worth b is in general smaller.

We will therefore follow this subject no further, and turn now

* Pogg. *Ann.* vol. xliii. p. 79.

† Pogg. *Ann.* vol. xliii. pp. 68 and 73. The great difference between the numbers of the first and second series is due to a difference in the unit made use of.

to the second point of comparison between theory and experience, namely, to the case *where the connecting wire remains the same, but the magnitude of the battery and the quantity of electricity distributed over it are changed.*

Here also the difficulty spoken of lies in our way. As we are ignorant of the nature of a portion of the action of the discharge, we are unable to say how this portion changes with the magnitude of the battery and the quantity of electricity, and cannot, therefore, from the effects observed at a particular place in the circuit, conclude with security as to the totality of the action. Only in regard to the heat developed in the continuous portion of the connecting wire can we with certainty assume that every alteration observed in any one portion is proportional to those which take place in all other portions.

Now when the connecting wire has a great reduced length, it may certainly be assumed that the greatest part of the total action is applied to heating it; and in this case also, if a divergence from the said proportionality should exhibit itself, the differences caused by it will be comparatively small, so that without any considerable inaccuracy we may assume that the heating observed at any portion is proportional to the corresponding total action.

Now the total action, according to equations (18) and (19), for an incomplete discharge is represented by

$$\frac{Q^2-Q_1^2}{S}\,.\,\text{const.},$$

and for a complete discharge, such as took place in the experiments of Riess, according to (18) and (20), by

$$-\frac{Q^2}{S}\,.\,\text{const.};$$

and this is the exact expression which Riess established experimentally for the heating in the connecting wire, inasmuch as the equation (21) completely expressed*, is

$$C=\frac{al'}{1+bl}\,.\,\frac{Q^2}{S}, \quad \ldots\ldots \quad (21a)$$

where a is a constant quantity†.

* Pogg. *Ann.* vol. xlv. p. 23.

† This coincidence between theory and experiment is also adduced by Helmholtz (his Memoir, p. 19); the development of his formula is not however

The cases heretofore considered referred to the incomplete discharge of an ordinary battery. We will examine only two other cases, namely, *the incomplete discharge of an ordinary battery, and the complete discharge of the so-named cascade battery of Franklin.*

With reference to the first, we possess experimental measurements by Riess*, who partially discharged a charged battery by bringing both its coatings into connexion with one another; so that the electricity which was formerly confined to one, now distributed itself over two. He changed the experiment by making both batteries to consist of different numbers of jars, and observed each time the heating of the one or the other connecting wire. The jars of each battery were all equal, but unfortunately the jars of one battery were different in size from those of the other. As the result of his observations, he states that all the quantities of heat generated at a constant place in the inner and in the outer connecting arc are completely expressed by the following formula†:—

$$C=\frac{aQ^2}{\left(\frac{n}{n'}+\frac{s'}{s}\right)ns}; \quad \ldots\ldots\ldots \quad (22)$$

for the sake of more easy comparison with my other formulas, I have here chosen different letters from those chosen by Riess. C expresses the observed heat, Q the quantity of electricity applied, s the superficial content of the inner coating of a jar of the first battery, and n the number of these jars; s' and n' the same quantities for the other battery, and finally a a constant, which is somewhat larger for the inner connecting arc than for the outer; this is explained by the fact that a little more electricity was distributed over the inner coating than over the outer.

We will now compare this heating with the increase of the potential.

From equation (18) we derive as potential of the first battery before the discharge, when the quantity of electricity is denoted

quite clear to me, inasmuch as he introduces a quantity which he names the quantity of work, and concerning which he says that it is proportional to the coated surface of the battery, without however explaining its meaning further, or giving any reason for this proportionality.

* Pogg. *Ann.* vol. lxxx. p. 214. † Pogg. *Ann.* vol. lxxx. p. 217.

by Q, and when for the total superficial content S its value ns is set, the expression

$$W = -\frac{Q^2}{ns} \cdot \frac{k}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In order to determine how the entire quantity of electricity Q is distributed by the discharge over both batteries, we know the condition, that on the coatings which are connected together the potential functions must be equal. Let V_1 and V_1' be the potential functions on the inner coatings after the discharge, and Q_1, Q_1' the quantity of electricity upon them, which it is our object to ascertain; we have then, according to (17),

$$V_1 = -\frac{Q_1}{ns} k,$$

$$V_1' = -\frac{Q_1'}{n's'} k',$$

where k' is the same magnitude for the second battery that k is for the first. If these expressions be set down as equal, and bearing in mind that

$$Q_1 + Q_1' = Q,$$

we obtain

$$\left.\begin{aligned} Q_1 &= \frac{\frac{ns}{k}}{\frac{ns}{k} + \frac{n's}{k'}} Q, \\ Q_1' &= \frac{\frac{n's'}{k'}}{\frac{ns}{k} + \frac{n's'}{k}}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

From this we obtain further, when W_1 denotes the entire potential of both batteries after the discharge,

$$W_1 = \frac{1}{2}(Q_1 V_1 + Q_1' V_1') = -\frac{\frac{1}{2}Q^2}{\frac{ns}{k} + \frac{n's'}{k'}}, \quad . \quad . \quad (25)$$

and thus we obtain the increase of the potential,

$$W_1 - W = \frac{\frac{1}{2}\frac{k^2}{k'} \cdot \frac{s'}{s} \cdot Q^2}{\left(\frac{n}{n'} + \frac{k}{k'} \cdot \frac{s'}{s}\right) ns}. \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The quantity $\frac{1}{2}\frac{k^2}{k'} \cdot \frac{s'}{s}$ is constant for the entire series of expe-

riments, and hence we can write

$$W_1 - W = \frac{A.Q^2}{\left(\frac{n}{n'} + \frac{k}{k'} \cdot \frac{s'}{s}\right) ns} \quad . \quad . \quad . \quad . \quad (27)$$

If this expression be compared with that found by Riess for the heating (22), it is seen that to make both proportional it is only necessary to assume that in the jars of both batteries, although they were not equal, the quantities k and k' possess nearly the same value; and this assumption is peculiarly justified by the fact that Riess found by direct measurement, that when they were combined the electricity distributed itself over the batteries in proportion to their surfaces, and this, according to (24), could only be the case when k was $= k'$*.

Riess altered his experiments so that he increased the length of the connecting wire, and observed the consequent diminution of heat at a certain place. The results of these observations coincide in general with those already mentioned, and we will therefore pass over them here, and also some other experiments described in the same paper.

In regard to the cascade battery, we possess experiments from Dove† and Riess‡. It consists, as is known, of a number of single jars or complete batteries, insulated and so connected with each other, that the exterior coating of the first is connected with the interior coating of the second, the exterior of the second with the interior of the third, and so forth. Only the inner

* As the quantities k and k', according to the above, depend principally upon the *thickness of the glass* of both batteries, it appeared to me of interest to ascertain these thicknesses; while this Memoir was still in the press, I therefore requested M. Riess to make the measurement, and am indebted to his kindness for the following communication. In the small jars (those of the second battery) the thickness of the glass varies considerably, its mean thickness being $1\frac{1}{3}$ Paris line. The large jars (those of the first battery) he was unable himself to measure as they were closed above, and he has therefore measured two others which he had made at the same time with the former, and with the intention of being used along with them. The glass in these is nearly uniform and $1\frac{1}{4}$ line thick. As an absolute equality of the glass thicknesses was not to be expected under the circumstances, and as it is not a necessary consequence of the assumed equality of k and k', inasmuch as the latter are also to a certain, although subordinate extent, dependent on the shape and magnitude of the jars which are different in both batteries, I believe that the coincidence of the numbers $1\frac{1}{3}$ and $1\frac{1}{4}$ is sufficiently exact.

† Pogg. *Ann.* vol. lxxii. p. 406. ‡ Pogg. *Ann.* vol. lxxx. p. 349.

coating of the first and the outer coating of the last are free; and, in charging, these are treated as if both surfaces formed a single battery.

The principles according to which the charge of such a compound battery must be determined, are for the most part already given by Green*. Let the quantities of electricity distributed over the two coatings of the single batteries, and the corresponding potential functions be expressed in order by—

$$\left.\begin{array}{l} Q_1,\ Q'_1;\ Q_2,\ Q'_2;\ Q_3,\ Q'_3\ \&c. \\ V_1,\ V'_1;\ V_2,\ V'_2;\ V_3,\ V'_3\ \&c. \end{array}\right\} \quad . \quad . \quad . \quad (28)$$

Now if positive electricity be communicated by a conductor to the inner coating of the first battery, the outer coating of the same can only receive its negative charge from the interior of the second battery, which will thus become positively charged; we therefore have

$$Q'_1 = -Q_2;$$

and further, as in two bodies which are connected by a conductor the potential functions are equal, we have for the same two coatings

$$V'_1 = V_2;$$

and two equations exactly similar apply to every other pair of connected coatings, so that the following series of equations is given,

$$\left.\begin{array}{l} Q'_1 = -Q_2,\ Q'_2 = -Q_3,\ Q'_3 = -Q_4\ \&c. \\ V'_1 = \ \ V_2,\ V'_2 = \ \ V_3,\ V'_3 = \ \ V_4\ \&c. \end{array}\right\} \quad . \quad (29)$$

Besides this, the four quantities Q, Q', V and V', stand for each of the batteries in such a relation to each other that from every two of the same the two others may be determined. In accordance with the remark at the conclusion of a former paper of mine, if we denote the said four quantities by q, q', V and V', the equation (15), which applies to the case where $V'=0$, can, if we neglect the members of a higher order in respect to k, be changed into the following more general one,

$$V - V' = -\frac{k}{s}\frac{q-q'}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where instead of $\frac{q-q'}{2}$ we may also write q, or $-q'$.

* An Essay on the Application of Mathematical Analysis to the theories of Electricity and Magnetism, Art. 8.

Further, we have quite generally the equation

$$q+q'=-(\alpha V+\beta V'), \quad . \quad . \quad . \quad . \quad . \quad (31)$$

wherein α and β are two positive constant quantities, which, like k, depend upon the nature of the jar, and hence in the case before us, where we have simply to do with equal surfaces, possess throughout the same value. For a battery of n jars, and with the quantities Q and Q′ of electricity, both equations pass into

$$V-V'=-\frac{k}{ns}\frac{Q-Q'}{2} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$Q+Q'=-n(\alpha V+\beta V'). \quad . \quad . \quad . \quad . \quad (33)$$

By means of these equations, in connexion with equation (29), if any two of the quantities (28) be given, all the others may be found.

The experiments conducted by Dove and Riess consist in both cases of two different series. In the first, the number of jars in all the connected batteries was the same, but the number of batteries was varied; in the second, on the contrary, the number of the batteries made use of remained constant, namely, always *two*, but in each of these batteries the number of jars was varied. Both series of experiments present many difficulties to a comparison with the theory. In the second series, however, these are less considerable than in the first, and we will therefore commence with the consideration of the second.

The experiments were arranged as follows:—the batteries were both insulated, and the inner coating of the first was connected with the conductor of the electric machine, the outer of the second with a Lane's unit jar. The number of sparks from the unit jar gave the quantity of electricity communicated to the second outer coating, and we can set the potential function upon this coating after the passage of each spark equal to zero, the potential function of the quantity remaining in the unit jar being neglected.

There are therefore, as above required, two of the quantities (18) known, and to deduce the others from these we can proceed from the second outer coating to the second inner one, to the first outer, and finally to the first inner coating. If the quantity of electricity as measured by the unit jar be $-Q$, and the num-

ber of jars be denoted by n_1 and n_2, all members which, in regard to k, are of a higher order than the first being neglected, we obtain the following series of expressions:—

$$\left.\begin{aligned} Q'_2 &= -Q & V'_2 &= 0 \\ Q_2 &= \left(1+\alpha\frac{k}{s}\right)Q & V_2 &= -\frac{k}{n_2 s}Q \\ Q'_1 &= -\left(1+\alpha\frac{k}{s}\right)Q & V'_1 &= -\frac{k}{n_2 s}Q \\ Q_1 &= \left\{1+\left[2\alpha+(\alpha+\beta)\frac{n_1}{n_2}\right]\frac{k}{s}\right\}Q & V_1 &= -\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{k}{s}Q. \end{aligned}\right\} \quad (34)$$

The potential of the whole compound battery is

$$W=\frac{1}{2}(Q_1V_1+Q'_1V'_1+Q_2V_2+Q'_2V'_2); \quad . \quad . \quad (35)$$

and this gives

$$W=-\left\{1+\left[2\alpha+(\alpha+\beta)\frac{n_1}{n_2}\right]\frac{k}{s}\right\}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{k}{2s}Q^2; \quad . \quad (36)$$

or, neglecting the member of the second order in respect to k,

$$W=-\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{k}{2s}Q^2. \quad . \quad . \quad . \quad . \quad . \quad (37)$$

As after the discharge the potential is equal to zero, $-W$ is the increase of the potential due to the discharge; and if, as before, we assume that, other things being the same, the heating at any single place in the connecting wire is proportional to the total action, we can write

$$C=A\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{Q^2}{2s}, \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where C is the heat generated, and A is a constant.

When we compare this formula with the results of the observations, we find in the first place the proportionality between the generated heat and the square of the quantity of electricity corroborated. In regard, however, to the dependence of the heat on the number of jars n and n_2, Dove gives for this a different formula. Let the entire superficial content of the coatings

of both batteries, that is $n_1 s$ and $n_2 s$, be denoted by S_1 and S_2, (38) then passes into—

$$C = A \cdot \left(\frac{1}{S_1} + \frac{1}{S_2}\right)\frac{Q^2}{2}; \quad \ldots\ldots \quad (39)$$

and instead of this, Dove gives the formula

$$C = A \cdot \frac{Q^2}{\sqrt{S_1 \cdot S_2}} *. \quad \ldots\ldots \quad (40)$$

The results of his experiments agree well with this formula, but the later results of Riess, on the contrary, agree better with mine, as is shown by the table below.

By both observers n_2 was first preserved constant, and n_1 so varied that

$$n_1 = n_2,\ = 2n_2,\ = 3n_2 \text{ and } = 4n_2$$

successively; in the next place n_1 was preserved constant, and n_2 so varied that

$$n_2 = n_1,\ = 2n_1,\ = 3n_1 \text{ and } = 4n_1$$

successively. In order better to compare the results, I have taken as unit in both cases the heat observed in the first experiment, where $n_1 = n_2$, and reduced the other quantities of heat generated to this. In the case of Riess, who gives two values for each experiment, I have taken the mean of both.

I. n_1 variable, n_2 constant.

n_1.	Heat generated.			
	Calculated		Observed	
	According to Dove's formula.	According to formula 39.	By Dove.	By Riess.
n_2	1	1	1	1
$2n_2$	0·71	0·75	0·72	0·76
$3n_2$	0·58	0·67	0·59	0·69
$4n_2$	0·50	0·63	0·51	0·66

* Pogg. *Ann.* vol. lxxii. p. 419.

II. n_2 variable, n_1 constant.

n_2.	Heat generated.			
	Calculated		Observed	
	According to Dove's formula.	According to formula 39.	By Dove.	By Riess.
n_1	1	1	1	1
$2n_1$	0·71	0·75	0·71	0·78
$3n_1$	0·58	0·67	0·60	0·72
$4n_1$	0·50	0·63	0·50	0·68

It will be seen that in the first table a satisfactory agreement exists between the numbers of the third and fifth columns. In the second table the differences are certainly more considerable; when, however, we reflect how difficult it must be practically to fulfil the theoretic conditions required by the formula, particularly that of complete insulation, and that even in this case the formula is to be regarded as only approximately true, the differences are not calculated to create a doubt as regards the theory; and it must be also remarked here that all the numbers of the fifth column are greater than what my formula gives them, while to approximate to that of Dove they must be smaller.

Still less suited to a comparison with the theory is the second of the above-mentioned series of experiments. In these, three or four batteries, or single jars, were used as elements, which while charging were united to form a cascade battery. The discharge was then effected with the first alone, or with the first and second, and so forth; the amount of heat generated in the connecting wire being each time observed.

Certain difficulties present themselves here, from which it must be remarked the other experiments were not totally free, but could not here exert the same amount of influence as in the cases now under consideration. Among these the fact deserves particular attention, that every new battery added necessitated an increase in the length of the connecting arc. In the series of experiments just considered, when the number of jars of the battery was increased, the new jars were set close beside the old

ones; and although the system of bodies exposed to the action of the electricity was thus increased, still the increase could scarcely be regarded as an actual lengthening of the connecting arc, and was therefore, as in former cases, neglected. In the series of experiments now before us, however, each additional new battery was placed *behind* the others, so that its intervening wire must be regarded as an actual lengthening of the connecting arc.

From this it follows that the assumption formerly made, when the connecting arc remained the same, that the heat generated at a single place is proportional to the totality of action, cannot be extended to the case where, by increasing the elements of a cascade battery, the total action is increased; but that, on the contrary, the heat generated in this case ought to be somewhat under the quantity which would flow from the above assumption. Now, as according to the above equations, the total action, or the increase of the potential, if the members of a higher order in respect to k be neglected, is proportional to the number of elements discharged, an electrical thermometer introduced into the circuit, where the number of elements is gradually increased, ought to show temperatures which are something less than the series of whole numbers expressive of the gradual increase of the elements.

In Dove's experiments* this difference is indeed not observable, inasmuch as with four batteries he found the heat generated to be expressed by the whole numbers 1, 2, 3, 4. The experiments of Riess†, on the contrary, show considerable divergences, for with four jars he found the numbers to be as 1 to 3 instead of 1 to 4; and with three jars he found, instead of 1 to 3, the rates of 1 to 2·5. A definite law cannot of course be expected here, inasmuch as the results must greatly depend on the nature of the battery made use of, and of its connexions.

The correctness of the assumption made above, that every combination of every two elements is to be regarded as an actual lengthening of the circuit, is particularly evident from the fact, that, according to the observations of both physicists, the heating of the wire connecting the elements takes place in the same manner as in the principal connecting arc; and that by intro-

* Pogg. *Ann.* vol. lxxii. p. 408. † Pogg. *Ann.* vol. lxxx. p. 351.

ducing a bad conductor at one of these connexions, the heating of any portion of the principal arc is diminished in almost the same degree as if the said bad conductor had been introduced into the principal arc itself.

Dove seems to have otherwise explained the circumstance, that with four batteries, four connecting wires being present, the heat generated in each is greater than in the single connecting wire of a single battery. After having remarked that with four batteries he obtained a length of spark sixteen times as great as with one, but that when two interruptions existed at the same time in two of the connecting wires the sum of the lengths of the sparks obtained was less than the length obtained with one interruption in a single wire, he proceeds*, "While the heat generated in a combined battery in each of the four connecting wires is fourfold, the sixteen-fold length of stroke refers to the total combination as to an entire connecting wire." In this case he seems to lay down the principle, that *in the four wires together the heat generated will be sixteen-fold.* Still plainer, although without definite numbers, this opinion is expressed at another place, where the point under consideration is that the length of wire melted by a cascade battery increases with the increase of the elements discharged. He says there†, "Now as the same increase takes place in the intervening wires, when the number of the combined batteries is gradually increased, with four batteries we can melt four lengths of wire at the same time, every one of which is greater than that which can be melted by a single battery."

Quite apart from the question whether Dove, in expressing himself thus, really entertained the above opinion, as this may readily be the conclusion arrived at by the reader of his paper, and as, if the opinion were well-founded, it would contradict our theory, I think it incumbent on me to say a few words in connexion with the subject.

The total heat generated cannot be greater than the increase of the potential due to the discharge, and this is, as already mentioned, about *four* times instead of *sixteen* times as great with four batteries as with one. Now if an electric air-thermometer, the wire of which has a great reduced length, be succes-

* Pogg. *Ann.* vol. lxxii. p. 415. † Pogg. *Ann.* vol. lxxii. p. 412.

sively introduced in the four connecting arcs under favourable circumstances, it will be observed that the heat generated at each time approaches the fourfold of that generated by a single battery. This, however, will no longer take place if in the four connecting arcs four thermometers are simultaneously introduced, inasmuch as in this case the effect which in the former experiment was concentrated upon a single thermometer is now spread over all four. The same is derived immediately from the observation already mentioned, that a bad conductor introduced at any place of connexion lessens the heating at the other places also. Hence when we have four thermometers in the circuit at the same time, the heat observed in each must be much less than that observed if only a single one be used, inasmuch as the three others by their resistance weaken the action of the current; the same will naturally be the case with four wires raised to the melting-point.

Let us sum up the result of all the cases which have hitherto been investigated, the greater number, it must be confessed, are too complicated to permit of a strict comparison with the theory; so far, however, as a comparison is possible, it has always proved favourable to our principal proposition, and I am aware of no established fact which speaks against the proposition. I believe therefore it may be regarded as corroborated by experience.

[J. T.]

Article II.

On the Processes which have taken place during the formation of the Volcanic Rocks of Iceland. By R. Bunsen.

[From Poggendorff's *Annalen*, 1851. No. 6.]

THE following memoir contains a short review of the principal results which have been obtained during the course of a more comprehensive investigation of the volcanic phænomena of Iceland. A complete treatment of the subject must not be expected in this place, more especially as results, which, like those here described, can only be arrived at by means of an extended experimental inquiry, require a kind of demonstration which to be intelligible cannot be given in few words. Neither do the limits of the present memoir admit of my entering into a detailed consideration of the inferences which might be deduced from these investigations with regard to the general question of the formation of plutonic rocks.

I have been induced to anticipate the publication of my entire researches by that of this abstract, from the wish to draw the attention of chemists and geologists to a series of very simple relations which it will undoubtedly be possible to trace throughout the whole class of eruptive rocks, and which may perhaps furnish a not unacceptable groundwork for further research in this direction.

I. *Genetic Relations existing between non-metamorphic Rocks.*

The rocks formed from volcanic masses ejected in a state of igneous fusion consist of a mixture of silicates, which during their eruption formed a homogeneous liquid. It was not until after solidification that these masses assumed the form of plutonic rocks, in which the perfect homogeneity of the original mass is rarely met with. It is well known that the act of solidification is accompanied by a separation of distinctly different minerals, which are either imbedded in a uniform matrix, the

constituents of which have not yet passed into the state of definite compounds, or are grouped together in crystalline aggregates without any matrix. Such purely crystalline aggregates are not abundant among the products belonging to the most recent eruptive periods. Among these are found far more frequently such rocks as consist of an amorphous and a crystalline mass, the former corresponding with the solvent, and the latter with the dissolved parts of the solidifying eruptive mass. The petrographic character which manifests itself in this condition of the existing rocks, depends immediately and essentially upon the chemical constitution of the original homogeneous mass of liquid silicates. The mere investigation of the minerals which have been formed in the rock cannot lead to any knowledge of the constitution of the mass itself, which has wholly or partially separated into crystalline minerals. If, therefore, we are desirous not to restrict ourselves to a mere natural history description of the different varieties of rocks, but to go back to the actual conditions of their origin, it is evident that the average composition of a rock, without reference to the more recently formed minerals which it may contain, can alone constitute the starting-point of such an investigation. The extensive volcanic system situated near the northern polar circle, and constituting the island of Iceland, affords the best opportunity of obtaining these data, because it is not associated with any calcareous or siliceous sedimentary beds which could have exercised any disturbing influence upon the normal composition of the eruptive volcanic rocks, and because these rocks, which in turn present themselves as the representatives of universally-occurring formations, are for the most part amorphous, or at least almost always bear the stamp of such a uniform and intimate mixture as to render possible the determination of their average composition.

Although the great diversity displayed in the mineralogical and chemical composition of the non-metamorphic rocks of Iceland would appear at first sight to exclude any idea of a recognizable uniformity in their origin, still there is one character which is found, upon more close observation, to connect together in the most intimate manner all these products, from the most recent lavas to the most ancient eruptive masses, however widely different their mineralogical constitution may be.

The rocks of Iceland, and probably those of most other volcanic systems, may be classified into two principal groups, which, notwithstanding the gradual transition by which they are united, may still be easily recognized in their extreme members, and distinguished as *normal-trachytic* on the one hand, and as *normal-pyroxenic* on the other. The individual members, frequently presenting great petrographic differences, maintain in each of these groups a peculiar, but perfectly constant composition*.

The first distinguishing characteristic of the normal trachytic rocks is, that they represent almost exactly a mixture of bisilicates of alumina and alkali, in which lime, magnesia, and protoxide of iron are either wholly wanting or present only in insignificant quantities. The following analyses show the composition of the principal members of this group. The variety of rock 1 constitutes the chief mass of the trachytic cone of Baula†. Its colour is white, frequently with a yellow or bluish tinge; it is rough, moderately granular, and has a great number of small irregular cavities.

The variety 2, from Strútrháls, near Kalmanstúnga, four Danish miles south-east of Baula, forms a perpendicular, massive, irregularly-fissured precipice, whose base is washed by the Nordhlingafljót. This rock is traversed by a trap-dyke. It is of a brilliant white, has a distinctly crystalline granular structure, and in other respects resembles that previously mentioned. The variety 3, from Laugarfjall, near the Great Geyser, is in large columns; it forms a compact, bluish-gray mass, large fragments of which present an indistinct angular fracture, and small fragments a splintery fracture with isolated felspathic concretions, but imperfectly distinguishable from the matrix, and an extremely small number of minute crystals of hornblende. The variety 4 is from a massive trachyte, occurring in the form of crags in the Arnarhnipa (Eagle's Rock), on the banks of the Laxá, between Hruni and Stórinúpr. It is not columnar, and resembles the rock of Strútrháls, but is yellower, has a finer

* I understand by the former the trachytic rocks richest in silica, and by the latter the basaltic and doloritic rocks with the smallest quantity of silica.

† Forchhammer considers this rock to consist of a hitherto unknown variety of felspar, which he calls Baulite.

granular structure, and a somewhat earthy compact appearance. 5 is a clinkstone from Falkaklettur, near Kalmanstúnga, in separate slabs, consisting of a compact, grayish-brown, dull mass, speckled or striped with peroxide of iron, and in which there are here and there traces of felspathic concretions. 6 is a blackish-gray, homogeneous, stony lava, with a great number of small irregular cavities. It is apparently of very recent formation, and occurs at Hrafntinnuhrygyr, near Krafla, at the far north-east of Iceland. 7 is a black homogeneous obsidian, possessing the characters peculiar to this rock, and alternating in beds with the above lava:—

	1.	2.	3.	4.	5.	6.	7.
Silica . .	75·91	77·92	75·29	78·95	76·42	76·38	75·77
Alumina . .	11·49	12·01	12·94	10·22	9·57	11·53	10·29
Protoxide of iron	2·13	1·32	2·60	2·91	5·10	3·59	3·85
Lime . .	1·56	0·76	1·01	1·84	1·53	1·76	1·82
Magnesia .	0·76	0·13	0·03	0·14	0·20	0·40	0·25
Potash . .	5·64	3·27	5·42	1·76	1·94	1·88	2·46
Soda . . .	2·51	4·59	2·71	4·18	5·24	4·46	5·56
	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Universally-occurring non-metamorphic rocks, containing more silicic acid than these, are not met with in Iceland.

The normal-pyroxenic rocks, which as basic silicates of alumina and protoxide of iron, in combination with lime, magnesia, potash and soda, form the most extreme members towards the opposite end of the series, present a similar correspondence in their average composition. Here likewise again we find that rocks belonging to the most different periods possess the same mean composition, as is shown by the following analyses. 8 gives the composition of the older coarse, granular, stratified trap rock of Esjagebirge, which, when it alternates with tuff and trap conglomerate, is generally found in immense columns situated perpendicularly to the stratification. 9 is a similar, but somewhat more compact rock, constituting the north-eastern coast cliffs of the island Vidhey. 10 is a compact, lighter-coloured, fine, granular, massive basaltic rock, which traverses the palogonite tuff of the Hagafjall on the right bank of the Thjórsá, near Stórinupr.

11 is a massive non-columnar basaltic rock from Skardhsfjall, near Hecla, which, like the hills of tuff in the same locality, is thrown up parallel to the cone of Hecla, and which presents in some cavities a scoriaceous surface, showing signs of previous liquidity. 12 is a hand specimen of an extremely homogeneous ancient lava from Hecla. 13 is a light gray lava, filled with cavities, forming the precipice of Almånnagjá, and whose principal mass consists of an intimate and uniform mixture of about equal parts of a felspathic silicate free from iron, and a dark-coloured ferruginous silicate in granular crystalline particles. Small augitic concretions may be distinguished in the latter mineral, possessing the most deceptive resemblance to olivine:—

	8.	9.	10.	11.	12.	13.
Silica	50·05	47·48	49·17	47·69	49·37	47·07
Alumina . . .	18·78	13·75	14·89	11·50	16·81	12·96
Protoxide of iron .	11·69	17·47	15·20	19·43	11·85	16·65
Lime	11·66	11·34	11·67	12·25	13·01	11·27
Magnesia . . .	5·20	6·47	6·82	5·83	7·52	9·50
Potash	0·38	0·60	1·67	0·48	0·20	0·58
Soda	2·24	2·89	0·58	2·82	1·24	1·97
	100·00	100·00	100·00	100·00	100·00	100·00

As the proportion of the oxygen of the silicic acid to that of the bases is here, with slight variations, as 3 : 1·998, all these rocks may be regarded as a constant mixture of bibasic silicates, when we consider only their entire mass, without reference to the fact of their constituents being grouped into minerals of definite composition.

The quantity of silica almost always bears a constant proportion to the lime and magnesia, while the relation between the quantities of alumina and protoxide of iron is subject to considerable variations. The reason of this is not difficult to discover, for there are abundance of well-known phænomena which can throw light upon it. The silver dissolved in lead separates in that portion of the metal which solidifies first in a different proportion from that in which it separates afterwards; and a remelted bar of auriferous silver never gives the same per-centage of gold throughout its entire mass. The same inequality occurs in the solidifying silicates. The more easily solidifying consti-

tuent minerals separate first; those which solidify less easily do not separate until afterwards, in such a way as to render it unnecessary to suppose this result to be aided by the power of crystallization. It cannot therefore appear remarkable that an originally homogeneous melted mass should yield a rocky mass, richer in protoxide of iron and poorer in alumina at the commencement of its solidification, and towards the end richer in alumina and poorer in iron compounds. This may indeed be easily ascertained to be the case by experiment.

It is, in fact, by no means an uncommon circumstance in Iceland, that the upper and lower ends of one and the same undecomposed trachytic column, situated in the midst of an extensive mass of rock, present sensible differences in composition. The gray-coloured mass from the lower end of such a column in the neighbourhood of Kalmanstúnga had the composition 14, and that from the opposite end of a white colour the composition 15:—

	14.	15.
Silica	72·74	71·49
Alumina	10·53	12·71
Protoxide of iron	6·57	4·27
Lime	2·47	1·24
Magnesia	1·51	0·08
Potash	3·00	3·35
Soda	2·33	3·73
Water	2·30	2·79
	101·45	99·66

In like manner, the same mass of rock shows, even at distances of a few feet, sensible differences in the relative quantities of potash and soda.

If these variations in composition are really the result of an unequal separation of minerals, determined by conditions of temperature and pressure, it is easy to understand that a decrease in the quantity of one constituent must necessarily be attended by an increase of the other, and that consequently the sum of both can vary but little. All the above analyses prove this to be really the case. The resemblance is restored when the quantities of the especially-variable constituents are added together:—

	16.	17.	18.	19.	20.	21.
Silica	50·05	47·48	49·17	47·69	49·37	47·07
Alumina and protoxide of iron	30·47	31·22	30·09	30·93	28·66	29·61
Lime	11·66	11·34	11·67	12·25	13·01	11·27
Magnesia	5·20	6·47	6·82	5·83	7·52	9·50
Potash	0·38	0·60	1·67	0·48	0·20	0·58
Soda	2·24	2·89	0·58	2·82	1·24	1·97
	100·00	100·00	100·00	100·00	100·00	100·00

	22.	23.	24.	25.	26.	27.	28.
Silica	75·91	77·92	75·29	78·95	76·42	76·38	75·77
Alum. & protoxide of iron	13·62	13·33	15·54	13·13	14·67	15·12	14·14
Lime	1·56	0·76	1·01	1·84	1·53	1·76	1·82
Magnesia	0·76	0·13	0·03	0·14	0·20	0·40	0·25
Potash	5·64	3·27	5·42	1·76	1·94	1·88	2·46
Soda	2·51	4·59	2·71	4·18	5·24	4·46	5·56
	100·00	100·00	100·00	100·00	100·00	100·00	100·00

By taking the mean of these analyses, which form only a small part of those which have been made, compositions are obtained from which the results of the individual analyses differ but little, and which may be regarded as the mean values of the composition of the two liquid masses, the normal trachytic and the normal pyroxenic, the two great sources from which have originated, on the one hand, those Icelandic rocks containing the largest proportion of acid, and on the other hand, those containing the largest proportion of bases:—

	29. Normal trachytic composition.	30. Normal pyroxenic composition.
Silica	76·67	48·47
Alumina and protoxide of iron	14·23	30·16
Lime	1·44	11·87
Magnesia	0·28	6·89
Potash	3·20	0·65
Soda	4·18	1·96
	100·00	100·00

From these numbers we can at once calculate the mean re-

lation between the oxygen of the acid and that of the bases, which for the trachytic rock is as 3 : 0·596, and for the pyroxenic as 3 : 1·998. All the other unaltered rocks of Iceland which do not immediately belong to these two groups have such a composition, that the oxygen in their bases varies between 0·579 and 1·948 for 3 oxygen in the acid. There is strictly only one relation met with, and that is such as necessarily belongs to rocks consisting of intimate mixtures of these two extreme members. Consequently the question directly suggests itself as to whether the non-metamorphic rocks of Iceland have really resulted from such an admixture of fused masses, or in other words, whether there were only two separate foci, which have maintained the plutonic activity there observed from its first commencement up to the present day. This problem admits of a direct solution.

If we characterize by S the per-centage of silica in a mixed rock, by s the per-centage of silica in the normal trachytic, and by σ the per-centage of silica in the normal pyroxenic rocks, then

$$\frac{s-S}{S-\sigma}=a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which equation a represents the quantity of normal pyroxenic rock mass which must be mixed with one part of normal trachytic rock mass in order to give the composition of the mixed rock in question. This value a may indeed be calculated from any one of the constituents of the rock: silica is however selected for this purpose, as best adapted, because it can be most accurately determined and is the least variable constituent in the rocks. All the other constituents of the mixed rocks are then determined by means of a. For if the weight of the separate constituents in one part of the normal pyroxenic rock mass is taken as $p_0\, p_1 \ldots p_n$, and in the same manner the weight of the separate constituents in the unity of weight of the normal trachytic rock mass as $t_0\, t_1 \ldots t_n$, the weight of all the other constituents of the mixed rock may be ascertained by means of the equation

$$1=\frac{(ap_0+t_0)}{(a+1)}+\frac{(ap_1+t_1)}{(a+1)}+\ldots\frac{(ap_n+t_n)}{(a+1)}. \quad . \quad . \quad (2)$$

If, therefore, it is really the case that there exist in Iceland

only two great plutonic foci which have supported the volcanic action since its first commencement to the present time, and if the contents of these foci have respectively the compositions belonging to the above-considered extreme members, then the composition of all the non-metamorphic rocks of Iceland may be determined by means of one constituent, and most advantageously by the per-centage of silica. "

Pyroxenic rock to 1 part of trachyte, ...	0·000	0·0241	0·106	0·199	0·310	0·444	0·609	0·818	1·084	1·446	1·959	2·745	4·099	6·989	17·43	∞
Silica.....	76·67	76·00	74·00	72·00	70·00	68·00	66·00	64·00	62·00	60·00	58·00	56·00	54·00	52·00	50·00	48·47
Alumina and protoxide of iron. ..	14·23	14·61	15·73	16·86	18·00	19·13	20·26	21·39	22·52	23·65	24·78	25·92	27·04	28·17	29·29	30·16
Lime.....	1·44	1·68	2·43	3·17	3·90	4·65	5·38	6·13	6·87	7·61	8·35	9·08	9·83	10·57	11·31	11·87
Magnesia.....	0·28	0·43	0·91	1·38	1·84	2·31	2·78	3·25	3·72	4·19	4·65	5·12	5·59	6·06	6·53	6·89
Potash	3·20	3·14	2·96	2·78	2·60	2·42	2·23	2·05	1·87	1·69	1·51	1·33	1·15	0·96	0·79	0·65
Soda	4·18	4·14	3·97	3·81	3·66	3·49	3·35	3·18	3·02	2·86	2·71	2·55	2·39	2·24	2·08	1·96
	100·0	100·00	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0

If these numbers should be found to correspond with the composition of all the original varieties of Icelandic rocks which have not yet undergone a chemical metamorphism, it may be considered as proved that they are all either merely mixtures of the above-mentioned acid and basic extreme members, or those members themselves; and that the great mineralogical and petrographic differences which these rocks present, are only consequences of the varying proportions in which the mixture took place, and of the prevailing physical conditions under which the rocks reached their present situation and assumed their present form. Among the large number of analyses of Icelandic rocks which have been carried out in my laboratory, there is not one in which the results differ from the calculated composition obtained in the above-mentioned manner, to a greater extent than might reasonably be expected in such a calculation, based merely upon the mean results of analyses. The following experimental and calculated results were obtained for a granular whitish gray trachyte, containing hornblende and orthoclase crystals from Oexnadalr in the Norden Islands :—

	31. Found.	Calculated.
Silica	73·57	73·57
Alumina and protoxide of iron	17·19	15·89
Lime	1·41	2·58
Magnesia	0·81	1·01
Potash	2·19	2·92
Soda	4·83	3·93
	100·00	10·00

The calculation requires that there should be 0·1325 pyroxenic mass to 1·0 of trachyte. Another compact gray pyroxenic rock from Reyjadalr Foss, a small waterfall which is formed by the Nordbrá near Hvammr, shows a still greater correspondence. This rock was either massive or separated by columnar fissures, and contained small zeolitic druses filled with green earth.

	32. Found.	Calculated.
Silica	51·75	51·75
Alumina and protoxide of iron	28·39	28·31
Lime	10·49	10·65
Magnesia	5·90	6·13
Potash	1·01	0·96
Soda	2·46	2·20
	100·00	100·00

The calculation requires 7·597 pyroxenic mass to 1·0 trachyte. A compact blackish gray rock, with an indistinct angular fracture in small pieces, and a conchoidal fracture in large pieces, presenting a very uniform crystalline granular appearance, and occurring in the neighbourhood of Kalmanstúnga, which affords particularly good opportunities for the study of the mixed rocks, consisted of—

	33. Found.	Calculated.
Silica	53·08	53·08
Alumina and protoxide of iron	28·70	27·57
Lime	9·92	10·16
Magnesia	5·32	5·81
Potash	0·61	1·06
Soda	2·37	2·32
	100·00	100·00

The calculated relation of trachyte and pyroxenic masses in this rock is 1·0 to 5·117.

Experiment and calculation gave for a trap (clinkstone?) from Eskifjördhr, analysed by Damour,

	34. Found.	Calculated.
Silica	66·12	66·12
Alumina and protoxide of iron	24·05	20·20
Lime	3·24	5·34
Magnesia	0·46	2·76
Potash	1·29	2·24
Soda	4·84	3·34
	100·00	100·00

This rock represents a mixture of 1·0 trachyte and 0·5991 trap mass.

A clinkstone from Klettaberg near Kalmanstúnga, in separate slabs, consisted of—

	35. Found.	Calculated.
Silica	73·37	73·37
Alumina and protoxide of iron	17·26	16·09
Lime	2·49	2·66
Magnesia	1·52	1·05
Potash	3·01	2·90
Soda	2·35	3·93
	100·00	100·00

The relation of the normal trachytic and pyroxenic masses is in this instance 1 : 0·1325.

A great number of other analyses of this kind, which sometimes showed a greater difference and sometimes a still greater correspondence between the experimental and calculated results, lead to the same conclusion.

But even if these examples should leave any doubts of the existence of an intimate connexion between the various unaltered rocks occurring in Iceland, such doubts must be entirely removed by the evidence presented in favour of such connexion by a great number of trachytic and basaltic dykes. It is here very frequently possible to detect the intermixture of the extreme acid and basic members, not only by means of chemical analysis, but by direct observation. In one of the south-eastern valleys of

the Esja mountains, opposite Mossfell, the conglomeratic pyroxenic rock is traversed by a trachyte dyke, which consists in its interior of a pure white mass, becoming gradually darker and more ferruginous towards the surrounding rock. This is but one among many examples which might be brought forward. The analysis of the mass from this dyke, without taking into account the water which it contained, in consequence of especial circumstances, gave for the part in the centre of the dyke (36), for that situated nearer to the saalband (37), and for that part of the rock immediately surrounding the dyke (38) :—

	36.	37.	38.
Silica	78·95	66·18	50·25
Alumina	7·71	9·74	12·55
Protoxide of iron .	4·32	12·05	16·13
Lime	1·55	4·49	11·10
Magnesia	0·42	3·04	7·59
Potash	2·48	0·94	0·34
Soda	4·57	3·56	2·04
	100·00	100·00	100·00

A comparison of these results with the above average compositions, shows that the interior mass of the dyke has almost exactly the composition of the pure normal trachytic rocks, while, on the contrary, the surrounding mass has the composition of the normal pyroxenic rocks; and the part situated near to the surrounding rock is an admixture of both in the proportion of 0·5923 pyroxenic mass to 1·0 trachyte, for which the following calculated composition closely corresponds with that found by actual analysis :—

	39. Found.	Calculated.
Silica	66·18	66·18
Alumina and protoxide of iron .	21·79	20·15
Lime	4·49	5·32
Magnesia	3·04	2·74
Potash	0·94	2·25
Soda	3·56	3·36
	100·00	100·00

Upon closer examination, indeed, the most distinct indications are found of a lateral fusion of the neighbouring rock and the

substance of the dyke, gradually decreasing from the surface of contact towards the central part of the dyke, and to which they do not extend. These observations and experiments prove that it is not necessary to regard the chemically intermediate members between the extreme acid and basic members of the Icelandic group of rocks as resulting from simultaneous eruptions from the two great plutonic foci, but that even the already formed rocks may, during eruptions or injections, have furnished one part of the material for the formation of mixed rocks.

A confirmation of this law is likewise met with in applying it to the rock formations upon the lava streams of the Icelandic volcanoes. The final members, and the intermediate rocks resulting from their mixture, are not absent even among these most modern formations. The enormous lava streams which have rent the palagonitic ranges of hills of Krafla and Leirhnúkr, afford the most striking evidence of this, and Hecla not less so. However, the law is in this instance not so distinctly recognizable, because the volcanic mountains penetrated by these lava streams consist of metamorphic products of the palagonitic tuff, which, although it corresponds in composition with the normal trap mass, is subject to greater variation in the proportions of its constituents. The discrepancies accounted for by this circumstance were rendered still greater by the impossibility of obtaining for examination any other than hand specimens, which contained separate and not very uniformly mixed felspathic masses. But notwithstanding these disadvantages, the law is still found to obtain.

The blackish gray stony lava of Hrafntinnuhryggr (40) on the south-eastern foot of the Krafla, and the obsidian (41) alternating within beds, have exactly the composition as the normal trachytic mass.

	40.	41.
Silica	75·12	75·28
Alumina . . .	11·34	10·22
Protoxide of iron	3·92	4·24
Lime	1·73	1·81
Magnesia . . .	0·39	0·25
Potash . . .	1·85	2·44
Soda	4·39	5·53
Water	0·41	0·23
	100·00	100·00

The streams of obsidian occurring on the north-eastern declivity of Hecla approach very closely in composition to the above. They represent a mixture of 1·0 trachyte and 0·2325 pyroxenic substance.

	42. Found.	Calculated.
Silica	71·35	71·35
Alumina and protoxide of iron	17·33	17·32
Lime	1·24	3·41
Magnesia	0·19	1·53
Potash	4·23	2·73
Soda	5·66	3·75
	100·00	100·00

The extreme basic member is represented in the substance of the great lava streams which has flowed from this volcano in a W.N.W. direction to the banks of the Thjorsá.

	43. Found.	Calculated.
Silica	49·60	48·47
Alumina and protoxide of iron	28·81	30·16
Lime	13·07	11·87
Magnesia	7·56	6·89
Potash	0·20	0·65
Soda	1·24	1·96
	100·00	100·00

The lavas from the western foot of Hecla, examined in my laboratory by Dr. Genth, are represented as resulting from the mixture of the two extreme members by the following analyses:

	Lava at Háls. 44.		Efrahvols lava. 45.		Hecla ash of 1845. 46.	
	Found.	Calc.	Found.	Calc.	Found.	Calc.
Silica	55·95	55·95	59·45	59·45	56·76	56·76
Alumina and protoxide of iron	29·83	25·93	27·68	23·96	27·47	25·48
Lime	6·54	9·10	5·50	7·80	6·75	8·79
Magnesia	4·21	5·14	2·38	4·32	4·04	4·95
Potash	0·96	1·33	1·48	1·65	2·63	1·40
Soda	2·51	2·55	3·56	2·82	2·35	2·62
	100·00	100·00	100·00	100·00	100·00	100·00
	1 trachyte+2·77 pyroxenic mass.		1 trachyte+1·568 pyroxenic mass.		1 trachyte+2·402 pyroxenic mass.	

The order of the analyses corresponds with the succession of the streams examined, from the more ancient to the younger. They show that the flow from the two volcanic foci which maintain the activity of the volcano, is as irregular as this activity itself. On the formation of the lava streams of Thjorsá the pyroxenic focus alone acted, and in the formation of that at Háls the action of the pyroxenic focus preponderated over that of the trachytic. The contrary relation is observed in the Efrahvols lava; and during the most recent eruption in the year 1845, the action of the pyroxenic focus again preponderated over that of the trachytic.

The great resemblance which is everywhere found to exist between the rocky masses poured out from volcanoes, justifies the conjecture that these processes of rock formation are not limited to Iceland alone. A number of analyses of analogous kinds of rocks from other localities, executed in my laboratory, prove that this is really the case. The volcanic system of the high table-land of Armenia is in this respect especially remarkable. The mean results of analyses of rocks from that locality, for which we are indebted to the excellent researches of Abich, likewise give for the extreme acid member occurring there precisely the same composition as the Icelandic.

	Normal trachytic substance of Iceland.	The same from the high table-lands of Armenia*.			
		47.	48.	49.	50.
Silica	76·67	77·27	77·60	77·42	76·66
Alumina and protoxide of iron . .	14·23	14·14	13·72	14·83	15·17
Lime	1·44	1·31	1·40	2·73	1·25
Magnesia . . .	0·28	?	?	?	?
Potash	3·20	2·44	2·30	4·32	2·94
Soda	4·18	4·15	4·21		3·52
Loss by ignition .	0·00	0·51	0·61	0·57	1·12

This resemblance is recognizable, however, not only in the extreme acid member of the series, but is still more distinct

* *Ueber die geolog. Natur des Armen. Hochlandes*, von Dr. H. Abich. Dorpat, 1843.

47. Brown obsidian from the Lesser Ararat. 48. Obsidian porphyry from the Greater Ararat. 49. Obsidian from Kiotangdag. 50. Dioritic porphyry from Besobdal.

in the mixed rocks. A compact black basalt from the source of the Euphrates, which was analysed by M. Jäckel in my laboratory, represents a mixture of 1 trachytic and 0·7332 pyroxenic substance.

	51. Found.	Calculated.
Silica	64·76	64·74
Alumina and protoxide of iron	22·31	20·99
Lime	5·13	5·84
Magnesia	1·91	3·07
Potash	2·51	2·12
Soda	3·38	3·24
	100·00	100·00

All the other analyses which Abich gives in his paper likewise correspond in a surprising manner with the law:—

Rock at the summit of the Greater Ararat.

	52. Found.	Calculated.
Silica	70·14	70·14
Alumina and protoxide of iron	18·17	17·92
Lime	4·72	3·85
Magnesia	1·00	1·81
Potash	1·47	2·61
Soda	4·50	3·67
	100·00	100·00

1 trachyte and 0·3013 pyroxenic mass.

Dark Gray Rock from the summit of the Kasbeck.

	53. Found.	Calculated.
Silica	69·77	69·77
Alumina and protoxide of iron	18·27	18·13
Lime	5·13	3·99
Magnesia	1·65	1·89
Potash and Soda	5·18	6·22
	100·00	100·00

1 trachyte and 0·3239 pyroxenic mass.

Reddish-brown Rock from the summit of the Kasbeck.

	54. Found.	Calculated.
Silica	70·97	70·97
Alumina and protoxide of iron	18·13	17·45
Lime	4·24	3·55
Magnesia	1·54	1·62
Potash and Soda	5·12	6·41
	100·00	100·00

1 trachyte and 0·2533 pyroxenic mass.

Rock from the summit of the Elbruz.

	55. Found.	Calculated.
Silica	69·65	69·63
Alumina and protoxide of iron	19·85	18·25
Lime	4·40	4·02
Magnesia	2·27	1·92
Potash and Soda	3·83	6·18
	100·00	100·00

1 trachyte and 0·3314 pyroxenic mass.

Rock from Ararat.

	56. Found.	Calculated.
Silica	65·96	65·96
Alumina and protoxide of iron	22·18	20·28
Lime	4·27	5·39
Magnesia	2·13	2·79
Potash	1·34	2·23
Soda	4·12	3·34
	100·00	100·00

1 trachyte and 0·6124 pyroxenic mass.

Compact Porphyritic Rock from Ararat.

	57. Found.	Calculated.
Silica	65·27	65·27
Alumina and protoxide of iron	20·88	20·67
Lime	6·57	5·65
Magnesia	3·47	2·96
Potash and Soda	3·81	5·45
	100·00	100·00

1 trachyte and 0·6786 pyroxenic mass.

A somewhat decomposed porous Rock from Ararat.

	58. Found.	Calculated.
Silica	65·39	65·39
Alumina and protoxide of iron	22·65	20·61
Lime	7·40	5·61
Magnesia	3·00	2·92
Potash and Soda	1·56	5·47
	100·00	100·00

1 trachyte and 0·6666 pyroxenic mass.

Unknown Crystalline Rock from between Keschet and Kobi.

	59. Found.	Calculated.
Silica	61·25	61·25
Alumina and protoxide of iron	25·72	22·95
Lime	6·27	7·14
Magnesia	3·77	3·89
Potash and Soda	2·99	4·77
	100·00	100·00

1 trachyte and 1·207 pyroxenic mass.

Abich mentions only one variety of rock which approximates nearer in its composition to the normal pyroxenic substance. This is an amygdaloid basalt from Ararat, which may be regarded as consisting of 1 trachyte and 3·427 pyroxenic substance, thus:

	60. Found.	Calculated.
Silica	54·84	54·84
Alumina and protoxide of iron	27·96	26·56
Lime	9·28	9·51
Magnesia	3·72	5·39
Potash and Soda*	4·20	3·70
	100·00	100·00

While, therefore, on the one hand, the composition of the Caucasian rocks may be deduced (their per-centage of silica being known) from the composition of the Icelandic rocks, the Cau-

* The alkalies are here estimated from the deficiency.

casian rocks, on the other hand, afford a possibility of ascertaining, by means of calculation from the extreme acid members which concur in conjunction with the accompanying basic substances, the composition of the pyroxenic substance which filled the non-trachytic focus of volcanic rock-formations in Iceland. The various constituents of such a substance may be easily ascertained by means of the following equation derived from (2),

$$\frac{(a-1)\,A_n - t_n}{a} = p_n,$$

in which A_n represents, in per cents, the separate constituents of a mixed rock. This formula gives, for reasons which may readily be understood, results which are more accurate, in proportion as A_n differs more from the numerical value of the normal trachytic composition. This is the case only with the analysis (60). If therefore from this the value of p_n is calculated, we obtain—

	Caucasus. Calculated.	Iceland. Found.
Silica	48·47	48·47
Alumina and protoxide of iron	31·97	30·16
Lime	11·56	11·87
Magnesia	4·72	6·89
Alkalies	3·28	2·61
	100·00	100·00

It cannot therefore be doubted that both the extensive volcanic elevations constituting the high table-land of Armenia and the island Iceland have flowed from sources which were *chemically* identical. The idea that perhaps all the volcanic formations on the earth's surface have originated from the same source, or even, indeed, that it is from this very source that all the ferruginous and non-ferruginous plutonic rocks have originated by fusion together, is rendered the less improbable by the fact that the mineralogical differences existing between those Caucasian and Icelandic rocks which present the same mean composition, are not less marked than those observed among other ferruginous rocks of plutonic origin. It would therefore be very interesting to trace the genetic relations existing among the more ancient

formations, in a manner similar to that which I have adopted with regard to the volcanic rocks. It would be necessary to select for this purpose only the more extensive and largely developed rock systems, in order to avoid those disturbing influences which may have resulted from the contact of chemically metamorphosed exogenous rocks with those of plutonic origin. These disturbing influences manifest themselves even where volcanic rocks have penetrated in isolated elevations, through calcareous or siliceous sedimentary beds.

It still remains, after the above consideration of this subject, to point out the connexion which exists between the mode of formation of Icelandic and Caucasian rocks just described, and their mineralogical composition. This investigation affords the most interesting results, but it would not be compatible with the limits of this abstract to trace these various relations. Their consideration must therefore be deferred for the present.

II. *Genetic Relations of the Metamorphic Rocks.*

1. *Palagonitic Rocks.*

The metamorphic rocks, which constitute a by no means inconsiderable part of Iceland, are far more interesting than the unaltered rocks. The most remarkable member of this group is certainly the palagonitic tuff, which consists of a mixture of anhydrous and hydrated silicates. The anhydrous silicates belong exclusively to the pyroxenic rocks treated of above, and are never accompanied by trachytic masses, or even replaced by them; the hydrated silicates, which generally cement together the fragmentary rock, may, on the contrary, be regarded as a mixture or combination of two silicates, of which one is represented by the formula $3RO\,2SiO^3 + Aq$, and the other by $3Al^2O^3, SiO^3 + Aq$. These silicates appear to combine together in definite proportions; at least I consider it necessary to regard palagonite, for which I have proposed the formula $3RO\,2SiO^3 + 2Al^2O^3\,SiO^3 + Aq$, as well as a mineral occurring in the tuffs of Chatham island, one of the Galapagos, which I found to consist of $3RO\,2SiO^3 + Al^2O^3\,SiO^3 + Aq$, as being such combinations.

Without here entering into any special examination of the various members of the tuff rocks, it may be as well to mention that the palagonitic substance appears to occur everywhere as a characteristic constituent of these formations when the pyroxenic rocks of the volcanic period are especially well developed. Besides Iceland, where it occurs in extensive masses, it is met with in the more considerable basaltic elevations of Germany and France, in the Euganean islands, on Etna, the Azores, Canaries, Cape Verde Islands and the Galapagos Islands, probably also upon the volcanic islands of the South Sea. The following analyses will give an idea of the degree of uniformity in the chemical composition of this very generally occurring cementing substance of the volcanic tuff:—

Iceland, Seljadalr.

	61.	Oxygen found.	Oxygen calculated.
Silica	37·42	19·43	19·44
Alumina	11·17	9·47	9·72
Protoxide of iron	14·18		
Lime	8·76	5·12	4·86
Magnesia	6·04		
Potash	0·69		
Soda	0·65		
Water	17·15		
Foreign residue	4·11		
	100·17		

Iceland, Trollkonugil near Hecla.

	62.	Oxygen found.	Oxygen calculated.
Silica	39·98	21·16	19·96
Alumina	8·26	9·16	9·98
Protoxide of iron	17·65		
Lime	8·48	4·62	4·99
Magnesia	4·45		
Potash	0·43		
Soda	0·61		
Water	18·25		
Foreign residue	1·89		
	100·00		

Iceland, Palagonitic Sandstone near Reykjahildh.

	63.	Oxygen found.	Oxygen calculated.
Silica	35·09	18·57	18·31
Alumina	10·60	9·05	9·16
Protoxide of iron	13·65		
Lime	4·83	4·11	4·58
Magnesia	7·07		
Potash	0·25		
Soda	0·50		
Water	17·25		
Foreign residue	11·13		
	100·00		

Iceland, Laugarvatnshellir.

	64.	Oxygen found.	Oxygen calculated.
Silica	40·38	21·37	20·38
Alumina	10·79	9·10	10·19
Protoxide of iron	13·52		
Lime	8·56	5·19	5·09
Magnesia	6·35		
Potash	0·64		
Soda	0·61		
Water	16·98		
Foreign residue	2·32		
	100·16		

Iceland, Krísuvík.

	65.	Oxygen found.	Oxygen calculated.
Silica	37·95	20·09	20·60
Alumina	13·61	10·49	10·30
Protoxide of iron	13·75		
Lime	6·48	5·17*	5·15
Magnesia	7·13		
Potash	0·42		
Soda	1·72		
Phosphoric acid	0·43		
Water	12·68		
Foreign residue	7·25		
	100·419		

* After deducting the oxygen belonging to the phosphates and carbonates.

Iceland, Naefrholt near Hecla.

	66.	Oxygen found.	Oxygen calculated.
Silica	32·86	17·39	17·65
Alumina	7·31	8·47	8·83
Protoxide of iron	16·81		
Lime	6·80	5·03	4·41
Magnesia	6·13		
Potash	0·79		
Soda	1·98		
Water	11·38		
Foreign residue	16·36		
	100·42		

Iceland Tuff, containing Fossils, from Folsvogr.

	67.	Oxygen found.	Oxygen calculated.
Silica	28·53	15·03	15·20
Alumina	9·29	7·12	7·60
Protoxide of iron	9·40		
Lime	6·02	4·34	3·80
Magnesia	5·60		
Potash	0·96		
Soda	0·84		
Water	7·61		
Foreign residue	31·05		
	99·27		

Iceland Detritus from the Lava near Hruni.

	68.	Oxygen found.	Oxygen calculated.
Silica	37·11	19·64	18·74
Alumina	9·78	8·97	9·37
Protoxide of iron	14·67		
Lime	4·99	3·93	4·68
Magnesia	5·61		
Potash	1·57		
Soda	0·00		
Water	14·04		
Foreign residue	12·24		
	100·00		

Galapagos, Dyke Mass.

	69.	Oxygen found.	Oxygen calculated.
Silica	37·83	19·64	19·24
Alumina	12·95	9·03	9·62
Protoxide of iron	9·93		
Lime	7·49	5·00	4·81
Magnesia	6·54		
Potash	0·94		
Soda	0·70		
Water	23·00		
Foreign residue	0·96		
	100·34		

Galapagos, Rock forming a Crater.

	70.	Oxygen found.	Oxygen calculated.
Silica	36·15	18·77	18·31
Alumina	11·31	8·43	9·15
Protoxide of iron	10·48		
Lime	7·78	4·85	4·58
Magnesia	6·14		
Potash	0·76		
Soda	0·54		
Water	24·69		
Foreign residue	2·19		
	100·00		

The analysis of the cementing matter of tuffs, occurring upon the Cape Verde Islands, the Azores, Canaries, and in the basaltic formations of Germany, yielded results corresponding perfectly with the above.

The frequent occurrence of these rocks, and their alternation with the unaltered volcanic rocks by which they have been penetrated, indicates that the palagonitic tuff rocks have essentially contributed to the formation of the trachyto-pyroxenic mixed members of this group, and a closer examination of the phænomena of plutonic contact proves that such is really the case. The influence of the tuffs upon the formation of these rocks consequently demands an especial investigation. The following analyses give the composition of the varieties of tuff

examined, calculated for anhydrous substance and protoxide of iron :—

	Selja-dalr.	Troll-konugil.	Gala-pagos.	Reykja-hildh.	Laugar-vatn-shellir.	Krisu-vik.	Naefr-holt.	Foss-vogr.	Laxa.
Silica	48·29	51·20	50·18	49·69	50·71	47·63	46·29	47·78	51·36
Alumina	14·41	10·58	17·18	15·01	13·55	17·08	10·30	15·55	13·53
Protoxide of iron	16·47	20·34	11·85	17·40	15·44	15·53	21·30	14·17	18·27
Lime	11·31	10·85	9·94	10·01	10·75	8·14	9·58	10·09	6·91
Magnesia	7·79	5·70	8·68	6·83	7·89	8·95	8·64	9·39	7·76
Potash	0·88	0·55	0·93	0·35	0·81	0·52	1·11	1·61	2·17
Soda	0·85	0·78	1·24	0·71	0·76	2·15	2·78	1·41	0·00
	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Oxygen in SiO^3 and the bases

	3 : 2·032	3 : 1·703	3 : 1·992	3 : 1·906	3 : 1·835	3 : 2·178	3 : 2·039	3 : 2·099	3 : 1·746
Silica	48·29	51·20	50·18	49·69	50·71	47·63	46·29	47·78	51·36
Alumina and protoxide of iron	30·88	30·92	29·03	32·41	28·99	32·61	31·60	29 72	31·80
Lime	11·31	10·85	9·94	10·01	10·75	8·14	9·58	10·09	6·91
Magnesia	7·79	5·70	8·68	6·83	7·98	8·95	8·64	9·39	7·76
Potash	0·88	0·55	0·93	0·35	0·81	0·52	1·11	1·61	2·17
Soda	0·85	0·78	1·24	0·71	0·76	2·15	2·78	1·41	0·00
	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

These analyses lead to the unexpected result, that the substance of the melted palagonitic silicates almost exactly corresponds in its composition with the normal pyroxenic rocks, with the single exception of a somewhat greater variation here and there in the relative proportions of the several constituents. In fact, the following mean composition, deduced from all these analyses, scarcely shows any perceptible difference from that of the normal pyroxenic rocks, when we overlook the trifling excess in the quantity of magnesia and the somewhat smaller quantity of lime :—

	Palagonite.	Normal pyroxenic substance.
Silica	49·24	48·47
Alumina and protoxide of iron	30·82	30·16
Lime	9·73	11·87
Magnesia	7·97	6·89
Potash	0·99	0·65
Soda	1·34	1·96
	100·00	100·00

The proportion of the oxygen in the acid to that in the bases is for palagonite as 3 : 1·948, and for the pyroxenic rock as

3 : 1·998, which may be regarded as identical; and these numbers certainly differ less than the results of the several analyses from which these mean values are deduced. For this reason an almost uniform composition, frequently corresponding more closely with the calculation than when the normal pyroxenic substance is applied to this purpose, is obtained when the normal palagonitic substance is made the basis of the calculation. But the circumstance that the composition of these palagonitic substances deviates rather more from the mean results of analysis, explains at the same time in the most simple manner the slight discrepancies observed in some few of the lavas occurring in the palagonitic district of Iceland, the quantity of lime corresponding to the palagonitic composition decreasing slightly in proportion to the increase of magnesia. Even the almost perfect identity of the constitution of the palagonitic tuffs with that of the normal pyroxenic rocks, the absence of trachytic admixtures, and still more their gradual transition into the almost anhydrous substance of the pyroxenic rocks, which may be observed on a large scale, as well as in the separately-imbedded masses, render it in the highest degree probable that the formation of tuff is most intimately connected with the formation of the pyroxenic rocks. Observations which Darwin made in the Cape Verde Islands, and my own examination of specimens of the rocks to which they refer, for which I am indebted to the kindness of that distinguished investigator, have chiefly contributed to give me a clue to the remarkable processes concerned in the formation of palagonitic tuff.

In the neighbourhood of Porto Praja, a basaltic lava occurs, which has been poured out over a recent deposit of limestone. It may there be seen that the lava has, while in a liquid state, acted upon the underlying limestone over which it flowed, and become loaded with fragments of it. The product of this mutual action is a breccia-like conglomerate, in which the altered lava is mixed with a very pure mass of carbonate of lime. A closer examination of this mixture, which even in its exterior has quite the appearance of having been kneaded together in a pasty condition, excludes every possibility of supposing that the fragments of limestone accompanying the lava have originated from subsequent infiltration. The chemical change which has resulted

from the contact of the limestone with the lava does not leave any doubt as to the nature of the process by which the palagonite has been formed. Wherever the lava is in contact with the limestone, it is converted into a mass presenting all the mineralogical characters and chemical reactions of palagonite; and this metamorphism, characterized by a gradual transition into the unaltered rock, is more fully developed where the calcareous substance preponderates over the other constituent of the mass. The analysis of this metamorphic lava showed it to have the following composition, which scarcely differs from that of the pure palagonite:—

	71.	Oxygen found.	Oxygen calculated.
Silica	26·21	13·87	14·58
Alumina	8·62	7·31	7·29
Protoxide of iron	10·96		
Lime	4·79	4·33*	3·65
Magnesia	9·44		
Potash	1·81		
Soda	2·85		
Carbonic acid	5·10		
Water	14·62		
Residue	15·65		

I have been able to recognize perfectly analogous, although not identical relations, by observation and experiment on basaltic dykes traversing sedimentary limestone. Direct observation is therefore sufficient to prove that a palagonitic substance may result from the action of lime upon pyroxenic rocks at a high temperature. And, in fact, a number of tuffs from our basaltic formations, and from the volcanoes of the Galapagos islands, which I have had an opportunity of examining, prove in the most decisive manner that some masses of this kind have actually been formed in this way. The following analysis of a tuff from Chatham island, mixed in the most intimate manner with carbonate of lime, and forming a crater, furnishes a proof of this statement:—

* After deducting the oxygen of the carbonates.

	72.	Oxygen found.	Oxygen calculated.
Silica	34·516	18·27	17·54
Protoxide of iron	10·400	7·95	8·77
Alumina . . .	10·338		
Magnesia . .	7·801	4·49*	4·39
Lime	4·788		
Potash . . .	1·644		
Soda	1·525		
Phosphate of lime	0·336		
Carbonate of lime	4·320		
Water . . .	18·140		
Pyroxenic subst.	6·476		

If the formation of palagonite really takes place in the way pointed out, it must be possible to obtain this mineral artificially by similar means. And, in fact, this may easily be done by igniting an intimate mixture of 1 part finely-powdered basalt and 13 parts of slaked lime and elutriating the mass thus obtained with water. The product consists of a mixture of lime and palagonite, which may be recognized under the microscope by its peculiar characteristics. Nevertheless we are obliged to infer that the greater number of palagonites, and especially those of Iceland, have not originated from such a reaction of pyroxenic rocks and limestone, from the fact that carbonate of lime scarcely ever occurs as a constituent of the *undecomposed* palagonites of Iceland, and also because the per-centage of lime in the mineral itself, calculated for anhydrous substance, does not equal the per-centage of the same constituent in the normal pyroxenic rocks; for which reason it is not well possible to imagine that palagonite, poor in lime, can originate from pyroxenic rocks rich in lime, in consequence of a further addition of this constituent. On the contrary, it might be expected that alkalies would give rise to the formation of palagonite more readily than alkaline earths, as they do not necessarily require to alter the proportion of the constituents in palagonitic pyroxenic rocks. Experiment fully justifies this conjecture. The most beautiful palagonite powder, possessing all the mineralogical and chemical characters of the Icelandic, is obtained when finely-powdered

* After deducting the oxygen belonging to the phosphates and carbonates.

basalt is mixed with a large excess of fused hydrate of potash, and the basic alkaline silicate formed removed by means of water. The substance obtained after washing and elutriation is hydrated, and pulverulent when dry; it gelatinizes even with the weakest acids, is readily decomposed by carbonic acid and sulphuretted hydrogen, and has the following composition, corresponding with that of the purest Icelandic palagonite:—

	73.	Oxygen found.	Oxygen calculated.
SiO^3	30·764	16·28	17·1
Fe^2O^3	20·497	9·15	8·5
Al^2O^3	4·273		
CaO	8·016	4·56	4·2
MgO	4·600		
KaO	1·826		
NaO	0·532		
H^2O	30·047	26·70	

The residue after elutriation consists of a mixture of silicates, whose mean composition differs from that of palagonite only by about a one-fifth smaller per-centage of silica, which remains in the alkaline water used for washing the substance, from which distinctly-formed zeolitic crystals are sometimes separated having the composition $3CaO\,2SiO^3 + Aq$. I shall again refer to these further on.

This conversion of pyroxenic rocks into palagonite is accompanied by a very remarkable phænomenon. A considerable quantity of pure hydrogen is disengaged, resulting from the oxidation of protosilicates of iron to persilicates taking place at the cost of the equivalent of water contained in the hydrate of potash. The consequence of this is, that there is no trace of protoxide of iron in the palagonites, and the protoxide of iron in the pyroxenic rocks always appears as peroxide in the palagonitic tuffs. The protoxide and peroxide of manganese behave in a manner similar to the protoxide of iron, either free or combined with silica, passing, with disengagement of hydrogen, into manganate of potash, which reaction may be partly the cause of the frequent incrustations and dendritic deposits which are met with in the palagonitic rocks. The occurrence of metallic copper, evidently reduced from chloride of copper, in the palagonite

tuffs of the Faroe Islands, and elsewhere, admits of the most simple explanation by the reducing action of this hydrogen.

It is not very easy at first sight to conceive from whence the alkali might have been derived which could thus give rise to the formation of tuff in Iceland. Considering the variable per-centage of alkali in the pyroxenic rocks compared with the more constant proportions of their other constituents, we might be induced to assume that a separation of these alkalies took place in the rocks while in a state of igneous fusion. The idea of such a separation of alkalies is indeed not to be unconditionally rejected. It is well known that most salts are decomposed at high temperatures. If their acids are considerably more volatile than the bases, either basic salts are formed, or the bases are liberated while the acid is volatilized. The compounds of sulphuric, carbonic, nitric, arsenious acids, &c. undergo, with few exceptions, this decomposition. If, on the contrary, the acid is less volatile than the base or the salt itself, as is the case with the ammoniacal salts, then it is the base which is volatilized while the acid remains behind. The alkaline silicates may very probably suffer the latter kind of decomposition; for if silica and caustic potash or soda are heated together upon a platinum wire by means of a galvanic current to near the melting-point of the metal, the alkali is volatilized by a temperature at which the silica does not even begin to melt. It would appear therefore that in lavas which are heated to a temperature so elevated as to become liquid enough to be shot out in large parabolic curves from the opening of the crater, it is not only possible, but even probable, that a separation of alkalies by heat takes place, the more especially if it is remembered that carbonic acid or aqueous vapour, never wanting in such volcanic processes, must have given rise to the formation of hydrated alkalies and carbonates, compounds which are so volatile that their sublimation may be directly observed in some technical operations. There are indeed instances in which the separation and volatilization of the alkaline constituents even of silicates may be detected. In the coal and flux used in the blast furnaces in England, which are remarkable for their intense heat, the whole quantity of the alkali present is in combination with silica. Nevertheless, at the foot of these furnaces, where for years together a temperature prevails nearly

equal to the melting-point of platinum, such a sublimation of carbonated alkali, together with cyanide of potassium, takes place, that these products are in some instances collected by hundred weights. Unless this separation and sublimation of alkalies be attributed exclusively to the reducing influence of the coal, it would not be inadmissible to assume that similar processes take place in the immediate neighbourhood of those volcanic foci which are surrounded by the substance of pyroxenic rocks in a state of igneous liquidity. Such conditions therefore may possibly have contributed largely to the formation of palagonite. However, the enormous extent of the Icelandic tuff-rocks renders it very improbable that we have here to do with a mode of origin which would in any case bear the character of a merely local one. It is therefore certainly more in accordance with the strict principles of scientific inquiry to dispense with any merely hypothetical explanation of this question, and to rest satisfied with the assumption, justified both by experiment and observation, that in the volcanic periods there was, besides the trachytic and pyroxenic foci, a third focus in a state of activity, which has now become extinct, and whose contents consisted of silicates rich in alkalies and sufficiently superbasic to break up under the influence of water into palagonitic substance and soluble compounds, which were washed away. The occurrence of the palagonitic cementing substance, scarcely ever absent in Iceland from among the conglomerates and masses of fragmentary rock destitute of fossils and accompanying eruptive rocks, is readily accounted for by means of this assumption. It is indeed a direct and necessary consequence of the pouring out of such masses of rock over the surface; and the palagonitic tuffs containing fossils are nothing more than the products of submarine accumulations of sediment, which received the material for their palagonitic cement from the above-mentioned eruptive silicates, rich in alkalies, while in a state of metamorphism.

2. *Zeolitic Formations.*

The zeolitic amygdaloid rocks are most intimately connected with those belonging to the palagonitic and pyroxenic classes. They constitute, indeed, the intermediate metamorphic members which connect together these two classes. As the relative mean

compositions of the latter are almost exactly one and the same, the chemical relation of the zeolitic rocks to the form which they originated, can no longer be ascertained by means of calculation. However, even a superficial observation of the geognostic relations in which they occur, leaves no doubt as to the nature of their formation. Near Silfrastadir, as well as at innumerable other places in Iceland, the zeolitic amygdaloids, having somewhat the character of conglomerates, are seen to pass through gradual transitions on the one side into solid trap rock, and on the other into palagonitic tuff, so intimately blended indeed that the concretions and fissures may be traced from the trap through the amygdaloid and into the tuff rocks. At Silfrastadir, where the trap, rising in rocky precipices above the beds of tuff, admits of a closer examination of these relations, the formation of zeolites appears most perfectly developed at the contact of the two kinds of rock characterized by this gradual transition, and decreases towards the compact rock in proportion as the visible traces of a mutual action disappear more and more, so that finally it is only in fissures and isolated cavities that the fine druses of chabasite are found which are characteristic of the amygdaloid formations of that locality. This phænomenon occurs everywhere in Iceland. It may be observed even in the most recently erupted lavas. One of the most remarkable examples of this kind occurs on the Krafla. The frequently more arenaceous than tuffaceous beds of this volcano, if indeed a range of tuff mountains penetrated by craters and lava and traversed by furmaroles may be so termed, are intersected upon the north-western declivity of the mountain by what appears to be a very recent lava, which has issued not from the opening of a crater, but from horizontal fissures in beds. The neighbouring palagonitic rock has, from the point of contact with these beds of lava, suffered a most remarkable metamorphism, which may be best observed under a microscope of thirty or forty-fold magnifying power. The substance of the anhydrous rock, without exactly having been melted, is separated into two siliceous masses, of which one is dark and ferruginous, and the other brilliantly white and free from iron. The former appears as a homogeneous matrix in which the latter is imbedded; both are amorphous. Nearer towards the lava, where the igneous action was more consider-

able, the rock assumes the character of the porous basaltic amygdaloid conglomerates, which are so frequently met with in Iceland as intermediate members between the palagonitic and pyroxenic rocks. The darker ferruginous matrix, which under the microscope has entirely the appearance of green-bottle glass, but when examined by the naked eye in large pieces appears more to resemble certain conglomeritic pyroxenic rocks, presents spheroidal cavities with smooth surfaces, either empty or filled with granules of the non-ferruginous silicate. When this latter crystalline mass, consisting of zeolitic substance, does not completely fill the cavities, druses of zeolitic crystals have been formed, or isolated zeolitic crystals.

This separation into ferruginous and non-ferruginous silicates may be effected artificially in the palagonite and palagonitic tuffs by the most simple means. When pieces of these substances about the size of hazel-nuts are rapidly heated in the flame of a spirit-lamp, or before the blow-pipe, until they are red-hot externally, all the phases of this metamorphism may be most distinctly traced by means of a microscope of forty-fold magnifying power, from the exterior vitrified crust to the interior scarcely decomposed nucleus. In one zone, which, even by its vitrified state, presents the most evident indications of previous strong ignition, there may frequently be recognized a mass filled with amygdaloid and drusy cavities which resembles most perfectly the basaltic amygdaloid rocks which dip beneath the trap on the Esja near Hruni, and at numerous other parts of Iceland. This resemblance extends so far, that even the crusts covering these artificial crystal druses are identical in outward appearance with those of the natural rocks. Even the manner in which the crystals are situated upon the walls of the druses is precisely the same. Sometimes brilliant chabasite crystals, with the striation peculiar to this mineral, are observed in the ignited mass separated from the uncrystalline matrix by a crystalline mass of chabasite, followed by a crust resembling a saalband.

The extraordinary abundance of zeolitic amygdaloid rocks in Iceland is accounted for in the most simple manner by these experiments, for the conditions requisite for their formation could scarcely be found anywhere combined in a manner more favourable than they are there. Even a hasty glance at the

lofty perpendicular cliffs of the mountains on the coast, consisting chiefly of pyroxenic substance, furnishes a distinct picture of this stupendous metamorphism. Trap dykes of more than a thousand feet in height are not unfrequently seen there traversing the entire rocks, sometimes massive, sometimes stratified, and branching out through the enormous horizontally extended trap beds in such a way as to leave not the slightest doubt that these masses, intersecting and covering the tuff rocks, are no other than the product of melted matter which was erupted through these dykes. The results of the igneous action of these injected beds of trap stand in a most intimate relation to the magnitude of the heating and heated beds, when especial influences have not prevailed to modify them.

In the amygdaloid rocks resulting from the metamorphism of tuff beds, whose original aggregation may frequently be distinctly enough recognized in the sometimes angular, sometimes rounded, imbedded masses, such a gradual transition into the unaltered trap rocks may in some instances be traced, that there is an absolute want of any line of demarcation between the two. The passage of a crumbling hydrated rock into one which is perfectly anhydrous, with all those characteristic stages of zeolite formation presented by a fragment of ignited palagonite, may here be observed on a large scale. It consequently cannot admit of a doubt that it was neither neptunian nor plutonic agencies alone which gave rise to the formation of zeolitic rocks in Iceland. On the contrary, we have here to do with a long series of different phases of metamorphic development, the products of which present themselves in the amygdaloid rocks. A truly plutonic rock possessing a superbasic composition, suffers a neptunian metamorphism into palagonite and palagonitic tuff, either at the place of its original eruption, or during the transport of its mechanically broken fragments. New plutonic masses break through this altered rock frequently after a long period of rest, and by a second act of metamorphism, now plutonic, convert it into zeolitic amygdaloid. Finally, a third neptunian metamorphosis, caused by gaseous exhalations and aqueous vapour, results from this change, and of which, as the last stage of all these processes, I shall speak subsequently. However simple and intelligible these phænomena may be as regards the forma-

tion of zeolitic amygdaloids, the origin of the zeolitic minerals imbedded in the substance of the trap beds and the more compact basalt, where it is obviously necessary to assume the former prevalence of very much more elevated temperatures than those which even, according to the above experiment, are compatible with the formation of zeolites from palagonite, must still appear very enigmatical. But even this phænomenon admits of being accounted for by means of an experiment, which would appear completely to clear up the mystery attaching to the origin of hydrated siliceous minerals imbedded in plutonic rocks. When, for instance, a finely powdered mixture of 0·2 part lime and 1·0 silica is introduced into 9 parts of caustic potash, melted in a silver capsule, and the whole allowed to cool slowly, after having been kept for some time at a strong red heat in a muffle, upon treating the mass with water, a network of prismatic crystals, frequently 4 to 5 lines in length, is found partly attached to the sides of the capsules. These crystals are hydrated silicate of lime mixed with some carbonate of lime, and represented by the formula $3CaO\,2SiO^3 + Aq$:—

	74
Silica	27·215
Lime	22·241
Potash	0·733
Water separated at 228°·2 . .	36·915
Water separated by ignition .	9·508
Carbonate of lime	2·603
	99·215

The artificial preparation of this beautifully crystallized hydrated silicate, and still more the altogether uncommon mode of its formation, are of very great interest in a geological point of view. We have here to consider, not merely the fact that a hydrated silicate is formed at a red heat, but the still more important fact, that it is not destroyed again under this condition, although, after it has once been separated from the surrounding mass, it loses four-fifths of its water at 228°·2, and the remainder at a temperature below red heat. It most undoubtedly follows from this, of its kind isolated, phænomenon, the further discussion of which, with all its experimental and theoreti-

cal consequences, I must defer for the more extended consideration of this subject, that the palagonitic and zeolitic metamorphism, which we generally find to succeed one another, may even take place at the most elevated temperatures under the simultaneous and subsequent influence of water. For it is only necessary to add to the red-hot superbasic mixture of caustic potash some powdered basalt, in order to obtain by subsequent treatment with water a mixture of palagonitic substance with those zeolitic crystals of hydrated silicate of lime. And indeed zeolitic druses, traversed by a palagonitic tuff, presenting none of the indications of such a second plutonic metamorphism, are actually found in Iceland, and very abundantly in the Faroë Islands. I have, for instance, a specimen of this kind from the Faroë Islands, which consists of concentric radiated desmin substance, enclosing a nucleus of unaltered palagonitic tuff, and surrounded externally by unaltered tuff.

After these experiments and observations, the occurrence of olivine and augite in sharply defined crystals, together with zeolitic minerals in the midst of a hydrated palagonitic mass, is easily accounted for. Those anhydrous minerals are crystalline plutonic products, which were not influenced in their composition by the subsequent neptunian metamorphism. They are consequently found in their original form, together with the zeolitic and palagonitic products of this metamorphism. Similar phænomena present themselves at the Pferdekopf in the Rhön mountains, with the single exception, that there the products of the last stage of rock formation, caused by aqueous vapour and volcanic gases, predominate. The formation of zeolitic minerals in pyroxenic rocks becomes equally intelligible through the aid of these experiments and observations. They may be produced in the melted rock when this is sufficiently alkaline and superbasic, as in their artificial preparation. And, in fact, the palagonitic constituent so characteristic of the metamorphosis of superbasic silicates is scarcely ever wanting in the zeolitic pyroxenic rocks. It is indeed the amorphous part of basalt which gelatinizes with acids, and which it is customary to regard as the zeolitic substance of this species of rock.

3. *Formation of Rocks by Pneumatolytic Metamorphism.*

The processes considered under this head refer to manifold products which result from the action of volcanic gases and vapours upon the rocks which have been already treated of. They are of no less importance than the latter, and acquire for geologists an especial interest from the fact, that the processes to which they owe their origin admit of direct observation. In order to comprehend the various reactions which take place during these processes, it must be remembered that the mass of most rocks consists of a mechanical mixture of solid rock with water; and that the action of the melted masses coming in contact with these beds impregnated with water was twofold, and took place at two distinctly different periods. It must have commenced with a vaporization of the water; and it was not until after this had ceased that the hitherto prevailing temperature, determined by the existing pressure, could increase and be raised so high that the thorough plutonic reaction between the heating and heated rocks commenced. All those apparent contradictions which are presented by the phænomena of plutonic contact admit of easy explanation by the aid of this physical necessity. For this purpose three possible cases must be kept in view. The first is when a melted eruptive mass, possessing the lowest possible temperature and moving very slowly, comes in contact with a rock which receives a very rapid supply of water from fissures and spring strata. Here all the conditions are combined, which would tend to prevent any trace of a direct igneous action upon the substance of the adjoining rock. The first result of the contact is the formation of a solid crust, frequently of a glassy, scoriaceous and basaltic character, such as may be observed in many places where basalt has penetrated, and especially when in the form of dykes. This solid crust may be compared to a badly conducting furnace-wall, through which the equalization of temperature in the adjoining rock, maintained constantly at the boiling temperature of water, can only take place slowly. A plutonic metamorphism or interfusion of the adjoining rock would here be a physical impossibility. The second case, frequent in Iceland, but more rare in our basalts, is that in which the last-mentioned conditions are either absent or comparatively

insignificant in amount, and its former existence is indicated by all the signs of a previous state of ignition. The nature of this action is in part essentially determined by the substance of the rock heated. The readily fusible palagonitic tuff is converted into basalt and zeolitic conglomerate, limestone to a superbasic silicate—the material of the palagonitic formations—sandstone to a vitrified mass resembling hornstone, in which the eruptive rock frequently pours through capillary rents and fissures in the form of an extravasated interfused mass.

Besides these two modes of action, there is a third, that, namely, in which the boiling-point of the water is raised by enormous pressure, even to a red heat, in consequence of which direct interfusion products of red-hot melted rock with red-hot liquid water are formed. There are, indeed, phænomena in Iceland which appear scarcely to admit of any other explanation than this. I shall return to the consideration of this point at some future opportunity.

These remarks will suffice to show that the greater number of processes of plutonic interfusion and metamorphosis must be preceded by a vaporization of water. The mechanical effects of this change present themselves in the volcanic concussions and eruptions, the chemical effects in the manifold exercise of fumarole action. Consequently the study of this action, and the products resulting from it, is of especial interest for the theory of volcanoes. The volcanic actions continuing after eruptions and manifesting themselves in solfataras, geysers, and thermal springs, furnish the material, which renders it possible, by means of direct observation and experiment, to penetrate to the source of all these phænomena connected in the most intimate manner with internal volcanic activity.

The most important data for such an investigation consist in a knowledge of the composition of the exhalations, which, as consequences of the great volcanic catastrophes, issue from the earth in fumaroles.

Besides aqueous vapour, which constitutes the chief part of these exhalations, they contain as gaseous constituents only carbonic acid, hydrochloric acid, sulphur vapour, sulphuretted hydrogen, sulphurous acid, free hydrogen, and, together with these as foreign admixtures not properly volcanic, nitrogen, oxygen

and ammonia. I have never been able to detect the minutest trace of carbonic oxide or carburetted hydrogens, although I was enabled to employ methods of investigation which would have detected the presence of only a few thousandths of those gases.

The chemical character of the fumarole action depends upon the preponderance of one or other of these gaseous constituents. The hydrochloric acid fumaroles, which not unfrequently occur on a large scale near the Italian volcanoes, and are then generally accompanied by a very considerable sublimation of chloride of sodium, appear to be of less importance in Iceland. I was only able to detect traces of hydrochloric acid in a free state in the crater fumaroles a few months old, which owed their origin to the last eruption of Hecla, as well as in the exhalations of vapour from the lava which was then erupted. As the volcano when I visited it shortly after the last eruption was already so far in a state of rest that there were no violent exhalations of vapour, the gases could only be drawn up by means of an air-pump from the quietly smoking crater fissures, which were tolerably open to the access of air. The aqueous vapour condensed while drawing up the gases always contained sensible quantities of free hydrochloric acid, which is put down as an uncertain quantity in the following analyses:—

	75. 1. Fumarole in the great crater of Hecla.	76. 2. Fumarole in the great crater of Hecla.	77. 3. Fumarole of the lava stream of 1845.
Nitrogen	81·81	82·58	78·90
Oxygen	14·21	16·86	20·09
Carbonic acid	2·44	0·56	1·01
Sulphuretted hydrogen .	0·00	0·00	0·00
Sulphurous acid . . .	1·54	0·00	0·00
Uncertain quantity of hydrochloric acid . . .			
Carbonic oxide	0·00	0·00	0·00
Carburetted hydrogen .	0·00	0·00	0·00
	100·00	100·00	100·00

It follows from these analyses that carbonic and hydrochloric acids constitute one part of these crater gases, and that they are sometimes accompanied by sulphurous acid, which decreases so

much in the fumaroles of the lava streams that it can no longer be detected in the gases, and scarcely in the condensed vapours. The composition of the solid and liquid products of the fumaroles in the newly-erupted crater of Hecla in 1845, and that of the lava stream which flowed from the lowest crater, correspond perfectly with this fact. The moist gravel surrounding the melted masses of sulphur in the interior of the uppermost and largest crater had the following composition:—

	78.
Sulphur	58·272
Sulphate of lime	0·796
$2Al^2O^3 + Al^2Cl^3$	0·425
Protochloride of iron	0·282
Chloride of calcium	0·650
Chloride of magnesium	0·056
Chloride of potassium	0·452
Chloride of sodium	0·024
Chloride of ammonium	0·005
Water	9·402
Decomposed lava gravel	29·636
	100·000

With the exception of the ammoniacal salts derived from the atmosphere, these are the same products which may be artificially obtained by the humid reaction of the sulphurous and hydrochloric acids present in the crater fumaroles and the rock of the crater. On the other hand, the only incrustation of salt scantily spread over the bottom of the highest crater, and which, from the unaltered condition of the rock upon which it was deposited, could only have been formed by sublimation, possessed an entirely different composition. Such an incrustation consisted of—

	79.
Chloride of sodium	5·65
Sulphate of lime	63·41
Sulphate of magnesia	12·68
Sulphate of soda	16·78
Sulphate of potash	0·88
	99·40

Since the sulphates as such are not capable of sublimation, it must be assumed that this saline incrustation was originally volatilized in the form of chlorine compounds, which were subsequently converted into sulphates by means of sulphurous acid in the presence of aqueous vapour and atmospheric air. In the fumaroles of the lower lava streams, on the contrary, characterized by only a small per-centage of sulphurous acid, the chlorine compounds again predominate, as is shown by the following analyses of products collected there a few months after the last eruption:—

	80.	81.
Chloride of ammonium . .	81·68	74·32
$4Fe^2O^3 + Fe^2Cl^3$	5·04	6·75
$4Al^2O^3 + Al^2Cl^3$	3·73	0·28
Chloride of magnesium . .	1·69	5·45
Chloride of calcium . . .	0·53	4·63
Chloride of sodium . . .	1·73	2·33
Chloride of potassium . .	0·53	0·70
Silica	0·95	0·25
Water and stony residue .	3·12	5·29
	99·00	100·00

The mode in which this mixture of salts was formed is the same as that of the crater products just treated of, except that in this instance it was not the atmospheric air alone which yielded the ammonia on the formation of chloride of ammonium, but, as I have shown* in another paper, likewise the vegetation† partially covering the ground overflowed by lava.

With regard to the origin of the hydrochloric acid in the crater gases there can be no doubt. Chloride of sodium, which occurs so frequently as a sublimate in volcanoes, is decomposed at very high temperatures, in the presence of aqueous vapours, by silicates into that acid and soda, which unites with the silicates causing the decomposition. It is not necessary to assume that the chlorine compound suffers this decomposition apart from the lava.

* Liebig's *Annalen*, vol. lxv. p. 70.

† According to my experiments, a square metre of meadow land yields on dry distillation a quantity of ammonia corresponding to 223·3 grms. of chloride of ammonium.

It is very easy to ascertain that the rock formed from the lava which flowed in 1845 from Hecla, and which yielded the sublimates frequently impregnated with traces of free hydrochloric acid, itself contains a sensible quantity of basic chlorine compounds in its mass. 100 parts of lava from the eruption crater contained in fact 0·246 chlorine, and the same quantity taken from the end of the stream 0·447.

From the nature of the origin of hydrochloric acid fumaroles, it necessarily follows that they present the character of permanent phænomena only when the direct volcanic activity, together with the degree of temperature indispensably necessary for this process, has not receded to any considerable depth below the surface; for otherwise the hydrochloric acid acting so powerfully upon the substance of the rocks, would very soon give rise to the formation of chlorides which are destitute of that degree of volatility which would enable them to reach the surface under the influence of a moderate temperature. It is for this reason that fumaroles are observed as the direct results of great volcanic eruptions soon after they cease, and continuing in a permanent state of activity only where lava has reached the surface in the form of scoriaceous ejections, which have lasted for a considerable time.

If these conditions are wanting, the seat of the change recedes further below the surface, where it may long continue in a state of activity in foci of mineral waters, as may even be inferred from the composition of thermal spring waters of Iceland, with regard to whose formation there cannot for one moment be a doubt, as they may easily be made artificially by the action of the volcanic gases upon the Icelandic rocks.

The sulphuretted fumaroles, on the contrary, have an entirely different origin, and are far less ephemeral, lasting sometimes for centuries after the great volcanic eruptions. Iceland presents, in its stupendous solfataras and geysers, which are the two principal stages of this fumarole process, the most abundant material for its thorough investigation. Nevertheless, I must in this place restrict myself to the statement of those results of my investigations upon this subject which relate to the general connexion of these phænomena with the original processes of volcanic activity.

The gases which penetrate forcibly through the muddy soil of the solfatara fields, or break through the more solid rock in violent streams of vapour, must in this case likewise form the starting-point of the investigation. The solfataras of Krísuvík present the most considerable of all the gaseous eruptions of this kind. That jet of vapour, which issues from the loose strong ground of the upper ridge some few hundred feet above the principal group of springs in the valley, pours forth with a hissing noise an immense stream of vapour, the tension of which is sufficient to project stones the size of the hand to a height of several feet. This stream of vapour contains 82·30 steam and 17·20 of gas, having the following composition:—

	82.
Carbonic acid	87·43
Sulphuretted hydrogen . .	6·60
Hydrogen	4·30
Nitrogen	1·67
Carbonic oxide	0·00
Carburetted hydrogen . .	0·00
	190·00

The composition of this vapour as it issues is—

	83.
Aqueous vapour	82·30
Carbonic acid	15·47
Sulphuretted hydrogen .	1·17
Hydrogen	0·76
Nitrogen	0·30
	100·00

According to a measurement, which however must only be regarded as in the highest degree approximative, this spring alone yields, during twenty-four hours, 223 cubic metres of sulphuretted hydrogen, 12 cubic metres of pure hydrogen, and a quantity of steam, the total effect of which is equal to the power of thirty horses.

Close to this spring there is another, scarcely inferior in magnitude, and exhaling gas, having almost exactly the same composition as the last, namely,—

	84.
Carbonic acid	88·24
Sulphuretted hydrogen .	6·97
Hydrogen	4·10
Nitrogen	0·69
Carbonic oxide	0·00
Carburetted hydrogen . .	0·00
	100·00

About a quarter of an hour's walk from this place, at the spot where in coming from Reykjavík the first large exhalation of vapour occurs in the bottom of the valley itself, there are at the edge of a piece of meadow ground, generally used by travellers for pitching their tents, a number of large pools of boiling mud, between which vapour is observed to break out with remarkable violence. Although the small space of solid ground then surrounding it was continually covered with hot clouds of vapour, it was possible to penetrate to the opening from which the vapour issued by means of the crusts of gypsum which had been formed between the boiling pools, and to collect the gas for analysis with an appropriate apparatus. Its composition was—

	85.
Nitrogen	0·50
Carbonic acid	79·07
Sulphuretted hydrogen .	15·71
Hydrogen	4·72
Carbonic oxide	0·00
Carburetted hydrogen . .	0·00
	100·00

The enormous force with which these gases accompanied by masses of steam issue, would make it appear that these springs are the principal openings of the fissures and channels from out of which the fumarole gases are diffused through the surrounding rock, causing its metamorphism. As the dissolved products of this metamorphosis possess, as a predominating character, an acid reaction owing to the formation of sulphuric acid, no traces of carbonate of lime or of silica present themselves among the products of the decomposition of the rocks in which the solfataras occur. Since the carbonic acid takes no part in these de-

compositions, it is in this instance exclusively the sulphuretted hydrogen and sulphurous acid which, in the presence of heated water, bring about all those alterations of rocks, the most remarkable of which I have already pointed out in a previous paper on the pseudo-volcanic phænomena of Iceland *. The analysis of the gases which are evolved from the smoking ground in the fumarole district, or from small pools of water and mud, affords the most striking proof of this exclusive action of sulphuretted hydrogen. Thus, for instance, the considerable per-centage of carbonic acid is not diminished, while the quantity of sulphuretted hydrogen, in proportion to that of free hydrogen, decreases more and more. The following analyses of the gases taken from different small pools of boiling water situated in the midst of the Solfatara of Krísuvík, most distinctly show this decrease of sulphuretted hydrogen :—

	86.	87.
Nitrogen	1·80	1·44
Carbonic acid	88·54	86·92
Sulphuretted hydrogen	1·79	3·28
Hydrogen	7·87	8·36
Carbonic oxide	0·00	0·00
Carburetted hydrogen	0·00	0·00
	100·00	100·00

It may be as well, for the sake of completeness, to add here the analysis of such a gas from Reykjahlidh in the far north of Iceland, which was drawn from the smoking muddy soil of a large fumarole by means of an artificial stream of vapour, and was remarkable for its unusually large quantity of hydrogen :—

	88.
Nitrogen	0·72
Carbonic acid	30·00
Sulphuretted hydrogen	24·12
Hydrogen	25·14
Carbonic oxide	0·00
Carburetted hydrogen	0·00
	100·00

It is evident from these experiments how little ground there is for denying the presence of combustible gases in the exhala-

* Liebig's *Annalen*, vol. lxii. p. 1.

tions of volcanoes. The objections which are supposed to be fatal to the old volcanic theory of Davy entirely lose their value after these results. For if, in the spirit of this theory, it is assumed that the lavas, and the phænomena of ignition accompanying them, result from an oxidation of alkaline and earthy metals determined by a decomposition of water, it admits of being proved, quite in contradiction of the views which have hitherto been entertained, that the quantity of the hydrogen evolved from volcanoes bears a perfect relation to the magnitude of the streams of lava formed. A single one of the vapour springs of Krísuvík yields, according to the measurement quoted above, about 12 cubic metres of hydrogen in twenty-four hours. Assuming, then, that the remaining innumerable springs, together with the large fumaroles occurring there, yield together a quantity only one hundred times as great—which may safely be regarded as far less than the quantity of this gas which is actually evolved—we may by means of this assumption and simple calculation show, that the formation of lava which would be equivalent to such an evolution of gas within the period which elapses between two great eruptions, is sufficient to produce immense streams of lava. Nor is it any longer possible to attach any importance to the second of the principal objections which have been made to Davy's hypothesis, namely, that it is unusual to observe any sensible appearance of flames during great volcanic eruptions. For if, from the known composition of the first-mentioned fumarole gas, we estimate the temperature of its flame, we find it to be 305°·6; consequently a temperature which is far below the point of ignition of hydrogen.

These gases are therefore combustible only at a red heat, and even under the most favourable circumstances can only produce by such a combustion an increase of temperature amounting to 305°·6, which in a red heat must necessarily escape altogether observation by the eye.

Since, as I have already pointed out, the palagonitic metamorphism is likewise accompanied by a disengagement of hydrogen, it would appear almost impossible that there should be any doubt as to the source of this gas. However, the constitution of the mixture of gas examined affords in itself a direct proof that neither the formation of palagonite nor a decomposition of water

by means of alkaline or earthy metals, can have had any share in the generation of the volcanic hydrogen; for both of these processes presuppose the prevalence of a temperature in which carbonic acid cannot exist in contact with hydrogen without suffering a partial reduction to carbonic oxide. But not the minutest trace of this gas is found in the volcanic exhalations.

Besides sulphuretted hydrogen, it is especially sulphurous acid which determines the character of the action of solfataras. This gas likewise always appears associated with aqueous vapour. On account of the great readiness with which it dissolves in the condensed vapour, it cannot be collected in the gaseous form. But even the smell of the water condensed from such vapour springs, and the reaction which it gives with iodine, testify to the presence of a considerable quantity of this gas. As sulphuretted hydrogen and sulphurous acid mutually decompose each other with a separation of sulphur, the two gases can never present themselves together. However, they frequently occur in the same fumarole district, close together.

It has been attempted to connect the formation of volcanic gases, in which the presence of hydrogen has hitherto been entirely overlooked, in part with the decomposition of organic substances. But the gases which result from the spontaneous decomposition or dry distillation of organic remains do not present the remotest similarity to these exhalations. To prove this statement, it will be sufficient to bring forward here a few analyses which I have made of coal-gas, and of some natural exhalations of combustible gases, with regard to whose organic origin there can be no question.

Marsh gas from a Pond in the Botanical Garden at Marburg.

	89. Summer.	Winter.
Nitrogen	49·39	18·03
Oxygen	0·17	0·00
Carbonic acid	3·10	5·36
Marsh gas	47·37	76·61
Hydrogen	0·00	0·00
Carbonic oxide	0·00	0·00
Olefiant gas	0·00	0·00
	100·00	100·00

Gas from a Brine-spring yielding Mineral Oil near Hanover.

	90.
Nitrogen	25·12
Oxygen	0·00
Carbonic acid . . .	14·41
Sulphuretted hydrogen	3·18
Marsh gas	56·61
Mineral oil vapour .	0·68
Hydrogen	0·00
Carbonic oxide . .	0·00
	100·00

Gas from the detonating Salt of Wieliczka.

	91.
Nitrogen	10·35
Oxygen	2·98
Carbonic acid . . .	2·00
Marsh gas	84·60
Hydrogen	0·00
Carbonic oxide . .	0·00
Olefiant gas . . .	0·00
	98·93

Marsh gas from the Coal-seams at Obernkirchen.*

	92.	
Nitrogen	7·16	
Oxygen	0·45	
Carbonic acid	2·61	
Marsh gas	97·53	[87·53?]
Hydrogen	0·00	
Carbonic oxide	0·00	
Olefiant gas	0·00	
	100·00	

* From the same boring from which Bischoff took gas for analysis.

Gas from the Thermal Springs at Aix-la-Chapelle.

	93. Kaiserquelle.		94. Corneliusquelle.	
	Gas evolved freely.	Gas diffused through water.	Gas evolved freely.	Gas diffused through water.
Nitrogen	66·98	9·00	81·68	7·79
Oxygen	0·00	1·23	0·00	0·00
Carbonic acid	30·89	89·40	17·60	92·21
Sulphuretted hydrogen	0·31	0·00	0·00	0·00
Marsh gas	1·82	0·37	0·72	0·00
Hydrogen	0·00	0·00	0·00	0·00
Carbonic oxide	0·00	0·00	0·00	0·00
Olefiant gas	0·00	0·00	0·00	0·00
	100·00	100·00	100·00	100·00

	95. Quirinusbad.	96. Rosenquelle.
Nitrogen	6·41	9·14
Oxygen	0·08	0·00
Carbonic acid	93·25	90·31
Sulphuretted hydrogen	0·00	0·00
Marsh gas	0·26	0·55
Hydrogen	0·00	0·00
Carbonic oxide	0·00	0·00
Olefiant gas	0·00	0·00
	100·00	100·00

Gas diffused in the Sulphuretted Water of Nenndorf.

	97. Medicinal spring.	98. Spring under the roof.	99. Bath spring.
Nitrogen	17·30	19·71	23·91
Oxygen	0·00	0·00	0·00
Carbonic acid	69·38	68·29	72·63
Sulphuretted hydrogen	11·86	11·72	3·29
Marsh gas	1·46	0·28	0·17
Hydrogen	0·00	0·00	0·00
Carbonic oxide	0·00	0·00	0·00
Olefiant gas	0·00	0·00	0·00
	100·00	100·00	100·00

Purified Coal-gas from an English Gas-work.

	100.
Nitrogen	1·89
Oxygen	0·00
Carbonic acid	2·83
Sulphuretted hydrogen	trace
Marsh gas	26·91
Hydrogen	35·13
Carbonic oxide	5·11
Elayle gas	2·70
Ditetryle gas	2·28
	100·00

These analyses, to which I could add a great number of others, sufficiently show that the volcanic gases are characterized by the absence of all combustible carbonaceous substances, while in the gaseous products of dry distillation, or spontaneous decomposition of organic remains, they are scarcely ever wanting.

If, in accordance with these facts, the solfatara gases can in no way be of organic origin, still it does not require any special hypothesis to account for their formation. The most simple experiment shows that when sulphur and aqueous vapour come into contact with heated pyroxenic rocks, all the conditions necessary for their formation are present. When the vapour of sulphur is passed over basalt, or any other of the pyroxenic rocks treated of above, at a red heat, a partial decomposition of the peroxide of iron in these rocks takes place, the sulphur being divided between its constituents. The oxygen of the oxide escapes in the form of sulphurous acid, and the metal remains in the rock as sulphuret. If afterwards steam is passed over the sulphuretted rock, still at a red heat, an abundance of sulphuretted hydrogen is disengaged, and magnetic oxide of iron is formed. If the temperature exceeds only very slightly a red heat, a part of this sulphuretted hydrogen is decomposed into its elements, and a sensible quantity of free hydrogen is found mixed with the sulphur vapour. Fragments of basalt, from the Stempelskopf near Marburg, heated to redness in sulphuretted hydrogen, and then treated with steam at a higher temperature, yielded a mixture of gases having the following composition:—

Sulphuretted hydrogen . . .	93·99
Hydrogen	6·01
	100·00

The phænomena upon which depends the activity of solfataras become, after these experiments, very easily intelligible. It is well known that almost all volcanic eruptions are accompanied by sublimation of sulphur. Consequently the zone from whence the sulphurous acid originates is situated where such masses of sulphur, whose occurrence may be easily accounted for by the action of volcanic heat upon decomposable sulphur compounds, come into contact in the state of vapour with red-hot pyroxenic rocks. When, at a subsequent period, the temperature communicated to this zone by the volcanic action sinks, a new phase of chemical action commences. The sulphur compounds of iron, and perhaps likewise of alkaline and earthy metals, which have been formed there, commence their action upon the aqueous vapour, and from this mutual action result sulphuretted hydrogen and the products of its decomposition—free hydrogen and sulphur vapour. It is therefore evident that these two processes are blended in each other, and meet in such a manner as necessarily to determine the irregular simultaneous exhalation of those gases at spots but little distant from each other in the same fumarole district. These processes likewise afford an explanation of the chronological course of the fumarole action. The sulphurous acid, whose appearance alone characterizes the initial stage of all these phænomena, is accompanied after a while by sulphuretted hydrogen, which by its reaction with the former gas gives rise to that succession of decompositions which characterize the true solfataras. Acid liquids saturate the rocks traversed by separated sulphur and torn up by aqueous vapour, converting these rocks, as I have already shown, whether they belong to the pyroxenic or trachytic group, into clay, by extracting from the silicates potash, soda, magnesia, lime, protoxide of iron, and frequently a part of the alumina, as sulphates.

This phase of destructive action is followed, in course of time, by a productive one, which increases in proportion as the source of sulphurous acid becomes extinct, and the gradually decreasing evolution of sulphuretted hydrogen recedes to greater depths.

In consequence of these changes, the acid reaction of the water with which the rocks are impregnated is converted into an alkaline reaction, resulting from the formation of alkaline sulphurets at the cost of the now alone-acting sulphuretted hydrogen. Simultaneously with the disappearance of the acid reaction, commences the action of the free carbonic acid upon the rocks, and the alkaline bicarbonates resulting from this action provide a solvent for the silica, by which means, and in accordance with the most simple laws, which I have already described, those wonderful geyser structures are formed, giving rise to the stupendous phænomena of the Icelandic eruptive springs.

The springs of carbonic acid finally make their appearance as the terminal stage of all this series of phænomena. They generally survive the plutonic catastrophe the longest, and appear to be exclusively limited to the Western Islands.

The fumaroles on the craters of Hecla were, when I had an opportunity of examining them more carefully shortly after the eruption in 1845, in that state which I have described as being the first stage of the secondary volcanic action. Not the slightest trace of sulphuretted hydrogen could then be detected either by the smell or by means of reagents, while, together with the abundant sublimation of sulphur, the presence of sulphurous acid could be recognized by its smell at a considerable distance from the craters. On approaching a lighted cigar to the fumaroles, those thick clouds of smoke were indeed observed which Piria has pointed out to be an indication of the presence of very minute traces of sulphuretted hydrogen. However, as it is very easy to ascertain that sulphur itself will cause the same phænomenon when subliming with aqueous vapour, it is doubtful whether even a trace of sulphuretted hydrogen accompanied the exhalations from the crater at that time. In the year 1843, I observed exactly the same phænomena in the crater of Vesuvius, when, after a long period of rest, it again began to show signs of activity, and lava, solidifying to scoriæ, commenced to pour forth from the crater-cone with periodic explosions of vapour. The last eruptions of the Krafla and Leirhnúkr, during the last century, likewise appear, according to the certainly very scanty records preserved, to have been accompanied by such phænomena.

In the extensive solfataras surrounding these latter two volcanoes, the second phase of the secondary volcanic action presents itself at the present time on the most stupendous scale, while the volcanic activity is already in a state of considerable decrease. As at Krísuvík, in the south-west of Iceland, the exhalations of sulphurous acid are very much inferior in magnitude to the sulphuretted hydrogen, which here escapes from the smoking clay soil and the boiling pools of mud in preponderating quantity.

With regard to the third phase of these operations manifesting itself in the phænomena of geysers, it can scarcely anywhere be studied in its chronological relations more favourably than at the famous springs of Haukadalr, which bear the name of the Great Geysers. The crater of these springs, which has acquired the material of its coating of siliceous tuff, like most of these springs, from the readily decomposable palagonitic tuff, rests upon a surface which is still in a state of fumarole action, situated at the north-western edge of the spring cone, a section of which is shown by a ravine. The streams of vapour, which there burst through the fumarole clay, coincide most perfectly, in their external appearance and in their actions, with the springs which are met with in the solfataras of Krísuvík and Reykjahlidh, with the single exception that at the geyser there is no trace of sulphurous acid or any sensible deposition of sulphur. A glance at the following composition of the gases, taken from this fumarole district, must, in fact, remove every doubt as to the identity of the origin of all these phænomena:—

	101.
Nitrogen	84·11
Carbonic acid	8·92
Hydrogen	6·59
Sulphuretted hydrogen	0·38
Carbonic oxide	0·00
Marsh gas	0·00
Oxygen	0·00
	100·00

The proportion of hydrogen to sulphuretted hydrogen and to free carbonic acid, furnishes here also a standard for estimating the consumption of the two latter gases, which, as may be seen,

is far greater in this instance than in the solfataras. These few simple and easily-intelligible processes of volcanic gas formation contain the key to a whole series of metamorphic changes, which may be comprised under the name of the pneumatolytic, as a universal and frequently-occurring class of phænomena. The composition and mode of formation of the acid and alkaline thermal waters of Iceland, which may easily be ascertained even by direct experiment, follow as simple consequences from these processes; and the argillaceous formations, which are found on the saalbands of the trachytic and pyroxenic dykes, but on the most extended scale in the beds of conglomeritic amygdaloids and tuffs, penetrated by plutonic rocks, have in part resulted only from a stupendous repetition of those very same processes of decomposition, which we may daily observe going on at the surface of the Icelandic solfataras. Without here referring at length to the less interesting modes of rock formation, which are determined by the separate action of aqueous vapour and hydrochloric acid, I will restrict myself, in order not to exceed the limits of this paper, to a brief statement of some few alterations of rocks which take place under the influence of the solfatara gases, and with which the long series of rock metamorphisms concludes. While the palagonitic metamorphosis gives rise to the formation of hydrated products, in which the relative proportion of the normal pyroxenic mass appears scarcely altered, the pneumatolytic metamorphism is accompanied by a loss of substance, by the decomposed rocks extending in the first instance to the alkalies and alkaline earths, and then to the oxides of iron and the silica. The influence of the fumarole gases, which no volcanic rock is able to withstand, not even the most acid trachyte, may be traced through all the phases of a progressive decomposition in the rocks of the solfatara districts. The first commencement of this action manifests itself as a bleaching of the rock; the dull appearance thus acquired is then followed by a disintegration of the mass, which advances until the rock becomes easily pulverizable; and finally, when the action is completed, there remains a plastic argillaceous mass almost entirely free from iron, which perfectly resists any further change, and after drying presents a very friable character, and gives a bright streak. The characteristic collateral products of

this process, whose special modes of formation I have already fully discussed in another place*, are crystals of iron pyrites, silica in the form of hyalite, hydrated oxide of iron and anhydrous oxide of iron, formed from it by long-continued boiling with water, as well as in some instances carbonate of lime and gypsum.

The springs of the Great Geyser have formed their siliceous incrustations over a surface of palagonitic tuff, which, as I have already mentioned, is penetrated by solfatara gases. From this palagonitic bed, covered with siliceous tuff and boulders, rises the small trachytic chain of mountains called Laugarfjall, which extends along by the side of the springs in a north-eastern direction. The traces of a not inconsiderable geyser action may be followed on the declivities of these hills as far as their trachytic ridge of rocks, and which, with the exception of a few isolated and unimportant springs of vapour, is almost entirely extinct. Below these is *one* spring, issuing from the trachytic rock itself, which it has converted into a white, earthy, friable mass, of a dull appearance, or even into a plastic clay. The following analyses of the original and the decomposed trachyte, show it was especially the alkalies which were extracted from the rock, while water was taken up:—

	102. Undecomposed trachyte.		103. Decomposed trachyte.
Silica	75·48		75·84
Alumina	12·97		13·71
Protoxide of iron	2·61	Peroxide of iron	3·21
Lime	1·01		0·70
Magnesia	0·03		0·14
Potash	5·43		1·24
Soda	2·72		1·94
Water	0·32		2·18
	100·57		98·96

The earthy mass passes finally under the increased action of the fumarole gases into a fat, pliant pipe-clay, in which the iron of the original rock is found in the form of small crystals of iron pyrites, the formation of which, as I have already shown, depends upon a very simple process of decomposition. The com-

* Liebig's *Annalen*, vol. lxi. p. 1.

mencement of this pneumatolytic process is not exclusively limited to the more concentrated evolutions of vapour from the solfataras; on the contrary, it extends not unfrequently beyond the widely-spread masses of trachytic rocks. It is more especially where these rocks penetrate the pyroxenic rocks, or are penetrated by the latter, consequently in the nearest proximity to the foci from whence, as I have shown above, the solfatara gases originate, that all those characteristic indications of the commencement of such fumarole actions make their appearance. The yellowish or bluish gray colour of the trachyte is replaced by a white; the rock assumes a duller appearance; and even if the decomposition does not usually advance so far that a perceptible loss of alkalies manifests itself, still a number of small and chiefly microscopic crystals of iron pyrites and a not inconsiderable per-centage of water may be detected in the rock as characteristic indications of the already commenced action of the solfatara gases. But these indications present themselves much more frequently, and in a more marked manner, in the saalband of the trachytic dykes, where the metamorphoses and abundant formation of iron pyrites in the adjoining rock point out the course which was taken by these gases, whose evolution was the secondary result of the great elevations of trachyte.

The basic palagonites and the pyroxenic rocks are still more readily decomposed under the influence of the heated water and the gases which it contains in solution than the acid trachytic rocks are. The dark substance of the rock assumes in this case also a lighter colour in the first instance, and breaks up into an earthy mass, which gradually becomes richer in water and poorer in alkaline bases and protoxide of iron, until finally it is entirely converted into a white, bluish gray, yellow or red clay, filled with small crystals of sulphur in beds, and not unfrequently containing admixtures of gypsum. It sometimes happens that all the stages of this metamorphism may be observed in one fragment of pyroxenic rock taken from the spot where the solfatara occurs. The frequently quite unaltered nucleus is found to pass towards the exterior into a plastic argillaceous mass, which consists of separate layers of white, gray, yellow, or brownish red colour alternating with each other, sometimes containing iron

pyrites, and sometimes free from it. The recently mentioned palagonitic substance of the tuff surrounding the solfatara of Krísuvík consists of—

	104.	Oxygen found.	Oxygen calculated.
Silica	37·95	20·09	20·60
Alumina . . .	13·61	10·49	10·30
Peroxide of iron	13·75		
Lime	6·48	5·47	5·15
Magnesia . .	7·13		
Potash . . .	0·42		
Soda	1·72		
Water . . .	12·68		
Phosphoric acid	0·43		
Residue . .	7·25		
	101·42		

It may be seen that the composition corresponds almost completely with the formula proposed for palagonite, $3RO\ 2SiO^3 + 2R^2O^3\ SiO^3 + Aq$, and consequently approximates very closely to the composition of the pyroxenic rocks. A fumarole clay formed from this tuff contained nearly 30 per cent. of very finely developed crystals of iron pyrites; and the argillaceous matter separated from them consisted of—

	105.	Oxygen found.	Oxygen calculated.
Silica	49·84	26·38	25·75
Alumina . . .	26·78	12·52	12·88
Lime	0·38	1·89	2·15
Magnesia . .	1·09		
Potash . . .	0·26		
Soda	0·10		
Protoxide of iron	5·73		
Sulphuret of iron	1·53		
Gypsum . . .	0·55		
Water . . .	14·95		
	101·22		

The mixture of which the argillaceous mass consists corresponds with the formula $RO\ 2SiO^3 + 2R^2O^3\ SiO^3 + Aq$. Exactly the same decomposition is observed in the pyroxenic rocks. At the north-eastern foot of the Námarfjall, near Reykjahlidh, there are

a number of large boiling pools of mud, which throw up their blackish gray argillaceous contents to a height of 10 or 15 feet, and heap it up in circular crater-like enclosures. One of these pools still shows at some depth traces of the adjacent pyroxenic lava stream, which bounds the solfataras on the north-east, and whose decomposed substance forms the argillaceous mud which fills the pool. The following analysis of this mud shows that lava rocks suffer the same decomposition as the palagonite when exposed to the action of the solfatara gases :—

	106.
Silica	55·62
Alumina	12·77
Peroxide of iron	1·91
Lime	1·56
Magnesia	0·36
Potash	0·43
Soda	1·18
Water	5·53
Sulphur	0·92
Sulphate of lime	3·45
Iron pyrites	16·27
	100·00

When we find the solfatara action lasting for centuries after the comparatively insignificant eruptions of modern volcanoes, we may certainly expect to meet with traces of similar processes resulting from the far more stupendous catastrophes which gave rise to the formation of the older trachytic and pyroxenic rocks. And in reality these may be observed, with all their characteristic peculiarities, in the plutonic dykes or injected masses occurring in Iceland, and traced from thence under circumstances which do not leave the least possibility of entertaining the remotest idea of their being the results of changes which rocks suffer under the influence of a simple process of extraction by surface water. The pneumatolytic changes, indeed, frequently take place without any removal of constituents from the altered rocks. The pyroxenic dyke masses, passing in their saalbands into a clay mixed with pyrites, carbonate of lime and gypsum, present themselves in a state which does not admit of their being distin-

guished either mineralogically or chemically from those products of decomposition which are observed in the rocks of the solfataras, and in mere appearance they are totally different from those decomposition crusts presented by fissures in the interior of the same rock, and communicating with spring strata, whether the decomposition may be less advanced, or still more so than in these dykes. A basaltic dyke, which has penetrated through the adjoining older trap on the north-eastern coast of Vidhey in the harbour of Reykjavík, presents a fine example of this kind. The trap rock contains a considerable per-centage of water, and possesses exactly the composition of the normal pyroxenic mass. Where it is in contact with the basalt dyke, a crust resembling trachyte is observable, which has a pitchy-black appearance, conchoidal fracture, and glassy lustre. Dried in the air at the ordinary temperature, it contains only 0·84 per cent. of water, and presents entirely the appearance of a fresh obsidian-like glass. The following analysis shows that this crust is actually nothing more than melted normal pyroxenic substance, having the same composition as other trap, from the lateral fusion and rapid cooling of which it has originated:—

	107.
Silica	47·58
Protoxide of iron	17·51
Alumina	13·78
Lime	11·36
Magnesia	6·48
Soda	2·90
Potash	0·60
Water	0·84
	101·05

The blackish gray substance of the basalt dyke itself has a dull earthy appearance, contains pyrites, together with calcareous spar disseminated through its compact mass, and resembles in the most deceptive manner the rocky nuclei which are sometimes found in imbedded pyroxenic masses which have been already converted by lying in the solfataras into pyritic clay on their surface, while in the centre the decomposition is only in its initial stage. The composition of this altered basaltic mass gives

a full explanation of the nature of the metamorphosis which has taken place.

	108.
Silica	46·47
Alumina	14·71
Protoxide of iron	14·29
Lime	8·04
Magnesia	4·98
Soda	1·53
Potash	0·87
Carbonate of lime	5·35
Pyrites	1·04
Water	3·58
Gypsum	trace
	100·86

We here observe the remarkable fact, that when the per-centage of lime and protoxide of iron, which has suffered from the metamorphic action, is again added to the original silicate, we obtain almost exactly the composition of the normal pyroxenic substance, thus:—

	109.	
Silica	49·17	
Alumina	15·57	31·35
Protoxide of iron	15·78	
Lime	11·68	
Magnesia	5·27	
Soda	1·62	
Potash	0·92	
	100·00	

The composition, which, as in the normal pyroxenic substance, corresponds to the relation between the quantities of oxygen in the acid and bases of 3 : 1·936, proves that the change which has taken place in the rock cannot have been connected with any sensible extraction, and that consequently it was not currents of water, but the gases and vapours dissolved in the water which attacked and caused the metamorphism of the rock. The remarkable trachyte dyke at the south-eastern foot of the Esja-mountains, whose interfusion with the adjoining pyroxenic rock I

have already described above, has exercised precisely the same influence upon the surrounding rock, which has thus for some considerable distance round been converted into a pitchy black mass, nearly resembling obsidian, sometimes dull and sometimes almost brilliant in appearance, containing calcareous spar and zeolites intimately disseminated throughout, and which, to judge from its zeolitic admixture and frequently still recognizable imbedded earthy fragments of pyroxenic rock, is the product of the metamorphosis of a palagonitic tuff. Analysis gave the following mean composition for this rock:—

	110.
Silica	47·47
Alumina	11·85
Protoxide of iron	15·24
Lime	5·76
Magnesia	7·17
Soda	1·93
Potash	0·32
Carbonate of lime	8·45
Water	2·61

The absence of pyrites and gypsum proves that it was carbonic acid alone which caused the metamorphism in the moistened rock. Here likewise, if the per-centage of lime contained in the original rock is restored from the disseminated calcareous spar, we obtain as the composition of the anhydrous rock,—

	111.
Silica	50·25
Alumina	12·54
Protoxide of iron	16·15
Lime	11·09
Magnesia	7·59
Soda	2·04
Potash	0·34
	100·00

By this means we again obtain a composition for the original rock, which is almost exactly the same as that of the normal pyroxenic substance, and indeed with the oxygen relation 3 : 1·81,

which approximates remarkably near to the corresponding relation in palagonite, 3 : 1·95. The palagonite rock of Laugarvatnshellir possesses almost exactly the same composition, calculated for anhydrous substance, as this rock, which is so totally different from it in all its characters :—

	112.
Silica	50·71
Alumina	13·55
Protoxide of iron	15·44
Lime	10·75
Magnesia	7·98
Soda	0·76
Potash	0·81
	100·00

If this carbonate of lime, amounting to more than 8 per cent., had been deposited by percolating calcareous water, and did not result from the penetration of carbonic acid into the moist rock, it would be inconceivable why the per-centage of lime in the disseminated calcareous spar should give almost exactly the composition of the original rock; and further, why such an enormous process of extraction should have failed to exercise any influence upon the so readily decomposable alkalies; and finally, how such a deposition of lime could have taken place at all in the compact rock, entirely destitute of cavities. Without attempting here to follow up these relations more closely, it may suffice to bring forward a few more examples of these fumarole actions which are frequently met with in Iceland, without any connexion with the course of the spring strata, and commencing from the plutonic dykes spread out into the surrounding rocks.

At the spot where, on the road from Hruni to Stórinupr, the Laxa is first met with, there rises on the south-eastern bank of this river the trachytic precipice Arnarhnipa, which is traversed by a pyroxenic dyke, whose edges are converted by fumarole action into a soft lavender-blue clay containing pyrites and carbonate of lime, and possessing the following composition :—

	113.
Silica	47·05
Alumina	10·91
Protoxide of iron	12·66
Lime	11·79
Magnesia	7·73
Soda	1·23
Potash	0·75
Carbonate of lime.	1·01
Pyrites	0·20
Water	6·67
Gypsum	trace
	100·00

On account of the unequal distribution of the pyrites in layers, it is not possible to compare this product of decomposition with the original rock from which it is derived. It perfectly resembles, not only in its composition, but likewise in its natural appearance, a rock decomposed by solfatara action, while it does not present the slightest similarity to the decomposed mass resulting from the mere action of water upon the fissures of pyroxenic rocks. Further towards the centre of the dyke the rock is more difficult of fracture, has a less decomposed appearance, and is impregnated throughout with calcareous spar. The composition of this substance was found to be—

	114.
Silica	50·82
Alumina	11·10
Protoxide of iron	12·97
Lime	4·34
Magnesia	3·90
Soda	1·93
Potash	0·31
Carbonate of lime	8·16
Pyrites	0·26
Water	5·05
Gypsum	trace
	100·00

In this instance, likewise, we meet with the remarkable fact that the metamorphic substance of the dyke has not lost any of

its constituents by extraction, and that the calcareous spar and pyrites present in it have not been introduced from outwards, but were formed by simple re-arrangement of the constituents upon the spot. For if the radicals contained in the calcareous spar and the pyrites are restored to the rock as lime and protoxide of iron, we obtain the composition of a tracheo-pyroxenic rock perfectly corresponding with theory:—

	115. Found.	Calculated.
Silica	56·48	56·48
Alumina and protoxide of iron	26·62	25·65
Lime	9·78	8·91
Magnesia	4·33	4·99
Potash	0·34	1·38
Soda	2·15	2·59
	100·00	100·00

This constitution corresponds to a mixture of 1 part trachyte and 2·521 pyroxenic substance. The penetrated rock has therefore here also mixed with the penetrating mass. These examples, to which I could add a great number of others, may suffice to explain the fact, that the gases penetrating the rocks together with water and aqueous vapour, are capable of metamorphosing their substance on the spot without any removal of the products of decomposition.

It only remains for me at the close of this general view, to mention the phænomena which present themselves where the pneumatolytic and zeolitic metamorphoses take place together. The result of this state of things is the production of argillaceous amygdaloid conglomerates, extremely rich in zeolites, and which alternate in irregular order as thick beds or masses with the pyroxenic eruptive rocks. They may be regarded as pyrocaustic products of an immense subterranean fumarole action, in which sulphuretted hydrogen or sulphurous acid were less active than aqueous vapour, either alone or mixed with carbonic acid. The already-mentioned districts round Silfrastadir are especially remarkable in this respect. This small spot is situated in the valley of the Héradsvötn, which opens into that of the Skagafjördhr, and is enclosed by rocky precipices, consisting of

a compact trap rock alternating with zeolitic, amygdaloid and palagonitic tuff. The trap, which belongs to the gray coarse-grained variety occurring all over Iceland, passes very gradually into a tough, bluish-gray, zeolitic amygdaloid, presenting an almost earthy appearance, and whose substance consists, to the amount of almost one-third, of crystal druses and rough masses of chabasite. No perceptible line of separation between the two rocks can be detected in the fissures and imbedded masses. When the homogeneous mixture of this amygdaloid admits of an average composition being ascertained by analysis, it is found to have exactly the constitution of the trap into which it passes; and it follows from the accompanying analyses, calculated for anhydrous substance, that both rocks consist of pure normal pyroxenic or palagonitic substance.

	116. Trap.	117. Amygdaloid.
Silica	49·87	49·60
Alumina	14·66	13·98
Protoxide of iron . .	13·57	14·60
Lime	12·56	11·78
Magnesia	6·55	6·90
Potash	0·42	0·22
Soda	2·37	2·92
	100·00	100·00

An equally gradual transition may be observed from the amygdaloid into a red, friable, argillaceous bed, only a few feet thick, but extending for miles round, the substance of which, under the microscope, appears as an altered amygdaloid, in which the unaltered chabasite is found with all its characteristic peculiarities of distribution and concretion, surrounded by the decomposed rock. The succeeding beds of palagonitic tuff are again connected by gradual transitions throughout all the phases of a progressive decomposition in the most intimate manner with the above argillaceous bed, in such a manner, indeed, that separate concretions may be traced from the solid trap rock down into the unaltered beds of tuff. This metamorphosis, which may be imitated on a small scale with any piece of palagonite, has here been effected on a large scale in Nature, at the time when the erupted melted trap caused the fusion of the underlying pala-

gonitic beds, and by a vaporization of water gave rise to the formation of the red pneumatolytic amygdaloid. Here, likewise, we find the metamorphism commencing as a physical necessity from the melted trap masses terminating in a fumarole action, whose products most perfectly resemble those of the solfatara localities. These phænomena present themselves very frequently in Iceland, and still more frequently in the palagonitic trap formations of the Faroë Islands, so distinguished for their abundance of zeolites.

It follows, with the most indisputable necessity, from these last-mentioned facts, that this formation of zeolites does not depend upon a removal and deposition of substances, but results solely from a transformation of palagonitic rocks taking place upon the spot.

Article III.

On the Dependence of Radiant Heat in its passage throu gh Crystals upon the direction of transmission. *By* H. Knoblauch, *Professor of Natural Philosophy in the University of Marburg.*

[From Poggendorff's *Annalen*, 1852, No. 2.]

THE quantity of radiant heat which passes through diathermanous bodies, depends, as is known, upon the nature of the latter. Melloni was the first to raise the question, whether, *in one and the same body—in a crystal for instance—the quantity transmitted was different along the different axes.* In the results arrived at by Melloni and myself while experimenting with transparent rock-crystal, and with calcareous spar, no difference of the kind was detected.

Notwithstanding this, I have been induced to extend the inquiry to other crystals. A *brown specimen of quartz* was first subjected to experiment. It was a complete cube, having four of its sides parallel to the axis of the crystal, and the remaining two perpendicular to the same. The sun's rays appeared to be peculiarly well adapted to experiments of this nature, inasmuch as they offer the double advantage of being parallel among themselves and of possessing the highest possible intensity.

By means of the reflector of a heliostat, a sheaf of rays was thrown into a dark room; the direction of the sheaf, notwithstanding the changing position of the sun, did not materially alter during the time of observation. The rock-crystal was placed in the path of the rays, being fixed upon a disc which permitted of a rotation of 90° in a horizontal plane. The heat, after having passed through the cube, was permitted to fall upon the square forward surface of a thermo-electric pile, and the consequent excitation was measured by a multiplier connected with the pile. It is scarcely necessary to remark, that all incidental heat was carefully shut out, and those rays only were allowed to fall upon the pile which had first traversed the mass of the crystal.

Under these circumstances, it was observed that the calorific rays, when transmitted parallel to the axis of the crystal, produced a deflection of the needle of the multiplying galvanometer amounting to 34°·05*. When, however, the rays were transmitted in a line perpendicular to the axis of the crystal (the latter having been turned 90° round), the deflection produced was only 31°·19.

The ratio of these numbers, which, as in most of the cases hereafter cited, are the arithmetic means of twelve different observations, is as 100 : 92.

The quantity of heat, therefore, transmitted by a brown rock-crystal, in a line perpendicular to its axis, is less than the quantity transmitted in a direction parallel to the same.

There is no reason whatever to suppose that this difference is in any way attributable to accidental imperfections on the part of the crystal.

The same experiment was made with a cube of *beryl*; here also a weaker power of transmission was exhibited perpendicular to the axis than parallel to the same.

If, as in the former case, we express by 100 the quantity of heat transmitted parallel to the axis of the beryl, then the quantity transmitted perpendicular to the axis, as measured by the multiplier, will be expressed by the number 54. This result

* When accurate determinations with the thermo-electric pile are required, it is not sufficient to simply read off the deflection; for this is not due to the temperature which is to be measured alone, but also to accidental differences of temperature between both faces of the pile. The amount contributed by the latter source to the total effect must therefore be ascertained; a metallic screen is placed in the path of the rays which totally intercepts them, without altering the accidental difference spoken of. The deviation of the needle which remains, indicates the true zero, from which we must reckon in the subsequent determination. If, for example, the total deflection were 35°·05, and the needle, after the introduction of the metallic screen, remained standing at 1°, the action to be attributed to the incident rays would amount to 34°·05.

All the values contained in the present memoir have been determined in this way. Each of the numbers whose arithmetic mean we have taken is thus the result of two readings of the thermo-multiplier, one of which was simply to ascertain the true zero for the observation made at the time.

The deflections of the needle are to be considered as the measure of the quantity of heat as within the limit of 35° (according to a special examination of the instrument); the deflections were proportional to the forces which produce them.

does not however possess the same demonstrative value as that obtained in the case of rock-crystal, inasmuch as the mass of the beryl was in some measure unhomogeneous.

A cube of *tourmaline* cut, as regards its axis, exactly as the former, and subjected to the same conditions of experiment, showed an opposite deportment. In this case the quantity of heat transmitted perpendicular to the axis was greater than the quantity transmitted parallel to it.

The deflections of the galvanometer by which these quantities were measured are to each other in the proportion of 100 : 158, the former number denoting the quantity transmitted along the axis, and the latter the quantity transmitted in a line perpendicular to the same. The homogeneous character of the specimen experimented with, does not permit of the idea that the observed difference is due to any accidental irregularity of the mass of the crystal.

The foregoing experiments, therefore, prove *that, in certain crystals, radiant heat is transmitted in unequal quantities in different directions.*

It seemed to me of interest to pursue this inquiry further, by applying *polarized* calorific rays* instead of the natural ones. For this purpose the rays were caused to pass through a Nichol's prism, previous to falling upon the cube of rock-crystal. The prism was first so placed that the principal section, passing through its obtuse angles, was vertical, and hence the plane of polarization of the issuing rays horizontal.

When the calorific rays thus polarized were permitted to traverse the cube in the line of the crystallographic axis, a deflection of 8°·1 was produced. When the line of transmission was perpendicular to the axis, the deflection was 8°·3; the difference between these two values lies within the limits of the error of observation, and hence they must be regarded as equal.

Hence when heat, polarized in the manner indicated, is ap-

* Strictly speaking, the heat as applied in the foregoing cases was not in a natural state, but polarized elliptically by reflexion from the steel mirror of the heliostat.

But the peculiarity communicated in this way (the angle of incidence being so small), compared with the linear polarization to which our experiments refer, is of so vanishing a character, that, as far as our results are concerned, the light may be regarded as being in its natural state.

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plied, the differences observed in the case of the natural rays disappear.

The deportment of beryl was similar to that of rock-crystal. True, in the case of beryl a greater quantity was transmitted along the crystallographic axis than perpendicular to it; but the proportion, which in the case of the natural rays was as 54 : 100, became, when polarized heat was applied, as 70 : 100. The difference between these two numbers is to be ascribed to the irregularity of the mass already alluded to.

In the case of tourmaline, no difference was observable in the quantity of heat transmitted. The thermoscope exhibited the same deflection, whether the rays were transmitted parallel to the axis or perpendicular to it.

From this we may conclude, *that polarized calorific rays, whose plane of polarization coincides with the axes of the crystals mentioned, are transmitted in equal proportions in all directions through the mass.*

The Nichol's prism was next turned so that the plane of its principal section lay horizontal, the rays passing through it being therefore polarized in a vertical plane.

The deflection produced by these rays, after having traversed the brown rock-crystal in a line parallel to its axis, was $9^{\circ}{\cdot}68$. When however the line of transmission was perpendicular to the axis, the observed deflection was only $7^{\circ}{\cdot}03$,—the ratio of these numbers is that of 100 : 73.

Hence the difference already noticed with the unpolarized rays, not only exhibits itself here once more, but is shown in an increased degree. In the case of beryl, the ratio of the quantity transmitted, under the same conditions, parallel to the axis to that transmitted perpendicular to the same, was as 100 : 21. In the case of tourmaline, the proportion was as 100 : 219. The following table contains the results of these and of the former observations :—

Name of crystal.	Heat polarized in horizontal plane.		Natural rays.		Heat polarized in vertical plane.	
	Proportionate quantities of heat transmitted through the crystal					
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Brown rock-crystal..	100	100	100	92	100	73
Beryl	100	70*	100	54	100	21
Tourmaline	100	100	100	158	100	219

* Difference due to the accidental irregularities of the specimen examined.

A comparison of these numbers leads us to the conclusion, *that the differences between the quantities transmitted are greatest when the heat is polarized in the manner last described, less in the case of the natural rays, and vanish altogether when the rays are polarized in the manner first described.*

The essential difference between the two cases where polarized heat was applied is to be traced to the fact, that in the first case the plane of polarization coincides with the axis of the crystal, whereas in the second case the axis was perpendicular to the plane of polarization. The question now arises, whether differences exist in the quantities transmitted *along the axis, when the heat is polarized in different planes.* To solve this point the crystal was placed with its axis in the line of transmission, the principal section of the Nichol was set successively vertical and horizontal, and the quantities of heat transmitted in both cases, respectively, were measured and compared. No difference which would permit of the conclusion that the quantity transmitted along the axis was at all affected by the position of the plane of polarization, was observed*.

When the transmissive power in *different directions perpendicular to the axis of the crystal* was determined, no difference was perceptible, it mattered not whether the light was in its natural or in a polarized condition. For all these directions the same result was obtained as that already communicated for one of them.

The foregoing results rendered another question of interest to me, and that is, whether the calorific rays whose *quantitative* differences were exhibited in the foregoing experiments, have also *qualitative* differences impressed upon them by passing, in the one or the other direction, through the crystals above-mentioned†.

* Slight differences in the thermoscopic results were sometimes observed according as the principal section of the Nichol was vertical or horizontal; but this was solely due to the slight disturbing influence caused by the elliptic polarization of the rays already noticed. Even without the rock-crystal, beryl, or tourmaline, these differences showed themselves when the Nichol was turned; and the ratios of the observed values were not in the least altered by the introduction of the crystal in the manner indicated,—a result established in the most decided manner by exact measurements in the case of each.

† In my earlier experiments with transparent calcareous spar no such difference was observable.—Pogg. *Ann.* vol. lxxiv. pp. 185, 186.

To ascertain this, I made use of a process which on former occasions enabled me to detect very small differences in the properties of radiant heat. The principle of the procedure consists simply in ascertaining whether the calorific rays whose qualities were to be compared, possessed the same power of transmission through diathermanous substances.

The experiments commenced, as before, with *brown rock-crystal*. The deflection produced by the heat which traversed the crystal parallel to its axis, was first observed and found to be 34°·71. A diathermanous body, a plate of blue glass for example, was then introduced between the crystal and the pile; a portion of the heat was thus held back, the pile being excited by those rays only which had passed through the blue glass. The deflection became reduced in consequence to 17°·19. Hence the quantity of heat transmitted through the crystal and falling on the forward surfaces of the glass, is to the quantity which emanates from the latter in the proportion of 34°·71 : 17°·19, or of 100 : 51.

The calorific rays were now permitted to traverse the crystal in a line perpendicular to its axis. Their direct action upon the thermoscope produced a deflection of 33°·58. When the blue glass was introduced the deflection was reduced to 17°·21. According to this, the quantity of heat falling on the forward surface of the glass, is to the quantity transmitted through it in the ratio of 33·58 : 17·21, or as 100 : 51, exactly as in the former case.

In the same manner the influence of other diathermanous bodies, of yellow, red and green glass, was ascertained. The results of these experiments are contained in the following table; the quantity of heat transmitted through the rock-crystal and falling upon the diathermanous plate is, as before, expressed by 100, and each number is the mean of about six observations.

Diathermanous bodies.	Ratio of the quantity of heat which falls upon the diathermanous body to the quantity which passes through the latter, the heat having been first transmitted through rock-crystal	
	Parallel to the crystallographic axis.	Perpendicular to the crystallographic axis.
Blue glass	100 : 51	100 : 51
Yellow glass	100 : 79	100 : 81
Red glass.....	100 : 65	100 : 64
Green glass	100 : 14	100 : 13

In none of these cases does the observed difference exceed

what may be attributed to errors of observation. It remained thus doubtful whether the calorific rays which penetrate the rock-crystal parallel and perpendicular to the axis are to be regarded as qualitatively alike, or whether their differences had not merely escaped observation.

For the further examination of this point, I made use of heat similarly polarized to that which in the former experiments exhibited the greatest quantitative differences.

The Nichol's prism was set in the path of the rays with its principal section horizontal, and the heat, thus polarized in a vertical plane, was permitted to traverse the crystal. Having observed the deflection produced under these conditions, one of the glass plates already made use of was introduced, as before, between the rock-crystal and the thermic pile, and the new deflection was observed. The question to be decided was, whether the heat, under these circumstances, would exhibit any essential differences as to its capability of transmission through the before-named diathermanous substances, according as the line of transmission through the crystal was parallel or perpendicular to its axis. The following table exhibits the results of the observations. Here, as in the former instances, the ratio of the quantity of heat falling on the diathermanous body to the quantity transmitted through it, is given; the former being expressed by 100. Every number is the result of about thirteen observations.

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a vertical plane, falls upon the diathermanous body, to that which passes through the latter, the incident heat having first passed through a cube of rock-crystal	
	Parallel to the horizontal crystallographic axis.	Perpendicular
Blue glass	100 : 40	100 : 39
Yellow glass	100 : 73	100 : 78
Red glass	100 : 42	100 : 45
Green glass	100 : 7	100 : 11

Here, comparing the first series of ratios with the second, differences are observed which lie without the limits of error due to observation. Hence *calorific rays which have passed through the brown rock-crystal in different directions possess, in different degrees, the power of penetrating one and the same diathermanous body.*

Thus, for example, the heat which has passed through the rock-crystal perpendicular to the axis, passes in greater quantity through the green glass than that which has traversed the crystal

parallel to its axis; for while in the former case 11 out of every 100 incident rays are transmitted, in the latter case only 7 per cent. pass through.

It is thus proved that the rays in question *differ from each other in respect to quality.*

On the other hand, no perceptible difference is exhibited when the heat, instead of being polarized in a vertical plane, is polarized in a horizontal one.

Experiments conducted exactly as the foregoing, and differing only in the circumstance that the principal section of the Nichol instead of being horizontal was vertical, gave the following values, each of which is the result of seven observations:—

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a horizontal plane, falls upon the diathermanous body, to that which passes through the latter, the incident heat having first passed through a cube of rock-crystal	
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Blue glass	100 : 43	100 : 42
Yellow glass	100 : 77	100 : 77
Red glass.....	100 : 50	100 : 50
Green glass	100 : 10	100 : 9*

The numbers of the two columns differ only within the limits of the error of observation. Hence we conclude *that calorific rays whose plane of polarization is parallel to the axis of the rock-crystal are homogeneous, whatever be the direction in which they have traversed the crystal.*

Just as little do differences in the properties of the calorific rays which have traversed the crystal parallel to its axis exhibit themselves, whether the said rays have been polarized in a vertical or in a horizontal plane.

This is proved by the following table; the values contained in it are each the result of four single determinations.

Diathermanous bodies.	Ratio of the quantity of heat which falls upon a diathermanous body, to that which passes through the latter, being first transmitted along the axis of the rock-crystal	
	Polarized in vertical plane.	Polarized in horizontal plane.
Blue glass	100 : 40	100 : 40
Yellow glass	100 : 71	100 : 71
Red glass.....	100 : 53	100 : 53
Green glass	100 : 10	100 : 10

* The different tables cannot be compared in every respect with each other, as they are the results of observations made on different days, on which it was impossible to preserve external conditions perfectly constant.

The equality of these ratios proves that the rays in both cases possess the same capability of transmission through the same diathermanous substance.

As long as the direction of the rays was perpendicular to the axis of the crystal, there was no difference observed, it mattered not how the direction was otherwise altered. The result true for one of these directions, and mentioned in its proper place, was true for all of them.

Beryl was next submitted to precisely the same examination as that carried out in the case of rock-crystal.

In the first instance, I permitted the natural rays to traverse the crystal parallel and perpendicular to the axis, and tested their deportment in the manner already described.

After the full description given of the mode of experiment with rock-crystal, no obscurity can exist about it in the present instance. From the observations with beryl, we obtain the following values, each of which is the mean of five distinct observations:—

Diathermanous bodies.	Ratio of the quantity of heat which falls upon the diathermanous body, to that which passes through it, the heat having previously traversed a cube of beryl	
	Parallel to the crystallographic axis.	Perpendicular to the crystallographic axis.
Blue glass	100 : 28	100 : 29
Yellow glass	100 : 56	100 : 50

From these numbers we draw the inference, *that the heat which has passed through the beryl in the direction of its axis is transmitted through a diathermanous body in a different proportion from that which has traversed the crystal perpendicular to its axis.*

For, of the first-mentioned class of rays, 56 out of 100 are transmitted through the yellow glass, while of the latter class only 50 rays out of 100 pass through the same medium. The calorific rays, in this instance, are therefore to be regarded as *qualitatively unlike.*

The difference exhibited itself in a still more striking manner, where the heat, before it entered the crystal (the axis of which, as before, was horizontal), had been polarized in a vertical plane. The following table contains the results of this observation:—

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a vertical plane, falls upon the diathermanous body, to that which passes through the latter, the heat having previously traversed a cube of beryl	
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Blue glass	100 : 39	100 : 50
Yellow glass	100 : 54	100 : 30

In this instance, the difference, as regards the transmissive power of both groups of rays, exhibits itself very distinctly. One group (that parallel to the axis of the crystal) is transmitted in less quantity through the blue glass than the other, while the latter is transmitted through the yellow glass more sparingly than the former. The differences in the present case are, in themselves, greater than when unpolarized heat was made use of. Thus, in the case of unpolarized heat, out of 100 rays incident on the yellow glass, 56 were in one position transmitted, and 50 in the other position; but in the present case the numbers corresponding to these positions are 54 and 30.

All differences of this kind again disappear when the heat, instead of being polarized in a vertical plane, is polarized in a horizontal one. This is manifest from the following numbers, which differ from each other, in both compartments, only within the limits of the error of observation.

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a horizontal plane, falls upon the diathermanous body, to that which passes through the latter, the heat having previously traversed the beryl	
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Blue glass	100 : 43	100 : 43
Yellow glass	100 : 63	100 : 61

Hence in the case of beryl also, we find that the *calorific rays whose plane of polarization coincides with the axis of the crystal are homogeneous, although they may have passed through the crystal in different directions.*

The same similarity of deportment is exhibited by all rays which have traversed the crystal parallel to its axis, whatever be their plane of polarization. The following table contains the observations which refer to this point:—

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a vertical / horizontal plane, falls upon the diathermanous body, to that which passes through the latter, having previously traversed the beryl along its horizontal axis.	
Blue glass	100 : 35	100 : 35
Yellow glass	100 : 42	100 : 40

Those rays which pass through the beryl perpendicular to the axis exhibit no differences, however the direction may be varied in other respects. The difference of deportment between such rays and those which pass along the axis, has already been pointed out.

To the experiments heretofore described, another was added, by which the results already arrived at were partly corroborated and in part expanded. *Two cubes of beryl,* both cut in the manner already indicated, were so placed one after the other, that in one instance their axes were parallel and in another perpendicular to each other. In the former case both axes were vertical, in the latter one was vertical and the other horizontal. The horizontal calorific rays proceeding from the heliostat, traversed both crystals successively, forming an angle of 90° with each of the axes. Behind these cubes diathermanous bodies were introduced, as before, in order to ascertain the transmissive power of the rays which had passed through the cubes. The following table exhibits the ratios in which they passed through the blue or yellow glass. The rays incident upon the plates are, as in the former instances, set equal to 100, and to this standard the quantity transmitted is referred. Every number is the arithmetic mean of eight distinct observations.

Diathermanous bodies.	Ratio of the quantity of heat which falls upon the diathermanous body, to that which passes through the latter, having previously traversed both the cubes of beryl with their axes	
	Parallel.	Perpendicular.
Blue glass	100 : 7	100 : 20
Yellow glass	100 : 26	100 : 10

The differences here exhibited are too considerable to need particular discussion. *Thus in this example also we find essential differences between those calorific rays which are permitted to pass through the cubes with their axes parallel, and those which traverse the cubes when the axes are perpendicular to each other.*

The examination of *tourmaline* led to the same results as those observed with the other crystals. The results of the observations with natural rays are contained in the following table: each number is the mean of five distinct determinations:—

Diathermanous bodies.	Ratio of the quantity of heat which falls upon the diathermanous body, to that which passes through the same, having previously passed through the cube of tourmaline	
	Parallel to the crystallographic axis.	Perpendicular to the crystallographic axis.
Red glass	100 : 62	100 : 43
Green glass	100 : 29	100 : 36

The comparison of both series shows plainly *in how different degrees the rays which pass parallel to the axis, and those which pass perpendicular thereto, penetrate* the *same* diathermanous body. The former rays pass through the red glass in greater quantity than the latter; while, in the case of green glass, the latter rays have the preponderance.

In the same manner, but in an increased degree, these differences show themselves when the calorific rays *are polarized in a vertical plane and transmitted through the tourmaline with its axis horizontal.* The following greatly divergent values place this beyond all doubt:—

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a vertical plane, falls upon the diathermanous body, to that which passes through the latter, having previously traversed the tourmaline	
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Red glass	100 : 14	100 : 6
Green glass	100 : 19	100 : 41

The differences disappear in this case also *when the plane of polarization is horizontal, that is, when it is parallel to the crystallographic axis.* This is shown by the following numbers, whose differences lie within the limits of the error of observation:—

Diathermanous bodies.	Ratio of the quantity of heat which, polarized in a horizontal plane, falls upon the diathermanous body, to that which passes through the latter, having previously traversed the tourmaline	
	Parallel to the horizontal crystallographic axis.	Perpendicular to the horizontal crystallographic axis.
Red glass	100 : 19	100 : 16
Green glass	100 : 9	100 : 10

Just as little are the calorific rays which pass *along the axis of the tourmaline* distinguishable from each other, whatever may have been their plane of polarization. *Nor are differences observed among those which pass perpendicular to the axis, whatever be their direction in other respects.* But these two groups, when compared with each other, exhibit, as already shown, striking qualitative differences.

Experiments were next conducted with two *plates of tourmaline*, through which the calorific rays were permitted to pass, first when the axes of the plates were parallel, and afterwards with the axes perpendicular to each other. The following numbers exhibit the proportions in which the rays, in both cases, respectively, pass a diathermanous body. Each number is the arithmetic mean of eight distinct determinations:—

Diathermanous bodies.	Ratio of the quantity of heat which falls upon the diathermanous body, to that which passes through the latter, having previously traversed the tourmaline with their axes	
	Parallel.	Perpendicular.
Red glass	100 : 30	100 : 42
Green glass	100 : 19	100 : 11

The comparison of both series permits no doubt to exist as to the different capacities of transmission of both groups of rays. They are therefore to be regarded as *unlike*.

Besides calcareous spar, rock-crystal, beryl and tourmaline, *dichroite* was examined: so far as the examination of this crystal extends, qualitative differences, dependent upon the direction of transmission, have also been observed.

The principal results of this investigation may be thus summed up:—

1. Radiant heat passes through certain crystals, as brown rock-crystal, beryl, tourmaline and dichroite, in unequal quantities in different directions, and shows itself (for example in its deportment towards diathermanous bodies) to be of a different quality, according as it has traversed the crystal in one or another direction. These differences are connected with the polarization of the calorific rays, and in connexion with this point we find—

2. That calorific rays whose direction is perpendicular to the

axis of the brown rock-crystal, beryl or tourmaline, when their plane of polarization forms an angle of 90° with the axis of the crystal, are transmitted in quite different proportions from those observed when the line of transmission is parallel to the axis.

The rays traverse the crystal in all directions alike when their plane of polarization coincides with the axis of the crystal.

3. In the first case the greatest qualitative differences manifest themselves, in the second case no such differences have an existence.

4. When the rays are transmitted along the axis, no differences are observed either as regards quality or quantity, whatever may be the position of the plane of polarization.

5. Compared among themselves, the different directions perpendicular to the axis exhibit no differences of transmission in the three crystals above-mentioned.

When we reflect that (according to former investigations) the natural calorific rays, on passing through the crystals perpendicular to their axes, are decomposed by double refraction into two groups, and so polarized that the plane of polarization of the one group coincides with the axis of the crystal, while that of the other group is perpendicular to the same; further, that this does not take place when the direction of transmission is along the axis; this, in connexion with what has gone before, explains the deportment observed in the case of the natural calorific rays.

Those rays which proceed parallel to the axis of the crystal undergo a certain absorption which depends upon the nature of the crystals, and determines the quantity and character of the transmitted rays,—determines, for instance, their deportment towards diathermanous bodies. In passing perpendicular to the axis of the crystal, the two groups of rays are absorbed in different proportions. Those whose plane of polarization coincides with the crystallographic axis suffer the same absorption as the rays which pass along the axis; the other rays, however, will be absorbed either in a greater degree, as in the case of brown rock-crystal and beryl, or in a less degree, as in the case of tourmaline. Further, as this absorption is elective, that is to say, directed in different degrees towards the different rays of one and the same group, the composition of such a group will be modified, and hence its capacity to penetrate a diathermanous body will be

altered. In this respect also the rays whose plane of polarization coincides with the axis are not to be distinguished from those which pass parallel to the axis. The others however exhibit essential differences. If the two groups of rays remain unseparated, their total action being alone examined, as in the experiments with the natural rays, we must arrive at results which, as regards quantity and quality, lie between those obtained from observations made with the separate polarized groups, where in one instance no difference, and in another the maximum difference is exhibited. The values above quoted furnish abundant proof of this. [J. T.]

Article IV.

On the Conservation of Force; a Physical Memoir.

By Dr. H. Helmholtz.

[Read before the Physical Society of Berlin on the 23rd of July, 1847. Berlin. G. Reimer.]

Contents.

Introduction.
I. The principle of the Conservation of *vis viva*.
II. The principle of the Conservation of Force.
III. The application of the principle in Mechanical Theorems.
IV. The Force-equivalent of Heat.
V. The Force-equivalent of the Electric Processes.
VI. The Force-equivalent of Magnetism and Electro-magnetism.

Introduction.

THE principal contents of the present memoir show it to be addressed to physicists chiefly, and I have therefore thought it judicious to lay down its fundamental principles purely in the form of a physical premise, and independent of metaphysical considerations,—to develope the consequences of these principles, and to submit them to a comparison with what experience has established in the various branches of physics. The deduction of the propositions contained in the memoir may be based on either of two maxims; either on the maxim that it is not possible by any combination whatever of natural bodies to derive an unlimited amount of mechanical force, or on the assumption that all actions in nature can be ultimately referred to attractive or repulsive forces, the intensity of which depends solely upon the distances between the points by which the forces are exerted. That both these propositions are identical is shown at the commencement of the memoir itself. Meanwhile the important bearing which they have upon the final aim of the physical sciences may with propriety be made the subject of a special introduction.

The problem of the sciences just alluded to is, in the first place, to seek the laws by which the particular processes of

nature may be referred to, and deduced from, general rules. These rules,—for example, the law of the reflexion and refraction of light, the law of Mariotte and Gay-Lussac regarding the volumes of gases,—are evidently nothing more than general ideas by which the various phænomena which belong to them are connected together. The finding out of these is the office of the experimental portion of our science. The theoretic portion seeks, on the contrary, to evolve the unknown causes of the processes from the visible actions which they present; it seeks to comprehend these processes according to the laws of causality. We are justified, and indeed impelled in this proceeding, by the conviction that every change in nature *must* have a sufficient cause. The proximate causes to which we refer phænomena may, in themselves, be either variable or invariable; in the former case the above conviction impels us to seek for causes to account for the change, and thus we proceed until we at length arrive at final causes which are unchangeable, and which therefore must, in all cases where the exterior conditions are the same, produce the same invariable effects. The final aim of the theoretic natural sciences is therefore to discover the ultimate and unchangeable causes of natural phænomena. Whether all the processes of nature be actually referrible to such,—whether nature is capable of being completely comprehended, or whether changes occur which are not subject to the laws of necessary causation, but spring from spontaneity or freedom, this is not the place to decide; it is at all events clear that the science whose object it is to comprehend nature must proceed from the assumption that it is comprehensible, and in accordance with this assumption investigate and conclude until, perhaps, she is at length admonished by irrefragable facts that there are limits beyond which she cannot proceed.

Science regards the phænomena of the exterior world according to two processes of abstraction: in the first place it looks upon them as simple existences, without regard to their action upon our organs of sense or upon each other; in this aspect they are named *matter*. The existence of matter in itself is to us something tranquil and devoid of action: in it we distinguish merely the relations of space and of quantity (mass), which is assumed to be eternally unchangeable. To matter, thus regarded, we

must not ascribe qualitative differences, for when we speak of different kinds of matter we refer to differences of action, that is, to differences in the forces of matter. Matter in itself can therefore partake of one change only,—a change which has reference to space, that is, motion. Natural objects are not, however, thus passive; in fact we come to a knowledge of their existence solely from their actions upon our organs of sense, and infer from these actions a something which acts. When, therefore, we wish to make actual application of our idea of matter, we can only do it by means of a second abstraction, and ascribe to it properties which in the first case were excluded from our idea, namely the capability of producing effects, or, in other words, of exerting force. It is evident that in the application of the ideas of matter and force to nature the two former should never be separated: a mass of pure matter would, as far as we and nature are concerned, be a nullity, inasmuch as no action could be wrought by it either upon our organs of sense or upon the remaining portion of nature. A pure force would be something which must have a basis, and yet which has no basis, for the basis we name matter. It would be just as erroneous to define matter as something which has an actual existence, and force as an idea which has no corresponding reality. Both, on the contrary, are abstractions from the actual, formed in precisely similar ways. Matter is only discernible by its forces, and not by itself.

We have seen above that the problem before us is to refer back the phænomena of nature to unchangeable final causes. This requirement may now be expressed by saying that for final causes unchangeable forces must be found. Bodies with unchangeable forces have been named in science (chemistry) elements. Let us suppose the universe decomposed into elements possessing unchangeable qualities, the only alteration possible to such a system is an alteration of position, that is, motion; hence, the forces can be only moving forces dependent in their action upon conditions of space.

To speak more particularly: the phænomena of nature are to be referred back to motions of material particles possessing unchangeable moving forces, which are dependent upon conditions of space alone.

Motion is the alteration of the conditions of space. Motion,

as a matter of experience, can only appear as a change in the relative position of at least two material bodies. Force, which originates motion, can only be conceived of as referring to the relation of at least two material bodies towards each other; it is therefore to be defined as the endeavour of two masses to alter their relative position. But the force which two masses exert upon each other must be resolved into those exerted by all their particles upon each other; hence in mechanics we go back to forces exerted by material points. The relation of one point to another, as regards space, has reference solely to their distance apart: a moving force, therefore, exerted by each upon the other, can only act so as to cause an alteration of their distance, that is, it must be either attractive or repulsive.

Finally, therefore, we discover the problem of physical natural science to be, to refer natural phænomena back to unchangeable attractive and repulsive forces, whose intensity depends solely upon distance. The solvability of this problem is the condition of the complete comprehensibility of nature. In mechanical calculations this limitation of the idea of moving force has not yet been assumed: a great number, however, of general principles referring to the motion of compound systems of bodies are only valid for the case that these bodies operate upon each other by unchangeable attractive or repulsive forces; for example, the principle of virtual velocities; the conservation of the motion of the centre of gravity; the conservation of the principal plane of rotation; of the moment of rotation of free systems, and the conservation of *vis viva*. In terrestrial matters application is made chiefly of the first and last of these principles, inasmuch as the others refer to systems which are supposed to be completely free; we shall however show that the first is only a special case of the last, which therefore must be regarded as the most general and important consequence of the deduction which we have made.

Theoretical natural science therefore, if she does not rest contented with half views of things, must bring her notions into harmony with the expressed requirements as to the nature of simple forces, and with the consequences which flow from them. Her vocation will be ended as soon as the reduction of natural phænomena to simple forces is complete, and the proof given

that this is the only reduction of which the phænomena are capable.

I. *The principle of the Conservation of* vis viva.

We will set out with the assumption that it is impossible, by any combination whatever of natural bodies, to produce force continually from nothing. By this proposition Carnot and Clapeyron have deduced theoretically a series of laws, part of which are proved by experiment and part not yet submitted to this test, regarding the latent and specific heats of various natural bodies. The object of the present memoir is to carry the same principle, in the same manner, through all branches of physics; partly for the purpose of showing its applicability in all those cases where the laws of the phænomena have been sufficiently investigated, partly, supported by the manifold analogies of the known cases, to draw further conclusions regarding laws which are as yet but imperfectly known, and thus to indicate the course which the experimenter must pursue.

The principle mentioned can be represented in the following manner:—Let us imagine a system of natural bodies occupying certain relative positions towards each other, operated upon by forces mutually exerted among themselves, and caused to move until another definite position is attained; we can regard the velocities thus acquired as a certain mechanical work and translate them into such. If now we wish the same forces to act a second time, so as to produce again the same quantity of work, we must, in some way, by means of other forces placed at our disposal, bring the bodies back to their original position, and in effecting this a certain quantity of the latter forces will be consumed. In this case our principle requires that the quantity of work gained by the passage of the system from the first position to the second, and the quantity lost by the passage of the system from the second position back again to the first, are always equal, it matters not in what way or at what velocity the change has been effected. For were the quantity of work greater in one way than another, we might use the former for the production of work and the latter to carry the bodies back to their primitive positions, and in this way procure an indefinite amount of mechanical force. We should thus have built a *perpetuum*

mobile which could not only impart motion to itself, but also to exterior bodies.

If we inquire after the mathematical expression of this principle, we shall find it in the known law of the conservation of *vis viva*. The quantity of work which is produced and consumed may, as is known, be expressed by a weight m, which is raised to a certain height h; it is then mgh, where g represents the force of gravity. To rise perpendicularly to the height h, the body m requires the velocity $v = \sqrt{2gh}$, and attains the same by falling through the same height. Hence we have $\frac{1}{2}mv^2 = mgh$; and hence we can set the half of the product mv^2, which is known in mechanics under the name of the *vis viva* of the body m, in the place of the quantity of work. For the sake of better agreement with the customary manner of measuring the intensity of forces, I propose calling the quantity $\frac{1}{2}mv^2$ the quantity of *vis viva*, by which it is rendered identical with the quantity of work. For the applications of the doctrine of *vis viva* which have been hitherto made this alteration is of no importance, but we shall derive much advantage from it in the following. The principle of the conservation of *vis viva*, as is known, declares that when any number whatever of material points are set in motion, solely by such forces as they exert upon each other, or as are directed against fixed centres, the total sum of the *vires vivæ*, at all times when the points occupy the same relative position, is the same, whatever may have been their paths or their velocities during the intervening times. Let us suppose the *vires vivæ* applied to raise the parts of the system or their equivalent masses to a certain height, it follows from what has just been shown, that the quantities of work, which are represented in a similar manner, must also be equal under the conditions mentioned. This principle however is not applicable to all possible kinds of forces; in mechanics it is generally derived from the principle of virtual velocities, and the latter can only be proved in the case of material points endowed with attractive or repulsive forces. We will now show that the principle of the conservation of *vis viva* is alone valid where the forces in action may be resolved into those of material points which act

in the direction of the lines which unite them, and the intensity of which depends only upon the distance. In mechanics such forces are generally named central forces. Hence, conversely, it follows that in all actions of natural bodies upon each other, where the above principle is capable of general application, even to the ultimate particles of these bodies, such central forces must be regarded as the simplest fundamental ones.

Let us consider the case of a material point with the mass m, which moves under the influence of several forces which are united together in a fixed system A; by mechanics we are enabled to determine the velocity and position of this point at any given time. We should therefore regard the time t as primitive variable, and render dependent upon it,—the ordinates x, y, z of m in a system of coordinates, definite as regards A, the tangential velocity q, the components of the latter parallel to the axes, $u=\frac{dx}{dt}$, $v=\frac{dy}{dt}$, $w=\frac{dz}{dt}$, and finally the components of the acting forces

$$X=m\frac{du}{dt},\quad Y=m\frac{dv}{dt},\quad Z=m\frac{dw}{dt}.$$

Now according to our principle $\frac{1}{2}\,m\,q^2$, and hence also q^2, must be always the same when m occupies the same position relative to A; it is not therefore to be regarded merely as a function of the primitive variable t, but also as a function of the coordinates x, y, z only; so that

$$d(q^2)=\frac{d(q^2)}{dx}dx+\frac{d(q^2)}{dy}dy+\frac{d(q^2)}{dz}dz. \quad . \quad . \quad . \quad (1)$$

As $q^2=u^2+v^2+w^2$, we have $d(q^2)=2udu+2vdv+2wdw$. Instead of u let us substitute its value $\frac{dx}{dt}$, and instead of du its value $\frac{Xdt}{m}$, the corresponding values of v and w being also used, we have

$$d(q^2)=\frac{2X}{m}dx+\frac{2Y}{m}dy+\frac{2Z}{m}dz. \quad . \quad . \quad . \quad . \quad (2)$$

As the equations (1) and (2) must hold good together for all values whatever of dx, dy, dz, it follows that

$$\frac{d(q^2)}{dx}=\frac{2X}{m},\quad \frac{d(q^2)}{dy}=\frac{2Y}{m}\text{ and }\frac{d(q^2)}{dz}=\frac{2Z}{m}.$$

But if q^2 is a function of x, y, and z merely, it follows that X, Y, and Z, that is, the direction and magnitude of the acting forces, are purely functions of the position of m in respect to A.

Let us now imagine, instead of the system A, a single material point a, it follows from what has been just proved, that the direction and magnitude of the force exerted by a upon m is only affected by the position which m occupies with regard to a. But the only circumstance, as regards position, that can affect the action between the two points is the distance ma; the law, therefore, in this case would require to be so modified, that the direction and magnitude of the force must be functions of the said distance, which we shall name r. Let us suppose the coordinates referred to any system of axes whatever whose origin lies in a, we have then

$$md(q^2)=2\mathrm{X}dx+2\mathrm{Y}dy+2\mathrm{Z}dz=0. \quad . \quad . \quad . \quad (3)$$

as often as

$$d(r^2)=2xdx+2ydy+2zdz=0$$

that is, as often as

$$dz=-\frac{xdx+ydy}{z};$$

setting this value in equation (3), we obtain

$$\left(\mathrm{X}-\frac{x}{z}\mathrm{Z}\right)dx+\left(\mathrm{Y}-\frac{y}{z}\mathrm{Z}\right)dy=0$$

for any values whatever of dx and dy; hence also singly

$$\mathrm{X}=\frac{x}{z}\mathrm{Z} \text{ and } \mathrm{Y}=\frac{y}{z}\mathrm{Z},$$

that is to say, the resultant must be directed towards the origin of coordinates, or towards the point a.

Hence in systems to which the principle of the conservation of force can be applied, in all its generality, the elementary forces of the material points must be central forces.

II. *The principle of the Conservation of Force.*

We will now give the law for the cases where the central forces act, a still more general expression.

Let ϕ be the intensity of the force which acts in the direction of r, which is to be regarded as positive when it attracts, and as negative when it repels, then we have

$$\mathrm{X}=-\frac{x}{r}\phi;\ \mathrm{Y}=-\frac{y}{r}\phi;\ \mathrm{Z}=-\frac{z}{r}\phi; \quad . \quad . \quad . \quad (1)$$

and from equation (2) of the foregoing section, we have

$$md(q^2) = -2\frac{\phi}{r}(xdx + ydy + zdz)\,;\ \text{hence}$$

$$\frac{1}{2}md(q^2) = -\phi dr\,;$$

or when Q and R, q and r represent corresponding tangential velocities and distances,

$$\frac{1}{2}mQ^2 - \frac{1}{2}mq^2 = -\int_r^R \phi dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let us regard this equation more closely; we find at the left-hand side the difference of the *vires vivæ* possessed by m at two different distances. To understand the import of the quantity $\int_r^R \phi dr$, let us suppose the intensities of ϕ which belong to different points of the connecting line ma erected as ordinates at these points, then the above quantity would denote the superficial content of the space enclosed between the two ordinates r and R. As this surface may be regarded as the sum of the infinite number of ordinates which lie between r and R, it therefore represents the sum of the intensities of the forces which act at all distances between R and r. Calling the forces which tend to move the point m, before the motion has actually taken place, *tensions*, in opposition to that which in mechanics is named *vis viva*, then the quantity $\int_r^R \phi dr$ would be *the sum of the tensions* between the distances R and r, and the above law would be thus expressed:—The increase of *vis viva* of a material point during its motion under the influence of a central force is equal to the sum of the tensions which correspond to the alteration of its distance.

Let us suppose the case of two points operated upon by an attractive force, at the distance R; by the action of the force they will be drawn to less distances r, their velocity, and consequently *vis viva*, will be increased; but if they should be driven to greater distances r, their *vis viva* must diminish and must finally be quite consumed. We can therefore distinguish, in the case of attractive forces, the sum of the tensions for the distances between $r=0$ and $r=R$. $\int_0^R \phi dr$, as those which yet remain, but

those between $r=\mathrm{R}$ and $r=\infty$ as those already consumed; the former can immediately act, the latter can only be called into action by an equivalent loss of *vis viva*. It is the reverse with repulsive forces. If the points are situate at the distance R, as the distance becomes greater *vis viva* will be gained, and the still existing tensions are those between $r=\mathrm{R}$ and $r=\infty$, those lost are between $r=0$ and $r=\mathrm{R}$.

To carry our law through in quite a general manner, let us suppose any number whatever of material points with the masses m_1, m_2, m_3, &c. denoted generally by m_a; let the components of the forces which act upon these parallel to the axes be X_a, Y_a, Z_b, the components of the velocities along the same axes u_a, v_a, w_a, the tangential velocity q_a; let the distance between m_a and m_b be r_{ab}, the central force between both being ϕ_{ab}. For the single point m_n we have, analogous to equation (1),

$$\mathrm{X}_n=\Sigma\left[(x_a-x_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{du_n}{dt},$$

$$\mathrm{Y}_n=\Sigma\left[(y_a-y_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{dv_n}{dt},$$

$$\mathrm{Z}_n=\Sigma\left[(z_a-z_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{dw_n}{dt},$$

where the sign of summation Σ includes all members which are obtained by putting in the place of the index a the separate indices 1, 2, 3, &c., with the exception of n.

Multiplying the first equation by $dx_n=u_n dt$, the second by $dy_n=v_n dt$, the third by $dz_n=w_n dt$, and supposing the three equations thus obtained to be formed for every single point of m_b, as it is already done for m_n; adding all together, we obtain

$$\Sigma\left[(x_a-x_b)dx_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(u^2{}_a)\right]$$

$$\Sigma\left[(y_a-y_b)dy_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(v^2{}_a)\right]$$

$$\Sigma\left[(z_a-z_b)dz_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(cw^2{}_a)\right].$$

The members of the left-hand series will be obtained by placing instead of a all the single indices 1, 2, 3, &c., and in each case

for b also all the values of b, which are greater or smaller than a already possesses. The sums divide themselves therefore into two portions, in one of which a is always greater than b, and in the other always smaller, and it is clear that for every member of the one portion

$$(x_p - x_q)dx\,\frac{\phi_{pq}}{r_{pq}},$$

a member

$$(x_q - x_p)dx_p\frac{\phi_{pq}}{r_{pq}}$$

must appear in the other portion: adding both together, we obtain

$$-(x_p - x_q)\,(dx_p - dx_q)\frac{\phi_{pq}}{r_{pq}}:$$

drawing the sums thus together, adding all three and setting

$$\frac{1}{2}d\Big[(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2\Big] = r_{ab}dr_{ab},$$

we obtain

$$-\Sigma\Big[\phi_{ab}dr_{ab}\Big] = \Sigma\Big[\frac{1}{2}m_a d(q^2{}_a)\Big], \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$-\Sigma\Big[\int_{r_{ab}}^{R_{ab}}\phi_{ab}dr_{ab}\Big] = \Sigma\Big[\frac{1}{2}m_a Q^2{}_a\Big] - \Sigma\Big[\frac{1}{2}m_a q^2{}_a\Big], \quad . \quad (4)$$

where R and Q, as well as r and q, denote contemporaneous values.

We have here at the left-hand side again the sum of the tensions consumed, on the right the *vis viva* of the entire system, and we can now express the law as follows:—In all cases of the motion of free material points under the influence of their attractive and repulsive forces, whose intensity depends solely upon distance, the loss in tension is always equal to the gain in *vis viva*, and the gain in the former equal to the loss in the latter. Hence *the sum of the existing tensions and vires vivæ is always constant.* In this most general form we can distinguish our law as *the principle of the conservation of force.*

In the deduction of the law as given above, nothing is changed if a number of the points, which we will denote generally by the

letter d, are supposed to be fixed, so that q_d is constantly $=0$; the form of the law will then be

$$\Sigma[\phi_{ab}dr_{ab}]+\Sigma[\phi_{ad}dr_{ad}]=-\Sigma\left[\frac{1}{2}m_b d(q^2{}_b)\right] \quad . \quad . \quad . \quad (5)$$

It remains to be shown in what relation the principle of the conservation of force stands to the most general law of statics, the so-called principle of virtual velocities. This follows immediately from equations (3) and (5). If equilibrium is to set in when a certain arrangement of the points ma takes place, that is, if in case these points come to rest, hence $q_a=0$, they remain at rest, hence $dq_a=0$, it follows from equation (3),

$$\Sigma[\phi_{ab}dr_{ab}]=0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or in case that forces act upon them from points m_b without the system, by equation (5),

$$\Sigma[\phi_{ab}dr_{ab}]+\Sigma[\phi_{ad}dr_{ad}]=0. \quad . \quad . \quad . \quad . \quad (7)$$

In these equations under dr are understood alterations of distance consequent on the small displacements of the point m_a, which are permitted by the conditions of the system. We have seen, in the former deductions, that an increase of *vis viva*, hence a transition from rest to motion, can only be effected by an expenditure of tension; in correspondence with this, the last equations declare that in cases where in no single one of the possible directions of motion tension in the first moment is consumed, the system once at rest must remain so.

It is known that all the laws of statics may be deduced from the above equations. The most important consequence as regards the nature of the acting forces is this: instead of the arbitrary small displacements of the points m, let us suppose such introduced as might take place were the system in itself firmly united, so that in equation (7) every $dr_{ab}=0$, it follows singly,

$$\Sigma[\phi_{ab}dr_{ab}]=0, \text{ and}$$
$$\Sigma[\phi_{ab}dr_{ab}]=0.$$

Then the exterior, as well as the interior forces, must satisfy among themselves the conditions of equilibrium. Hence, if any system whatever of natural bodies be brought by the action of exterior forces into a certain position of equilibrium, the equi-

librium will not be destroyed—1, if we imagine the single points of the system in their present position to be rigidly united to each other; and 2, if we then remove the forces which the points exert upon each other. From this however it follows further: If the forces which two material points exert upon each other be brought into equilibrium by the action of exterior forces, the equilibrium must continue, when, instead of the mutual forces of the points, a rigid connexion between them is substituted. Forces, however, which are applied to two points of a rigid right line can only be in equilibrium when they lie in this line and are equal and opposite. It follows therefore for the forces of the points themselves, which are equal and opposed to the exterior ones, that they must act in the direction of the line of connexion, and hence must be either attractive or repulsive.

The preceding propositions may be collected together as follows:—

1. Whenever natural bodies act upon each other by attractive or repulsive forces, which are independent of time and velocity, the sum of their *vires vivæ* and tensions must be constant; the maximum quantity of work which can be obtained is therefore a limited quantity.

2. If, on the contrary, natural bodies are possessed of forces which depend upon time and velocity, or which act in other directions than the lines which unite each two separate material points, for example, rotatory forces, then combinations of such bodies would be possible in which force might be either lost or gained *ad infinitum*.

3. In the case of the equilibrium of a system of bodies under the operation of central forces, the exterior and the interior forces must, each system for itself, be in equilibrium, if we suppose that the bodies of the system cannot be displaced, the whole system only being moveable in regard to bodies which lie without it. A rigid system of such bodies can therefore never be set in motion by the action of its interior forces, but only by the operation of exterior forces. If, however, other than central forces had an existence, rigid combinations of natural bodies might be formed which could move of themselves without needing any relation whatever to other bodies.

III. *The application of the principle in Mechanical Theorems.*

We will now turn to the special application of the law of the constancy of force. In the first place, we will briefly notice those cases in which the principle of the conservation of *vis viva* has been heretofore recognized and made use of.

1. *All motions which proceed under the influence of the general force of gravitation*; hence those of the heavenly and the ponderable terrestrial bodies. In the former case the law pronounces itself in the increase of velocity which takes place when the paths of the planets approach the central body, in the unchangeableness of the greater axes of their orbits, their time of rotation and orbital revolution. In the latter case, by the known law that the terminal velocity depends only upon the perpendicular height fallen through, and that this velocity, when it is not destroyed by friction or by unelastic concussion, is exactly sufficient to carry the body to the same height as that from which it has fallen; that the height of ascent of a certain weight is used as the unit of measure in our machines has been already mentioned.

2. *The transmission of motion through the incompressibly solid and fluid bodies,* where neither friction nor concussion takes place. Our general principle finds for this case expression in the known fact, that a motion transmitted and altered by mechanical powers, diminishes in force as it increases in velocity. Let us suppose that by means of any machine whatever, to which mechanical force is uniformly applied, the weight m is raised with the velocity c, by another mechanical arrangement the weight nm may be raised, but only with the velocity $\frac{c}{n}$, so that in both cases the quantity of tension developed by the machine in the unit of time is expressed by mgc, where g represents the intensity of gravity.

3. *The motions of perfectly elastic, solid, and fluid bodies.* As condition of complete elasticity, we must to the ordinary definition, that the body which has been changed in form or volume completely regains its primitive condition, add, that no friction takes place between the particles in the interior. In the laws of these motions our principle was first recognised and most frequently made use of. Among the most common cases of its application to solids may be mentioned the collision of elastic bodies, the

laws of which may be readily deduced from our principle; the conservation of the centre of gravity, and the manifold elastic vibrations which continue without fresh excitement, until, through the friction of the interior parts and the yielding up of motion to exterior bodies, they are destroyed. In fluid bodies, liquid (evidently also elastic, but endowed with a high modulus of elasticity and a position of equilibrium of the particles) as well as gaseous, (with low modulus of elasticity and without position of equilibrium) motions are in general propagated by undulations. To these belong the waves on the surfaces of liquids, the motion of sound, and probably also those of light and radiant heat.

The *vis viva* of a single particle Δm in a medium which is traversed by a train of waves, is evidently to be determined from the velocity which it possesses at its position of equilibrium. The general equation of waves determines, as is known, the velocity u, when a^2 is the intensity, λ the length of the wave, α the velocity of propagation, x the abscissa, and t the time, as follows:—

$$u = a \cdot \cos\left[\frac{2\pi}{\lambda}(x - \alpha t)\right].$$

For the position of equilibrium u is $= a$, hence the *vis viva* of the particle Δm during the undulatory motion $\frac{1}{2}\Delta m a^2$ is proportional to the intensity. If the waves expand spherically from a centre, masses continually increasing in bulk are set in motion, and hence the intensity must diminish, if the *vis viva* is to remain the same. Now as the masses embraced by the waves increase as the square of the distance, the law follows as a consequence, that the intensities diminish in the reciprocal ratio.

The laws of reflexion, refraction and polarization of light at the limit of two media of different wave-velocity, are known to have been deduced by Fresnel from the assumption that the motion of the limiting particles in both media is the same, and from the conservation of *vis viva*. By the interference of two trains of waves we have no destruction of *vis viva*, but merely another distribution. Two trains of waves of the intensities a^2 and b^2 which do not interfere, give to all points on which they strike the intensity $a^2 + b^2$; if they interfere, the maxima possess the intensity $(a + b)^2$, that is, $2ab$ more, and the minima $(a - b)^2$, just as much less than $a^2 + b^2$.

The *vis viva* of elastic waves is only destroyed by such processes as we denominate absorption. The absorption of sonorous waves we find to be chiefly effected by concussion against yielding unelastic bodies, for example, curtains and coverlets; they may therefore be regarded as a communication of motion to the bodies in question, in which the motion is destroyed by friction. Whether motion can be destroyed by the friction of the air-particles against each other is a question which cannot yet perhaps be decided. The absorption of rays of heat is accompanied by a proportional development of heat; how far the latter corresponds to a certain equivalent of force, we will consider in the next section. The conservation of force would take place if the quantity of heat radiated from one body appeared again in the body into which it was radiated, provided that none was lost by conduction, and no portion of the rays escaped elsewhere. The theorem is certainly assumed in investigations upon radiant heat, but I am aware of no experiments which furnish the proof of it. As regards the absorption of light by imperfectly transparent or totally opake bodies, we are acquainted with three peculiarities. In the first place, phosphorescent bodies absorb the light in such a manner that they yield it up again afterwards. Secondly, most luminous rays, perhaps all of them, appear to excite heat. The obstacles to the belief in the identity of the luminous, calorific, and chemical rays of the spectrum have been lately disappearing more and more; the heat-equivalent of the chemical and luminous rays appears to be very inconsiderable in comparison to their intense actions upon the eye. If, however, the similarity of these differently acting rays does not permit of being established, then the end of the motion of light must undoubtedly be declared to be unknown. Thirdly, in many cases the light absorbed developes chemical action. We must here distinguish two species of action; first, where the mere incitement to chemical activity is communicated, as in the case of those bodies which induce catalytic action, for example, the action upon a mixture of chlorine and hydrogen; and secondly, those in which it is opposed to chemical action, as in the decomposition of salts of silver and the action upon the green portions of plants. In most of these processes however, the effect of light is so little known, that we are able to form no judgement regarding the magnitude of the forces developed. The latter appear to be

considerable in quantity and intensity only in the actions on the green portions of plants.

IV. *The Force-equivalent of Heat.*

The mechanical processes in which an absolute loss of force has been heretofore assumed, are—

1. *The collision of unelastic bodies.*—The loss is mostly connected with the change of form and the compression of the body struck; hence with the increase of the tensions, we also find, by the repetition of such shocks, a considerable development of heat; for example, by hammering a piece of metal; finally, a portion of the motion will be communicated as sound to the contiguous solid and gaseous bodies.

2. *Friction*, both at the surface of two bodies which move over each other, and also that arising in the interior from the displacement of the particles. In the case of friction certain small changes in the molecular constitution of the bodies take place, especially when they commence to rub against each other; afterwards the surfaces generally accommodate themselves to each other, so that this change becomes a vanishing quantity in the further course of the motion. In many cases such changes do not at all appear, for example, when fluids rub against solid bodies or against each other. Besides those already mentioned, thermic and electric changes always take place.

It is customary in mechanics to represent friction as a force which acts against the existing motion, and the intensity of which is a function of the velocity. This mode of representing the subject is only made use of for the sake of calculation, and is evidently an extremely incomplete expression of the complicated process of action and reaction of the molecular forces. From this customary manner of regarding the subject, it was inferred that by friction *vis viva* was absolutely lost; and the same was assumed in the case of unelastic collision. It is not however here taken into account that, disregarding the increase of the tensions caused by the compression of the body rubbed or struck, the heat developed is also the representant of a force by which we can develope mechanical actions; the electricity developed, whose attractions and repulsions are direct mechanical actions, and the heat it excites, an indirect one, has also been

neglected. It remains therefore to be asked whether the sum of these forces always corresponds to the mechanical force which has been lost. In those cases where the molecular changes and the development of electricity are to a great extent avoided, the question would be, whether for a certain loss of mechanical force a definite quantity of heat is always developed, and how far can a quantity of heat correspond to a mechanical force. For the solution of the first question but few experiments have yet been made. Joule* has measured the heat developed by the friction of water in narrow tubes, and that developed in vessels in which the water was set in motion by a paddle-wheel; in the first case he found that the heat which raises 1 kilogramme of water 1°, was sufficient to raise 452 kilogrammes through the height of 1 metre; in the second case he found the weight to be 521 kilogrammes. His method of measurement however meets the difficulty of the investigation so imperfectly, that the above results can lay little claim to accuracy†. Probably the above numbers are too high, inasmuch as in his proceeding a quantity of heat might have readily escaped unobserved, while the necessary loss of mechanical force in other portions of the machine is not taken into account.

Let us now turn to the further question, how far heat can correspond to an equivalent of force. The material theory of heat must necessarily assume the quantity of caloric to be constant; it can therefore develope mechanical forces only by its effort to expand itself. In this theory the force-equivalent of heat can only consist in the work produced by the heat in its passage from a warmer to a colder body; in this sense the problem has been treated by Carnot and Clapeyron, and all the consequences of the assumption, at least with gases and vapours, have been found corroborated.

To explain the heat developed by friction, the material theory must either assume that it is communicated by conduction as supposed by Henry‡, or that it is developed by the compression of the surfaces and of the particles rubbed away, as supposed by

* Philosophical Magazine, S. 3. vol. xxvii. p. 205.

† It must be remembered that the writer was acquainted with the earlier experiments only of Mr. Joule.—Ed.

‡ Mem. of the Society of Manchester, t. v. p. 2. London, 1802.

Berthollet*. The first of these assumptions lacks all experimental proof; if it were true, then in the neighbourhood of the rubbed portions a cold proportionate to the intense heat often developed must be observed. The second assumption, without dwelling upon the altogether improbable magnitude of action, which according to it must be ascribed to the almost imperceptible compression of the hydrostatic balance, breaks down completely when it is applied to the friction of fluids, or to the experiments where wedges of iron have been rendered red-hot and soft by hammering and pieces of ice melted by friction†; for here the softened iron and the water of the melted ice could not remain in a compressed condition. Besides this, the development of heat by the motion of electricity proves that the quantity of heat can be actually increased. Passing by frictional and voltaic electricity—because it might here be suspected that, by some hidden relation of caloric to electricity, the former was transferred from the place where it was originated and deposited in the heated wire—two other ways of producing electric tensions by purely mechanic agencies in which heat does not at all appear, are still open to us, namely, by induction and by the motion of magnets. Suppose we possess a completely insulated body positively electric, and which cannot part with its electricity; an insulated conductor brought near to it will show free $+E$, we can discharge this upon the interior coating of a battery and remove the conductor, which will then show $-E$; this latter can of course be discharged upon the exterior surface of the first or upon a second battery. By repeating this process, it is evident that we can charge a battery of any magnitude whatever as often as we please, and by means of its discharge can develope heat, which nowhere disappears. We shall, on the contrary, have consumed a certain amount of force, for at each removal of the negatively-charged conductor from the inducing body the attraction between both is to be overcome. This process is essentially carried out when the electrophorus is used to charge a Leyden jar. The same takes place in magneto-electric machines; as long as magnet and keeper are moved opposite to each other, electric currents are excited which develope heat in the connecting wire; and in-

* *Statique Chimique*, t. i. p. 247.

† Humphry Davy, Essay on Heat, Light, and the Combinations of Light.

asmuch as they constantly act in a sense contrary to the motion of the keeper, they destroy a certain amount of mechanical force. Here evidently heat *ad infinitum* may be developed by the bodies constituting the machine, while it nowhere disappears. That the magneto-electric current developes heat instead of cold, in the portion of the spiral directly under the influence of the magnet, Joule has endeavoured to prove experimentally *. From these facts, it follows that the quantity of heat can be absolutely increased by mechanical forces, that therefore calorific phænomena cannot be deduced from the hypothesis of a species of matter, the mere presence of which produces the phænomena, but that they are to be referred to changes, to motions, either of a peculiar species of matter, or of the ponderable or imponderable bodies already known, for example of electricity or the luminiferous æther. That which has been heretofore named the quantity of heat, would, according to this, be the expression, first, of the quantity of *vis viva* of the calorific motion, and, secondly, of the quantity of those tensions between the atoms, which, by changing the arrangement of the latter, such a motion can develope. The first portion would correspond to that which has been heretofore called free heat, the second with that which has been named latent heat. If it be permitted to make an attempt at rendering the idea of this motion still clearer, the view derived from the hypothesis of Ampère seems best suited to the present state of science. Let us imagine the bodies formed of atoms which themselves are composed of subordinate particles (chemical elements, electricity, &c.), in such an atom three species of motion may be distinguished,—1, displacement of the centre of gravity; 2, rotation round the centre of gravity; 3, displacement of the particles of the atom among themselves. The two first would be compensated by the forces of the neighbouring atoms, and hence transmitted to these in the form of undulations, a species of propagation which corresponds to the radiation of heat, but not to its conduction. Motions of the single particles of the atoms among themselves, would be compensated by the forces existing within the atom, and would communicate motion but slowly to the surrounding atoms, as a vibrating string sets a second in motion and thereby loses an equal quantity of motion

* Phil. Mag. S. 3. 1844.

itself; this description of motion seems to be similar to the conduction of heat. It is also clear, that such motions in the atoms may cause changes in the molecular forces, and consequently give rise to expansion or an alteration of the state of aggregation. Of what nature the motion is, we have no means whatever of ascertaining; the possibility of conceiving the phænomena of heat as being due to motion is, however, sufficient for our present object. The conservation of force in the case of these motions will hold good in all cases where hitherto the conservation of caloric has been assumed; for example, in all phænomena of radiation and conduction of heat from one body to another, and in the case of the appearance and disappearance of heat during changes of aggregation.

Of the different modes in which heat manifests itself, we have considered the cases where one body radiates into another, and where it is produced by mechanical force; further on we will examine the heat generated by electricity. It remains to consider the development of heat in chemical processes. It has been heretofore referred to the setting free of caloric which was previously latent in the combining bodies. According to this, we must ascribe to every simple body, and every chemical combination which is capable of entering into still further combinations of a higher order, a definite quantity of latent heat which is necessary to its chemical constitution. From this we derive the law, which has been also partially verified by experience, that when several bodies unite together to a chemical compound, the same quantity of heat is developed, no matter in what order the combination may have been effected*. According to our way of viewing the subject, the quantity of heat developed by chemical processes would be the quantity of *vis viva* produced by the chemical attractions, and the above law would be the expression for the principle of the conservation of force in this case.

As little as the conditions and laws of the generation of heat have been investigated, although such a generation undoubtedly occurs, this has been done with reference to the disappearance of heat. Hitherto we are only acquainted with cases in which chemical combinations have been decomposed, or less dense

* Hess, in Pogg. *Ann.* vol. l. 392; lvi. 598.

states of aggregation brought about, and thus heat rendered latent. Whether by the development of mechanical force heat disappears, which would be a necessary postulate of the conservation of force, nobody has troubled himself to inquire. I can only in respect to this cite an experiment by Joule*, which seems to have been carefully made. He found that air while streaming from a reservoir with a capacity of 136·5 cubic inches, in which it was subjected to a pressure of 22 atmospheres, cooled the surrounding water 4°·085 Fahr. when the air issued into the atmosphere, and therefore had to overcome the resistance of the latter. When, on the contrary, the air rushed into a vessel of equal size which had been exhausted of air, thus finding no resistance and exerting no mechanical force, no change of temperature took place.

We have now to examine the manner in which the views of Clapeyron† and Holtzmann‡ bear upon our own. Clapeyron starts from the notion that only by its passage from warm bodies to cold ones can heat be applied as a means of developing mechanical force, and that the maximum of the latter is gained when the transmission of the heat is effected between bodies of equal temperatures, the alterations of temperature being effected by the compression and dilatation of the heated bodies. This maximum must be the same for all natural bodies which by heating and cooling can produce mechanical force; for were it different, that body in which a certain quantity of heat was capable of producing the greatest action might be applied in the production of mechanical work, and a portion of the latter might be made use of to bring the heat back from the colder to the warmer source, and thus mechanical force to infinity might be gained, it being at the same time tacitly assumed that the quantity of heat cannot be changed by this process. The following is the analytical expression given by Clapeyron to this law:—

$$\frac{dq}{dv}\frac{dt}{dp}-\frac{dq}{dp}\cdot\frac{dt}{dv}=C,$$

* Philosophical Magazine, S. 3. vol. xxvi. p. 369.

† Scientific Memoirs, vol. i. Part 3 of 1st Series.

‡ *Ueber die Wärme und Elasticität der Gase und Dämpfe.* Manheim, 1845. Translated in full in vol. iv. Part 14., of Scientific Memoirs.

where q denotes the quantity of heat contained by one body, t its temperature, both expressed as functions of the volume v, and the pressure p. $\frac{1}{C}$ is the mechanical work which the unit of heat (which can raise 1 kilogramme of water 1° Centigrade) produces when it passes into a temperature 1° lower. This is stated to be identical for all natural bodies, but to vary with the temperature. For gases the formula is

$$C = v\frac{dq}{dv} - p\frac{dq}{dp}.$$

Clapeyron's inferences from the general validity of this formula, at least in the case of gases, have many analogies on their side which agree with experience. His deduction of the law can only be admitted when the quantity of heat is regarded as unchangeable. Further, his more special formulas for gases, which alone are supported by experiment, follow also from the formula of Holtzmann, as we shall immediately show. With regard to the general formula, he has only sought to show that the law which follows from it is at least not contradicted by experiment. This law is, that when the pressure on different bodies, taken at the same temperature, is a little increased, quantities of heat will be developed which are proportional to the expansibility of the bodies by heat. I will here merely draw attention to what must be regarded as at least a very improbable consequence of this law. Compression of water at the point of maximum density would develope no heat, and between this and the freezing-point it would develope cold.

Holtzmann sets out from the consideration that a certain quantity of heat which enters a gas can cause in it either an increase of temperature or an expansion. The quantity of work thus produced by the heat he assumed to be the mechanical equivalent of the heat; he calculated from the experiments of Dulong upon sound, that the heat which raises the temperature of 1 kilogramme of water 1° Centigrade, would raise a weight of 374 kilogrammes 1 metre. This method of calculation, regarded from our point of view, is only admissible when the entire *vis viva* of the heat communicated is actually returned as mechanical force, hence the sum of the *vis viva* and tensions, that is,

the quantity of free and of latent heat, is just the same in the more expanded gases as in the denser ones of the same temperature. According to this, a gas which expands without producing work must exhibit no change of temperature, which indeed appears to follow from the above-mentioned experiment of Joule; and thus the increase and diminution of temperature by compression and dilatation would, under ordinary circumstances, be due to the excitation of heat by mechanical force, and *vice versâ*. In support of the correctness of Holtzmann's law, a great number of consequences from it which agree with experiment might be adduced; more particularly the deduction of the formula for the elasticity of the vapour of water at different temperatures.

Joule determines from his own experiments the force-equivalent, which Holtzmann, from the experiments of others, reckons at 374, to be 481, 464, 479; while, by friction, the force-equivalent for the unit of heat he found to be 452 and 521.

The formula of Holtzmann for gases coincides with that of Clapeyron, only in the former the undetermined function C of the temperature is found, and by this means the complete determination of the integral is rendered possible. The former formula is

$$\frac{pv}{a}=v\frac{dq}{dv}-p\frac{dq}{dp},$$

where a is the force-equivalent of the unit of heat; the formula of Clapeyron is

$$C=v\frac{dq}{dv}-p\frac{dq}{dp}.$$

Both are therefore coincident, when $C=\frac{pv}{a}$; or, as $p=\frac{k}{v}\left(1+\alpha t\right)$ where α is the coefficient of expansion, and k is a constant, when

$$\frac{1}{C}=\frac{a}{k(1+\alpha t)}.$$

The values found by Clapeyron for $\frac{1}{C}$ agree pretty well with this formula, as is shown by the following table:—

Temperature.	Calculated by Clapeyron.			According to the formula.
	a	*b*.	*c*.	
0°	1·410	.....	1·586	1·544
35·5	.....	1·365	1·292	1·366
78·8	.....	1·208	1·142	1·198
100	.....	1·115	1·102	1·129
156·8	.....	1·076	1·072	0·904

The number *a* is calculated from the velocity of sound in air, the series *b* from the latent heat of the vapours of æther, alcohol, water, and oil of turpentine, *c* from the expansive force of the vapour of water at different temperatures. Clapeyron's formula for gas is, according to this, identical with Holtzmann's; its applicability to solid and liquid bodies remains as yet doubtful.

V. *The Force-equivalent of Electric Processes.*

Statical Electricity.—Machine electricity can act in two ways as the cause of the generation of force; in the first place, the electricity itself moves with the body that bears it, in obedience to its attractive or repulsive forces; secondly, by its motion *in* the body that bears it, heat is generated. The first mechanical actions have, as is known, been deduced from hypothesis of two fluids which attract or repel with a force inversely proportional to the square of the distance; so far as experiment has been compared with theory, both have been found to agree. According to our primitive deduction, the conservation of force must take place with such forces. We will therefore enter only so far into the more special laws of the mechanical actions of electricity as is necessary for deducing the law of the electrical development of heat.

Let e_I and e_{II} be two electric elements referred to a unit which at the distance 1 repels an equal quantity of similar electricity with the force 1. When the opposite electricities are distinguished by opposite signs, and the distance between e_I and e_{II} is called r, the intensity of the central force is

$$\phi = -\frac{e_I e_{II}}{r^2}.$$

The gain in *vis viva* caused by passing from the distance R to r is

$$-\int_{R}^{r}\phi dr=\frac{e_{I}e_{II}}{R}-\frac{e_{I}e_{II}}{r}.$$

When they pass from the distance ∞ to r, the expression is $-\frac{e_{I}e_{II}}{r}$. Let us denote this last quantity, which is the sum of the tensions consumed by the motion from ∞ to r and of the *vis viva* produced, in conformity with Gauss in his magnetical researches as the *potential* of the two electric elements for the distance r; the increase of *vis viva* due to any motion whatever is then equal to the excess of the potential at the end of the route over its value at the beginning.

Calling the sum of the potentials of an electrical element, with reference to the whole of the elements of an electrified body, the potential of the element towards the body, and the sum of the potentials of all the elements of an electrified body towards all the elements of another, the potential of the two bodies, we obtain again the gain in *vis viva* in the difference of the potentials, provided that the distribution of the electricity in the bodies is not changed, hence that the bodies are idio-electric. If the distribution undergo a change, the magnitude of the electric tensions in the body itself becomes altered, and the *vis viva* gained must then be different.

By all methods of electrifying, equal quantities of positive and negative electricity are generated; in the neutralization of the electricities between two bodies, one of which, A, contains as much positive electricity as the other, B, does of negative, half of the positive electricity goes from A to B, and half the negative from B to A. Let the potentials of the bodies upon themselves be W_a and W_b, their potential towards each other V, we then find the entire *vis viva* which has been gained, when we subtract the potentials of the moving electric masses on themselves and towards each of the other masses, before transfer, from the same potentials after the transfer has been effected. It is here to be remarked that the potential of two masses changes its sign when the sign of one of the masses is changed. Hence the following potentials come under consideration:—

1. of the moved $+\frac{1}{2}E$ from A

upon itself	$\frac{1}{4}(W_b - W_a)$
towards the moved $-\frac{1}{2}E$ . .	$\frac{1}{4}(V - V)$
towards the motionless $\frac{1}{2}E$ . .	$\frac{1}{4}(-V - W_a)$
towards the motionless $-\frac{1}{2}E$.	$\frac{1}{4}(-W_b - V)$.

2. of the moved $-\frac{1}{2}E$ from B

upon itself	$\frac{1}{4}(W_a - W_b)$
towards the moved $+\frac{1}{2}E$ . .	$\frac{1}{4}(V - V)$
towards the motionless $-\frac{1}{2}E$.	$\frac{1}{4}(-V - W_b)$
towards the motionless $+\frac{1}{2}E$.	$\frac{1}{4}(-W_a - V)$
Sum	$-\left(V + \frac{W_a + W_b}{2}\right)$

This quantity gives us therefore the maximum of the *vis viva* to be generated and the quantity of tension gained by electrifying.

To obtain, instead of these potentials, more familiar ideas, let us consider as follows. Suppose surfaces to be constructed for which the potential of an electric element, which lies in them, in regard to several other electrified bodies which are present, possesses equal values, and let us call these surfaces of equilibrium, then must the motion of an electric particle from any point whatever of one of them to any point of another of them, always increase the *vis viva* by the same quantity; a motion on the surface itself, on the contrary, will not alter the velocity of the particle. Hence the resultant of the whole of the attractive forces of the electricity for every single point of space must be perpendicular to the surface of equilibrium which passes through it, and every surface which is at right angles to this resultant must be a surface of equilibrium.

The equilibrium of electricity in a conductor cannot take place until the resultants of the whole of the attractive forces of its own electricity, and such other electrified bodies as may happen to be present, are perpendicular to its surface; because, were it otherwise, the electric particles must be moved along the surface. Consequently the surface of an electrified conductor is itself a surface of equilibrium, and the *vis viva* gained by an infinitely small electric particle in its passage from one conductor to another is constant. Let C_a denote the *vis viva* gained by the unit of positive electricity in its passage from the surface of the conductor A to an infinite distance, so that C_a is positive for positive charges; A_a the potential of the same quantity of electricity in regard to A when it occupies a certain point upon the surface of A; A_b the same in regard to B; W_a the potential of A upon itself; W_b the same of B; V that of A upon B, and Q_a the quantity of electricity in A, Q_b in B; the *vis viva* gained by the particle e in its passage from an infinite distance to the surface A is

$$-eC_a = e(A_a + A_b).$$

If instead of e we set successively all the electric particles of the surface A, and for A_a and A_b the corresponding potentials, and add all, we obtain

$$-Q_aC_a = V + W_a.$$

In like manner, for the conductor B,

$$-Q_bC_b = V + W_b.$$

The constant C must not only possess the same value for one and the same conductor, but also for separate conductors, if the latter, when connected together in a manner by which the distribution of their electricities is not sensibly changed, exchange no electricity with each other; that is, it must possess the same value for all conductors possessing the same free tension. As unit of measure for the free tension of an electrified body, we can make use of a quantity of electricity which, distributed over a sphere of the radius 1, placed beyond the distance where induction can take place, is in electric equilibrium with the said body. If the electricity be distributed uniformly over the sphere, the exterior action, as is known, will be the same as if the electricity was concentrated at its centre. Denoting the mass of

electricity by E, the radius of the sphere by R=1, for this sphere we have the constant

$$C=\frac{E}{R}=E;$$

that is, the constant C is equal to the free tension.

In accordance with this, the tensions of two conductors which contain equal quantities Q of positive and negative electricity are

$$-\left(V+\frac{W_a+W_b}{2}\right)=Q\left(\frac{C_a-C_b}{2}\right).$$

As C_b is negative, the algebraic difference C_a-C_b is equal to the absolute sum of both. If the capacity of discharge of B be very great, and consequently C_b nearly $=0$, the quantity of the electric tensions is $\frac{QC_a}{2}=-\frac{V+W_a}{2}$; if the distance between the two conductors be also very great, the above becomes $-\frac{1}{2}W_a$.

We have found that the *vis viva* generated by the motion of two electric masses is equal to the decrease of the sum $\frac{Q_aC_a+Q_bC_b}{2}$. This *vis viva* is gained as mechanical force, if the velocity of the electricity in the bodies be a vanishing quantity when compared with the velocity of propagation of the electric motion; we must obtain it as heat when this is not the case. The heat Θ developed by the discharge of equal quantities Q of the opposed electricities is therefore

$$\Theta=\frac{1}{2a}Q(C_a-C_b),$$

where a is the mechanical equivalent of the unit of heat, or when $C_b=0$, as is the case with batteries whose external coating is not insulated, and whose capacity of discharge is S, so that $CS=Q$,

$$\Theta=\frac{1}{2a}QC=\frac{1}{2a}\frac{Q^2}{S}.$$

Riess* has proved by experiment that, with different charges and different numbers of similarly constructed jars, the quantity of heat developed in every single portion of the same connecting

* Pogg. *Ann.* vol. xliii. p. 47.

wire is proportional to the quantity $\frac{Q^2}{S}$. He however denotes by S the surface of the coating of the jars. With similarly constructed jars, however, this must be proportional to the capacity of discharge. Vorsselmann de Heer* and Knochenhauer† have also deduced from their respective experiments, that the development of heat with equal charges of the same battery remains the same, however the connecting wire may be altered. The latter has proved the same law to exist in branches of the connecting wire. With regard to the quantity $\frac{1}{2a}$, we as yet possess no experimental data.

It is easy to explain this law if we assume that the discharge of a battery is not a simple motion of the electricity in one direction, but a backward and forward motion between the coatings, in oscillations which become continually smaller until the entire *vis viva* is destroyed by the sum of the resistances. The notion that the current of discharge consists of alternately opposed currents is favoured by the alternately opposed magnetic actions of the same; and secondly, by the phænomena observed by Wollaston while attempting to decompose water by electric shocks, that both descriptions of gases are exhibited at both electrodes. This assumption also explains, why in these experiments the electrodes must possess the smallest surface possible.

Galvanism.—With regard to galvanic phænomena, we have to distinguish two classes of conductors:—1. Those which conduct in the manner of metals, and follow the law of the tension series. 2. Those which do not follow this law. To the latter belong all compound liquids, which undergo during conduction a decomposition proportional to the quantity of electricity conducted.

We can classify the experimental facts in accordance with the above, 1, into such as take place between conductors of the first class only—the charging with different electricities of different metals which are in contact; and 2, those between conductors of both classes, the electric tensions in open, and the electric currents in closed circuits. By any combination whatever of conductors of the first class electric currents can

* Pogg. *Ann.* vol. xlviii. p. 292, and the remark of Riess, *ibid.* p. 320.

† Pogg. *Ann.* vol. lxii. p. 364; lxiv. p. 64.

never be generated, but electric tensions only. These tensions, however, are not equivalent to a certain quantity of force like those heretofore considered, which implied a disturbance of the electric equilibrium; galvanic tensions are, on the contrary, consequent upon the establishing of the electric equilibrium; no motion of the electricity can be generated by them further than a change of distribution of the electricity consequent upon a change of position. Let us suppose all the metals of the earth brought into connexion with each other, and the corresponding distribution of electricity established; by no other combination of the same metals can any one of them suffer an alteration of its tension until contact is established with a conductor of the second class. The idea of the force of contact, the force which is active at the place where two different metals touch each other, and which developes and sustains the different electric tensions of the latter, has not hitherto been rendered more determinate than it is here, because the attempt to embrace the phænomena resulting from the contact of conductors of the first and second classes was made at a time when the constant and distinguishing feature of the phænomena, namely the chemical process, was not yet properly recognized. From this indefinite mode of regarding the subject, it would certainly appear that the force of contact is such that by means of it infinite quantities of free electricity, and hence mechanical force, might be generated, if a conductor of the second class could be found which was not electrolysed during the conduction. It was precisely this circumstance which excited such a resistance to the contact theory, notwithstanding the simplicity and precision of the explanations which it furnished*. The principle which we have thus far advocated contradicts the idea of such a force directly, if it do not recognise also the necessity of the chemical processes. If this however be admitted, if it be assumed that the conductors of the second class do not follow the law of the tension series, just because they conduct by electrolysis, then the idea of the force of contact is capable of great simplification, and may at once be referred to attractive and repulsive forces. All phænomena exhibited by conductors of the first

* Faraday's Experimental Researches in Electricity, 17th Series; Phil. Trans. 1840, p. 1, No. 2071; and Pogg. *Ann.* vol. liii. p. 568.

class may evidently be deduced from the assumption that different chemical substances possess different attractive forces for both electricities, and that these attractions are only exerted at insensible distances; while the mutual attractions of the electricities themselves are exerted at measurable distances. The force of contact would, according to this, be the difference of the attractive forces which the metallic particles at the place of contact exert upon the electricities at this place; and electric equilibrium would occur when an electric particle which passes from the one to the other neither gains nor loses *vis viva* further. Let c_I and c_{II} be the free tensions of the two metals, $a_I e$ and $a_{II} e$ the *vires vivæ* gained by the electric particle e by its passage to the one or the other uncharged metal, then the force gained by its passage from one to the other charged metal is

$$e(a_I - a_{II}) - e(c_I - c_{II}).$$

In the case of equilibrium this must be $=0$, and hence

$$a_I - a_{II} = c_I - c_{II},$$

that is, the difference of tension in different portions of the same metal must be constant, and in different metals must follow the law of the tension series.

In galvanic currents, with respect to the conservation of force, we have specially to consider the following actions: the generation of heat, chemical processes, and polarization. The electrodynamic actions shall be considered under the head of magnetism. The development of heat is common to all currents; in regard to the two other actions, we can divide the currents into those which excite chemical decomposition only, those which excite polarization only, and those which excite both.

We will, in the first place, investigate the conditions of the conservation of force in those circuits in which the polarization is completely annulled, inasmuch as these are the only ones for which we have arrived at determinate quantitative laws. The intensity of a current I in a battery of n elements is given by the law of Ohm as

$$I = \frac{nA}{R},$$

where the constant A is the electromotive force of a single element, and R is the resistance of the circuit; A and R are in

this circuit independent of the intensity. During a certain interval the action of such a circuit will be changed in no respect save in its chemical conditions and quantity of heat; the law of the conservation of force would therefore require that the heat to be gained by the chemical processes which have taken place must be equal to the quantity actually gained. In a simple portion of a metallic conductor, with the resistance r, the heat developed in a certain time t is, according to Lenz*,

$$\vartheta = I^2 r t,$$

where as unit for r the length of wire is taken in which the unit of current developes the unit of heat in the unit of time. For branching wires, where the resistance in the single branches is denoted by r_a, the total resistance r is given by the equation

$$\frac{1}{r} = \Sigma\left[\frac{1}{r_a}\right],$$

the intensity I in the branch r_n by

$$I_n = \frac{Ir}{r_n};$$

hence the heat ϑ^n in the same branch,

$$\vartheta_n = I^2 r^2 \cdot \frac{1}{r_n} t,$$

and the total heat developed in all the branches

$$\vartheta = \Sigma[\vartheta_a] = I^2 r^2 \Sigma\left[\frac{1}{r_a}\right] t = I^2 r \cdot t.$$

Hence the total quantity of heat developed in any circuit whatever, where the conduction is effected through any number of branches, if the law of Lenz be true for fluid conductors, as found by Joule,

$$\Theta = I^2 R t = n A I t.$$

We have two kinds of constant batteries, those constructed according to the system of Daniell and those on Grove's principle. In the first the chemical process consists in the dissolution of the positive metal in an acid, while the negative one is precipitated from its saline solution. Let us assume as the

* Pogg. *Ann.* vol. lix. p. 203 and 407. From the Bulletin of the Acad. of Scien. St. Petersburg.

unit of intensity a current which in the unit of time decomposes an equivalent of water (say 1 gramme); then in the time t, the equivalent of the positive metal dissolved will be nIt, and the same quantity of the negative will be precipitated. Calling the heat which an equivalent of the positive metal developes by its oxidation and the solution of the oxide in the contiguous acid a_z, and the same for the negative a_c, then the quantity of heat to be developed chemically would be

$$=nIt(a_z-a_c).$$

The chemical would therefore be equal to the electrical if

$$A=a_z-a_c,$$

that is, if the electromotive force of two of the so-combined metals be proportional to the difference of the quantities of heat developed by their combustion and by their combination with acids.

In the elements constructed upon Grove's principle, the polarization is annulled by permitting the hydrogen separated to reduce a fluid rich in oxygen which surrounds the negative metal. To these belong the batteries of Grove and of Bunsen: amalgamated zinc, dilute sulphuric acid, fuming nitric acid, platinum or coal; further, those batteries in which chromic acid is made use of, and which have been subjected to exact measurement: amalgamated zinc, dilute sulphuric acid, solution of bichromate of potash with sulphuric acid, copper or platinum. The chemical processes are the same in the two batteries in which nitric acid is used, and likewise the same in the two in which chromic acid is used; from this it would follow, according to the deduction just made, that the electromotive forces must be also equal, which, indeed, by the measurement of Poggendorff, is proved to be exactly the case. The battery prepared from coal and chromic acid is very inconstant, and possesses a considerably higher electromotive force, at least at the beginning; this battery is therefore not to be included here, but belongs to those in which polarization is exhibited. In these constant circuits the electromotive force is independent of the negative metal; we can reduce them to the type of Daniell's battery if we regard as the negative element the particles of oxide of chromium and nitrous acid which are immediately in

contact with the platinum; so that thus regarded an element of Grove or of Bunsen would become a circuit formed between zinc and nitrous acid, and those constructed with chromic acid circuits between zinc and oxide of chromium.

The batteries with polarization may be divided into two classes, such as excite polarization, but which cause no chemical decomposition, and such as give rise to both. To the former, which produce an inconstant and quickly disappearing current, belong the simple circuit of Faraday*, with combinations formed of solutions of caustic potash, sulphuret of potassium, and nitrous acid; further, those of the more strongly negative metals in common acids, when the positive are not able to decompose the acid; for example, copper with silver, gold, platinum, coal in sulphuric acid, and so forth; of the compound circuits, all those in which decomposition cells are introduced, the polarization of which overpowers the electromotive force of the other elements. Exact quantitative experiments on the intensity of these circuits have, up to the present time, on account of the great variability of the currents, not been carried out. In general the intensity of these currents seems to depend upon the nature of the immersed metal; their duration increases with the magnitude of the surfaces and with the diminution of the intensity of the current; they can be renewed, even after they have almost disappeared, by moving the metal in the fluid or in the air, by which the polarization of the plate against which hydrogen has been liberated is annulled. To such actions the residual currents, which exhibit themselves on fine galvanometric instruments, are probably due. The entire process is therefore an establishing of an electric equilibrium of the particles of the fluid and of the metals; the particles of fluid appear sometimes to undergo a modification of arrangement; and on the other side chemical changes occur, in many cases, at the metallic surfaces†. In compound circuits, where the polarization of plates originally alike is caused by the action of currents from other elements, we can obtain the lost force of the primitive current in the form of a secondary current, by removing the exciting elements, and causing the polarized

* Experimental Researches in Electricity, 16th series; Phil. Trans. 184. p. 1; and Pogg. *Ann.* vol. lii. p. 163 and 547.

† Ohm, in Pogg. *Ann.* vol. lxiii. p. 389.

metals to form a circuit among themselves. The absence of special facts prevents us from making closer application of the principle of the conservation of force in this case.

The most complicated case is presented by those circuits in which chemical decomposition and polarization take place side by side; to these belong the circuits in which gas is developed. The current in these cases is strongest at the commencement, and sinks more or less quickly to a point at which it remains pretty constant. With single elements of this description, or with compound batteries composed of such elements, the current of polarization ceases with extreme slowness; it is easier, on the contrary, to obtain constant currents by combining constant elements with inconstant ones, particularly if the plates of the latter be comparatively small. Hitherto however but few measurements have been made with such circuits; from the few which I have been able to find by Lenz* and Poggendorff†, it follows that the intensity of such currents, when different resistances of wire are introduced, cannot be expressed by the simple formula of Ohm; for when the constants are calculated from the low intensities, the results for higher intensities are found to be too great. It is therefore necessary to regard the numerator or denominator, or both, as functions of the intensity; the facts hitherto known do not enable us to decide which of these cases really takes place.

In applying the principle of the conservation of force to these currents we must divide them into two classes, into inconstant or polarization currents, with regard to which, what we have already expressed regarding the pure currents of polarization, and constant or decomposing currents, is applicable. The same mode of treatment is applicable to the latter and to the constant currents in which no gas is developed; the quantity of heat generated by the current must be equal to that due to chemical decomposition. For example, in a combination of zinc with a negative metal in dilute sulphuric acid, suppose the quantity of heat liberated by an atom of zinc during its solution and the expulsion of the hydrogen $a_z - a_h$, then the quantity of heat developed in the time dt would be

$$I(a_z - a_h)dt.$$

* Pogg. *Ann.* liv. 229. † *Ann.* lxvii. 531.

If, now, the development of heat in all portions of the circuit were proportional to the square of the intensity, that is $I^2R dt$, we should have, as before,

$$I=\frac{a_z-a_h}{R},$$

which is the simple formula of Ohm. As this however is not applicable in the present instance, it follows that there are certain transverse sections in the circuit in which the development of heat is subject to another law, and whose resistance therefore is not to be regarded as constant. If, for example, the heat liberated in any cross section whatever be directly proportional to the intensity, which among others must be the case with the heat due to a change of aggregation, hence $\vartheta=\mu I dt$, we have

$$I(a_z-a_h)=I^2r+I\mu$$

$$I=\frac{a_z-a_h-\mu}{r}.$$

The quantity μ would therefore appear in the numerator of the formula of Ohm. The resistance of such a transverse section would be $r=\frac{\vartheta}{I^2}=\frac{\mu}{I}$. If however the heat-development be not exactly proportional to the intensity, or, in other words, the quantity μ not constant, but increasing with the intensity, we then obtain the case which corresponds to the observations of Lenz and Poggendorff.

The electromotive force of such a circuit, as soon as the current due to polarization has ceased, would, analogous to the constant circuits, be that between zinc and hydrogen: in the language of the contact theory, it would be that between zinc and the negative metal, lessened by the polarization of the latter in hydrogen. We must then regard this maximum of the polarization as independent of the intensity of the current, and differing for different metals exactly as the electromotive forces differ. The numerator of the formula of Ohm, calculated from measurements of intensity with different resistances, can, however, besides the electromotive force, contain a quantity which springs from the resistance at the points of transition, and which is perhaps different for different metals. That such a resistance exists follows out of

the principle of the conservation of force, from the fact that the intensities of these circuits are not to be calculated from the law of Ohm, as the chemical processes remain the same. In support of the view that in circuits where the polarization has ceased, the numerator of the formula of Ohm is dependent on the nature of the negative metal, I have been unable to find any certain observations. In order to set the current of polarization quickly aside, it is necessary to increase as much as possible the density of the current on the polarized plate, partly through the introduction of cells with a constant intensity, and partly by diminishing the surface of the plate. In the experiments of Lenz and Saweljev*, which bear upon this point, the constancy of the current was, according to their own statement, not attained; the electromotive forces calculated from their observations contain therefore those of the currents of polarization also. They found for zinc and copper in sulphuric acid 0·51, for zinc and iron 0·76, for zinc and mercury 0·90.

I may remark, in conclusion, that to prove the equality of the heat developed chemically and electrically, an experiment has been made by Joule†. His method of experiment is however open to many objections‡. He assumes, for example, for the tangent compass, the law of tangents as correct up to the highest degrees; he did not work with constant currents, but calculated the intensity from the mean of the first and last deflections; he also assumes the constancy of the electromotive force, the resistance of the cells and the gas development. Hess has already drawn attention to the divergence of his quantitative determinations of heat from numbers found by others. In a notice in the *Comptes Rendus*, 1843, No. 16, E. Becquerel is said to have corroborated the same law empirically.

We have above seen ourselves necessitated to refer the idea of a contact-force to simple forces of attraction and repulsion, in order to bring it into coincidence with our principle. Let us now endeavour to refer the electric motions between metal and fluid to the same cause. Let us imagine the particles of the

* Bull. de la Classe Phys. Math. de l'Acad. d. Scienc. de St. Pétersburg, t. v. p. 1; and Pogg. *Ann.* lvii. 497.

† Phil. Mag. 1841, vol. xix. p. 275; and 1843, vol. xx. p. 204.

‡ See note to page 131.—Ed.

compound atom of a fluid endowed with different forces of attraction for the electricities, and thus differently electric. When these portions of the atoms are separated at the metallic electrodes, each atom, according to the law of electrolysis, yields up a quantity of $\pm E$, which is independent of the electromotive forces. We can therefore imagine that in the chemical combination itself the atoms are combined with equivalents of $\pm E$, which follow the same laws as the stochiometric equivalents of the ponderable substances in different combinations. If now two different metals are immersed in a fluid, without a chemical process taking place, the positive components will be attracted by the negative metal, and the negative components by the positive metal. The consequence will be an altered direction and distribution of the different electric fluid particles, the recurrence of which we recognise in a current of polarization. The moving force of this current would be the electric difference of the metals, and to this must therefore the intensity at the commencement be proportional; its duration must, the intensities being equal, be proportional to the number of the atoms which spread themselves over the plate, and consequently to the surface of the latter. In the currents which are accompanied by chemical decomposition a permanent equilibrium between the fluid particles and the metals is not attained, because the positively charged surface of the metal is continually removed, being itself converted into a portion of the fluid, and hence a perpetual renewal of the charge must take place behind it. Every atom of the positive metal which, united to an equivalent of positive electricity, enters into the solution, and for which an atom of the negative component is separated neutrally, causes an acceleration of the motion once commenced, whenever the attractive force of the first atom for the $+E$, denoted by a_z, is greater than that of the latter, a_c. The velocity would in this way increase to an unlimited extent, did not the loss of *vis viva* by the development of heat increase also at the same time. It will therefore merely increase until this loss $I^2R\,dt$ is equal to the consumption of tensions $I(a_z - a_c)$, or until

$$I = \frac{a_z - a_c}{R}.$$

I believe that in this division of galvanic currents into those

which occasion polarization and those which give rise to chemical decomposition, as required by the conservation of force, is the only means whereby the difficulties of the theory of contact and of the chemical theory will be alike avoided.

Thermo-electric currents.—In these currents we must seek the origin of force in the actions discovered by Peltier at the place of contact, by which a current opposed to the given one is developed. Let us suppose the case of a constant hydro-electric current into the conducting wire of which a piece of another metal is soldered, the temperatures of the places of union being t' and t'', the electric current will then, during the element of time dt, generate in the entire conduction the heat I^2Rdt; besides this, at one of the points where the metals are soldered together, the quantity $q_I dt$ will be developed, and at the other the quantity $q_{II}dt$ absorbed. Let the electromotive force of the entire circuit be A, hence $AIdt$ the heat to be generated chemically, it then follows from the law of the conservation of force,

$$AI = I^2R + q_I - q_{II}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let the electromotive force of the thermo-circuit be B_t, when one of the soldered junctions possesses the temperature t, and the other any constant temperature whatever, for example 0°; then, for the entire circuit, we have

$$I = \frac{A - B_{t_I} - B_{t_{II}}}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When $t_I = t_{II}$, we have

$$I = \frac{A}{R}.$$

This set in equation (1) gives

$$q_I = q_{II},$$

that is, when the temperatures of the places of soldering are both the same and the intensity of the current constant, the heat developed and that absorbed must be equal, independently of the cross section. If we assumed that the process is the same in every point of the cross section, it would follow that the heat developed in equal spaces of different cross sections is proportional to the density of the current, and from this again, that the quantities generated by different currents in the whole of the transverse sections are directly proportional to the intensity of the current.

When the solderings are of different temperatures, it follows from equations (1) and (2), that

$$(B_{t_I} - B_{t_{II}})I = q_I - q_{II},$$

that is to say, with the same intensity of current both the force which generates and which absorbs the heat increases with the temperature, in the same proportion as the electromotive force.

I am thus far unacquainted with any quantitative experiments with which either of the inferences might be compared.

VI. *Force-equivalent of Magnetism and Electro-magnetism.*

Magnetism.—Through the attractive and repulsive forces which a magnet exerts upon other magnets, or upon soft iron, it is capable of generating a certain *vis viva*. As the phænomena of magnetic attraction may be completely deduced from the assumption of two fluids which attract or repel in the inverse ratio of the square of the distance, it follows, without going further than the deduction made at the commencement of this memoir, that in the motions of magnetic bodies the conservation of force must take place. For the sake of the following theory of induction, we must consider a little more closely the laws of these motions.

1. Let m_I and m_{II} be two magnetic elements, referred to a unit which, at the distance 1, repels an equal quantity of a similar magnetism with the force 1. Let the opposed magnetisms be distinguished by opposite signs, and let r be the distance between m_I and m_{II}, the intensity of their central force is

$$\phi = -\frac{m_I m_{II}}{r^2};$$

the gain of *vis viva* during the passage from an infinite distance to r is $-\frac{m_I m_{II}}{r}$.

2. Let us call this quantity the potential of the two elements, and extending the term potential to magnetic bodies as in the case of electricity, we obtain the gain in *vis viva* during the motion of two bodies whose magnetism does not change, for instance of steel magnets, when we subtract from the potential at the end of the motion its value at the commencement of the motion. The gain of *vis viva* during the motion of magnetic

bodies liable to be changed by induction, will, on the contrary, as in the case of electricity, be measured by the alterations of the sum

$$V+\frac{1}{2}(W_a+W_b),$$

where V denotes the potential of the bodies towards each other, W_a and W_b those of the bodies upon themselves. If B be an unchangeable steel magnet, the approximation of a body with changeable magnetism generates a *vis viva* equal to the increase of the sum $V+\frac{1}{2}W_a$.

3. It is known that the exterior action of a magnet can always be represented by a certain distribution of its fluids over its surface; we can therefore for the potential of the magnet, substitute the potentials of such surfaces; we then find, as in the case of conducting electric surfaces, for a perfectly soft mass of iron A, which is magnetized by a magnet B, the gain C in *vis viva* for the unit of quantity of the positive magnetism, during the passage from the surface of the iron to an infinite distance, given by the equation

$$-QC=V+W_a.$$

Now as every magnet contains equal quantities of north and south magnetism, hence Q in each $=0$, it follows for such a piece of iron, or for a piece of steel of the same form, position, and distribution of magnetism, whose magnetism therefore is completely bound by the magnet B, that

$$V=-W_a.$$

4. But V is the *vis viva* generated by a steel magnet during its approximation until its magnetism is completely bound; according to this equation it must be the same no matter what may be the magnet to which it approaches, provided always that the approximation is continued till the magnetism is completely bound, for W_a remains always the same. The *vis viva* of a piece of iron, on the contrary, which has been approximated till the same distribution is effected, is, as shown above,

$$V+\frac{1}{2}W=-\frac{1}{2}W,$$

hence only half as great as that of the already magnetized mass. It is to be remembered that W is in itself negative, hence $-\frac{1}{2}W$ always positive.

If a piece of unmagnetized steel be caused to approach the influencing magnet, supposing it, when removed, to retain the magnetism imparted to it, $-\frac{1}{2}W$ will then be lost in mechanical force, and for this the magnet thus created is in a condition to produce $-\frac{1}{2}W$ more than the piece of unmagnetized steel.

Electro-magnetism.—Electro-dynamic phænomena have been referred by Ampère to attractive and repulsive forces exerted by the elements of a current, the intensity of which depends only upon the direction and velocity of the current. His deduction does not comprehend the phænomena of induction; the latter, together with the electro-dynamic, have been referred by W. Weber to the attractive and repulsive forces of the electric fluids themselves, the intensity of which depends on the velocity of approximation or of removal, and the increase of this velocity. Up to the present time no hypothesis has been established by which these phænomena could be referred to constant central forces. The laws of induced currents have been developed by Neumann*, by extending the law of Lenz for entire currents to the ultimate particles of the same, and these laws coincide in the case of closed currents with the developments of M. Weber. In like manner the laws of Ampère and Weber for the electro-dynamic actions of closed currents coincide with Grassmann's deduction of the same from rotatory forces†. Experience gives us no further intelligence; because thus far experiment has been resorted to only in the cases of closed or nearly closed currents. We will therefore confine the application of our principle to closed currents, and show that from it the same laws follow.

It has been already shown by Ampère that the electro-dynamic actions of a closed current can be always represented by a certain distribution of the magnetic fluids on a surface which is bounded by the current. Neumann has therefore extended the idea of the potential to closed currents, by setting for a current the potential of such a surface.

5. When a magnet moves under the influence of a current, the *vis viva* gained thereby must be furnished by the tensions consumed in the current. During the portion of time *dt*,

* Poggendorff's *Annalen*, lxvii. 31. † *Ibid.* lxiv. 1.

according to the notation before made use of, these are, $AIdt$ in units of heat, or $aAIdt$ in mechanical units, where a is the mechanical equivalent of the unit of heat. The *vis viva* generated in the path of the current is aI^2Rdt, that gained by the magnet is $I\frac{dV}{dt}dt$, where V represents its potential towards the conductor through which the unit of current passes. Hence

$$aAIdt = aI^2Rdt + I\frac{dV}{dt}dt,$$

consequently

$$I = \frac{A - \frac{1}{a}\cdot\frac{dV}{dt}}{R}.$$

We can distinguish the quantity $\frac{1}{a}\frac{dV}{dt}$ as a new electromotive force, that of the induced current. It always acts against that which moves the magnet in the direction which it follows, or which would increase its velocity. As this force is independent of the intensity of the current, it must remain the same, when before the motion of the magnet no current existed.

If the intensity be changeable, the whole induced current during a certain time is

$$\int Idt = -\frac{1}{aR}\int\frac{dV}{dt}dt = \frac{1}{a}\frac{(V_I - V_{II})}{R},$$

where V_I denotes the potential at the beginning, and V_{II} at the end of the motion. If the magnet comes from a very great distance, we have

$$\int Idt = -\frac{\frac{1}{a}V_{II}}{R},$$

independent of the route or the velocity of the magnet.

We can express the law thus:—The entire electromotive force of the induced current, generated by a change of position of a magnet relative to a closed conductor, is equal to the change which thereby takes place in the potential of the magnet towards the conductor, when the latter is traversed by the current $-\frac{1}{a}$. The unit of the electromotive force is here regarded as that by which the arbitrary unit of current is generated in the unit of resistance,

the latter being that in which the above unit of current developes the unit of heat in the unit of time. The same law is deduced by Neumann, 1. *c.* § 9, only instead of $\frac{1}{a}$ he has an undetermined constant ϵ.

6. When a magnet moves under the influence of a conductor, towards which its potential for the unit of current is ϕ, and of a piece of iron magnetized by this conductor, towards which its potential for the magnetism excited by the unit of current is χ, we have then, as before,

$$aAI = aI^2R + I\frac{d\phi}{dt} + I\frac{d\chi}{dt};$$

hence

$$I = \frac{A - \frac{1}{a}\left(\frac{d\phi}{dt} + \frac{d\chi}{dt}\right)}{R}.$$

The electromotive force of the induced current due to the presence of the piece of iron is therefore

$$-\frac{1}{a}\frac{d\chi}{dt}.$$

If in the electro-magnet the same distribution of the magnetism be effected by the current n, as by the approximated magnet, then, in accordance with what has been stated in No. 4, its potential $n\chi$ towards the magnet must be equal to its potential towards the conducting wire nV, when V denotes the same for the unit of current: therefore $\chi = V$. Hence when an induced current is excited by means of the magnetization of the piece of iron by the magnet, the electromotive force is

$$-\frac{1}{a}\frac{d\chi}{dt} = -\frac{1}{a}\frac{dV}{dt},$$

and, as in No. 7 we have the whole current

$$\int I dt = \frac{\frac{1}{a}(V_{I} - V_{II})}{R},$$

where V_{I} and V_{II} are the potentials of the magnetized iron towards the conducting wire before and after magnetization. Neumann deduces this law from its analogy with the foregoing case.

7. When an electro-magnet has become magnetic under the influence of a current, through the induced current heat is lost;

if the iron be soft the same induced current will proceed in the opposite direction when the circuit is broken, and the heat will be gained again. If it be a piece of steel which retains its magnetism, the heat is permanently lost, and in place of it we gain mechanical magnetic force equal, as shown in No 4, to half the potential of the said magnet when the binding is complete. From analogy with the foregoing cases it does not appear to be improbable that, as Neumann has concluded, the electromotive force corresponds to its entire potential, and that a portion of the motion of the magnetic fluids on account of their velocity is lost as heat, which is gained in the magnet.

8. If two closed conductors of currents be moved towards each other, the intensity of the current may be changed in each. Let V be their potential towards each other for the unit of current, we must then, for the same reason as in the former case, have

$$A_I I_I + A_{II} I_{II} = I_I^2 R_I + I_{II}^2 R_{II} + \frac{1}{a} I_I I_{II} \frac{dV}{dt}.$$

If the intensity of the current in the conductor R_{II} be much less than that in R_I, so that the electromotive force of induction which is excited by R_{II} in R_I vanishes in comparison to the force A_I, and we can set $I = \frac{A_I}{R_I}$, we obtain from the equation

$$I_{II} = \frac{A_{II} - \frac{1}{a} I_I \frac{dV}{dt}}{R_{II}}.$$

The electromotive force of induction is therefore the same that a magnet would generate which possesses the same electrodynamic force as the inducing current. This law has been proved experimentally by W. Weber*.

If, on the contrary, the intensity in R_I be a vanishing quantity compared to that in R_{II}, we find

$$I = \frac{A_I - \frac{1}{a} I_{II} \frac{dV}{dt}}{R_I}.$$

The electromotive forces of the conductors upon each other are therefore equal, when the intensities of the currents are equal, whatever may be the form of the conductor.

Here again the total force of induction, which, during a certain

* Electro-dynamische Massbestimmungen, p. 71-75. See also Scientific Memoirs, Part XX. p. 189.

motion of the conductors towards each other, furnishes a current which itself is unchanged by the induction, is equal to the change of its potential towards the other conductor which is traversed by $-\frac{1}{a}$. In this form the law is deduced by Neumann from the analogy of the magnetic and electro-dynamic forces, l. c. § 10, and he extends it also to the case where the induction in motionless conductors is effected by the strengthening or weakening of the current. W. Weber shows the coincidence of his assumption, with regard to the electro-dynamic forces, with these theorems, l. c. p. 147–153. From the law of the conservation of forces we do not obtain any determination for this case; by the reaction of the induced upon the inducing current a weakening of the latter must occur, which corresponds to a loss of heat equivalent to that gained in the induced current. In the action of the current upon itself, the same relation must exist between the weakening of the current at the commencement, and the extra current. No further consequences can however be deduced here, inasmuch as the form of the augmentation of the currents is not known, and besides this the law of Ohm is not immediately applicable.

Of known natural processes those of organic existences are still to be considered. In plants the processes are chiefly chemical, and besides these a slight development of heat takes place, at least in some: but the principal fact is, that a vast quantity of chemical tensions is here stored up, the equivalent of which we again obtain as heat by the combustion of the plants. The only *vis viva* which we know to be absorbed in the accomplishment of this is that of the chemical solar rays; we are, however, totally at a loss for the means of comparing the force-equivalents which are thereby lost and gained. Animals present some points in this respect which we can lay hold of. These take in oxygen and the complicated oxidizable combinations which are generated by plants, and give back the same, for the most part burnt, as carbonic acid and water, but in part reduced to simpler combinations; hence they consume a certain quantity of chemical tensions, and generate in their place heat and mechanical

force. As the latter compared with the quantity of heat represents but a small quantity of work, the question of the conservation of force is reduced to this, whether the combustion and metamorphosis of the substances which serve as nutriment generate a quantity of heat equal to that given out by animals. According to the experiments of Dulong and Despretz, this question can be approximately answered in the affirmative.

In conclusion I must refer to some remarks of Matteucci's which have been directed against the views advocated in this memoir, and which appear in the *Bib. Univ. de Genève*, No. 16, 1847, 15 May, p. 375. He proceeds from the proposition, that according to the above views a chemical process could not generate so much heat where it at the same time developes electricity, magnetism, or light, as when this is not the case. He takes pains to show by a series of measurements which he adduces, that zinc, during its solution in sulphuric acid, generates just as much heat where the solution is effected directly by chemical affinity as when it forms a circuit with platinum; and that an electric current produces just as much chemical and thermic action while it deflects a magnet as when no such deflection is produced. That Matteucci regards these facts as objections, is due to his total miscomprehension of the views which he undertakes to refute, which will be at once evident from a consideration of our statement of the subject. He then brings forward two calorimetric experiments on the heat which is developed by the combination of caustic baryta with concentrated or dilute sulphuric acid, and on that generated by the same electric current in a wire immersed in gases of different cooling capacities, whereby the above mass and the wire were sometimes glowing and sometimes not. He finds the quantity of heat in the former cases not less than in the latter. When we, however, reflect upon the incompleteness of our calorimetric arrangements, it will not appear extraordinary that differences of cooling through radiation, which are due to the fact that this radiation, according to its luminous or non-luminous nature, passes with less or more difficulty through the surrounding diathermanous bodies, escape observation. In the first experiment of Matteucci the union of the baryta with sulphuric acid was effected in a non-diathermanous leaden vessel, where the luminous rays were

completely prevented from escaping outwards. The imperfections of Matteucci's methods in carrying out these measurements need not be further dwelt upon.

By what has been laid down in the foregoing pages, I believe I have proved that the law in question does not contradict any known fact in natural science, but in a great number of cases is, on the contrary, corroborated in a striking manner. I have endeavoured to state in the most complete manner possible, the inferences which flow from a combination of the law with other known laws of natural phænomena, and which still await their experimental proof. The object of this investigation was to lay before physicists as fully as possible the theoretic and practical importance of a law whose complete corroboration must be regarded as one of the principal problems of the natural philosophy of the future.

[J. T.]

Article V.

On the Connexion of Diamagnetism with Magnetism and Electricity. *By* M. W. Weber.

[From Poggendorff's *Annalen*, vol. lxxxvii. p. 145.]
Extracted from the Memoirs of the Royal Society of Sciences of Saxony, p. 483 to 578; also in a separate work, "Memoirs on Electro-dynamic Measurements," by M. Weber. Leipzig: Wiedemann, 1852.

I. *Theory.*

In treating of magnetism, a distinction is made between *permanent* and *variable* magnets; we regard, for example, a magnet of hard steel as a permanent magnet, and a magnet of soft iron as a *variable* one. Were the antithesis between both classes perfect (which, however, is as little the case as that between conductors and insulators in electricity), the magnetism of the permanent magnet could only be investigated through its *effects*, while that of the variable magnet might be investigated through its *causes* as well as through its *effects*. At all events, even though the antithesis be not perfect, the variable magnet is more favourable to a complete examination of the nature of magnetism than the permanent one.

In the same manner, in treating of diamagnetism, it might be attempted to classify *diamagnets* under the two heads *permanent* and *variable*; but then *we should have no mark by which a permanent magnet could be distinguished from a permanent diamagnet*, and thus the classification would lose all practical significance. In the investigation of diamagnetism, therefore, *variable diamagnets* only are to be considered, which permit of examination partly through their causes and partly through their effects.

Now it is known that the investigation of the magnetism of a magnet through its *effects* (produced on other bodies) leads us to the knowledge of the *ideal distribution* of the magnetic fluid on the surface of a magnet, regarding which Gauss has proved, that as far as the explanation of phænomena is concerned, it answers

completely to the *true internal condition* of the magnet. In many investigations it is a great advantage to find a way furnished by the ideal distribution towards the simple and complete union of all the observed actions, without the necessity of making any hypothesis regarding the interior of the body; more particularly when the causes of these actions remain unknown, and are still to be investigated. From the fact itself, however, that the knowledge of this ideal distribution, derived from observation, affords a satisfactory and complete view of the phænomena, it evidently follows that from *the observed phænomena alone* we cannot proceed further than to the knowledge of this ideal distribution, which however must of necessity be distinguished from the knowledge of the true internal state of the magnet; or, in other words, that proceeding from the *observed actions*, we are not in a condition to pronounce upon the actual distribution of the magnetic fluid within a magnet, or upon the actual number, strength, and arrangement of the electric currents contained within it.

The same holds good for the actions of a *diamagnet*; from the observation of its actions we might arrive at a knowledge of the ideal distribution of the magnetic fluid on the surface of the diamagnet, and thus find a substitute for the knowledge of its true internal state; but we could thus obtain no information regarding the true internal condition, or the real nature of diamagnetism itself, its *generation* and *modifications*. To come upon the trace of these, we must not limit ourselves to the consideration of the *actions*, and the ideal distribution which depends upon them; but it is necessary to call in the aid of some other consideration which is based upon a foundation independent of these actions.

All the possible causes of diamagnetism (like those of magnetism) may be divided in a general manner into *internal* and *external*. The external cause (like the actions) is given by *observation*: it is the same for magnetism and diamagnetism, namely, *a magnetizing or electro-magnetizing force, determinate in magnitude and direction*. If, besides this external cause, that which lies within the magnet itself were known, by the union of both diamagnetism itself would be completely accounted for; and, inversely, we find a way open to the determination of the

true internal cause, where, besides the known external cause, we become (through its actions) acquainted with the diamagnetism which is the resultant of both causes. Following up the way here indicated, and combining the known magnetizing force with the ideal distribution deduced from the observed actions, for iron as well as for bismuth, we learn that the same force of magnetization causes opposite ideal distributions in the cases of iron and bismuth; or, inversely, the same ideal distribution in iron and bismuth corresponds to oppositely directed magnetizing forces. The reason why opposed external causes produce the same effect in iron and bismuth must be referred to different internal causes within the iron and the bismuth themselves. To determine more accurately the difference between the internal causes of iron and bismuth, it is necessary to classify *all possible internal causes* which can produce effects explainable by the ideal distribution, and then to ascertain whether, among all that we can reckon, such are embraced as will enable us to render an account of the above antithesis between magnetic and diamagnetic bodies, subjected to the same external influences.

Classification of the internal causes which may be assumed as the sources of the actions explainable by an ideal distribution.

We can adduce *four* essentially different kinds of internal causes which are capable of producing effects explainable by an ideal distribution:—

1. The internal cause of such effects may be referred to the existence of two magnetic fluids, which are more or less *independent of the ponderable matter which carries them.*

2. They may be due to the existence of two magnetic fluids, which are only capable of moving *in connexion with their ponderable carriers* (rotatory molecular magnets).

3. They may be due to the existence of permanent molecular currents formed by the electric fluids, and which rotate *with the molecules.*

4. They may be due to the existence of electric fluids which can be thrown into *molecular currents.*

These four possible internal causes of the actions explainable by an ideal distribution on the surface are the only ones which are known, and which can be submitted to examination. The

first case forms the basis of the magnetic theory of Coulomb and Poisson. The *third* case forms the basis of the theory of Ampère on the connexion of magnetism with electro-dynamics. The *second* case may be reduced to the third, inasmuch as Ampère has proved that molecular magnets and molecular currents are alike in all their actions, and hence the latter may be substituted for the former. The *fourth* case, therefore, which has heretofore been unattended to, is the only one that remains to be considered.

For each of these four cases there exists a definite connexion between the character of the ideal distribution and the direction of the magnetizing force to which it corresponds. Calling that direction along the line of magnetization in which the north pole of a magnetic needle is driven the *positive*, and determining the centre of gravity of the north and south fluids from their ideal distribution, then for the *first* case, according to the theory of Poisson, we find that the former of these two centres of gravity, as compared with the latter, is situated in the *positive* direction. For the *third* case this connexion has been developed by Ampère, and it has been found that the same dependence of the ideal distribution upon the magnetizing force exists here also. From the reduction of the *second* case to the third, already mentioned, it is evident that the same dependence exists in the second case. Hence, in regard to this dependence, the *fourth* case alone remains open to consideration.

This fourth case assumes the existence of electric fluids which can be thrown into molecular currents. But the *possibility* of being thrown into molecular currents necessitates the inference, that in the single molecules, or around them, closed paths exist in which the said fluids can move without resistance; from which it follows that a *current-exciting force* (a force which acts upon the positive and negative fluids in opposite directions) in the direction of the path is necessary to cause the fluids actually to move along it. Now the facts of magneto-electricity prove that by the increase or diminution of a magnetizing force, a current-exciting force (electromotive) is obtained, which acts upon the electric fluids in opposite directions, and hence must throw them into current motion. By the fundamental law of magnetic induction the direction of this molecular flux is given, in its depend-

ence on the increase or diminution of the magnetizing force; and by the connexion of electro-dynamics and magnetism developed by Ampère for the third case, the ideal distribution is, in its turn, given in its dependence upon the molecular flux. We thus obtain, mediately, the connexion between the *ideal distribution and the increase or diminution of the magnetizing force* to which it corresponds.

But it is evident from the above, that at every moment when an increase or diminution of the magnetizing force takes place, such a molecular flux must be generated, and that these currents thus successively excited, if they do not of themselves again disappear, must *sum* themselves up. These currents however do not vanish of themselves, for Ampère has proved that *persistency* must be ascribed to molecular currents; that is, the electric fluids, in their circular motions around the ponderable molecules, suffer no such resistance as that encountered when the fluids traverse a ponderable conductor, to which resistance is to be attributed the speedy disappearance of the electric currents in these conductors. [From this persistency, which belongs of necessity to the molecular currents, it is manifest that the *possibility* of throwing the electric fluids into molecular currents is to be referred to the fact, that in the molecules, or around them, closed paths exist in which the said fluids move without resistance.] From this it follows, that a continuous increase of the magnetizing force is accompanied by a continuous accumulation of the magnetic fluids, according to the ideal distribution; and hence we infer that *every given strength of the magnetizing force has a definite moment of ideal distribution* corresponding to it. This *summation* however takes place only in the case of molecular currents, for in this case alone the electric fluids move without resistance. Other currents, which are excited by the same force at a greater distance, but which, on account of the resistance experienced, quickly disappear, produce magnetic effects on other bodies only during the moment of their excitation (through increased or diminished magnetizing force). These effects immediately vanish as soon as the force has become constant, and hence stand in no relation whatever to the magnitude of the existing magnetizing force; a relation however must exist if the actions of variable magnets and diamagnets are to be accounted

for, and hence molecular currents alone are available here. Developing with regard to these molecular currents, in accordance with the laws of magnetic induction, the dependence of the moment of ideal distribution upon the magnitude of the magnetizing force in operation, we find, that if in the line of magnetization the direction in which the north pole of a magnetic needle is driven be called the *positive*, and the centres of gravity of the north and south fluids, according to the ideal distribution dependent on the magnetizing force, be determined, the former of these in comparison with the latter is situated in the *negative* direction, which is *exactly the reverse of what takes place in the other three cases*. This enables us to render an account of the inner cause of diamagnetism.

Internal Cause of Diamagnetism.

This remarkable result may be applied to the founding of a theory of diamagnetic phænomena, which shall assign an origin to the forces which produce them, a subject hitherto unexplained. For such a theory it is not sufficient that the diamagnetic state of a body may be conveniently represented by an ideal distribution of the magnetic fluids over its surface, but it is essential that it shall render an account of the *forces* which produce the diamagnetic state, and also of the *laws* according to which these forces act.

From the above statement and consideration of the different possible ways in which a condition representable by an ideal distribution might be developed in a body, one case alone was found from which a law coinciding with the fundamental phænomena of diamagnetism resulted. It follows from this, that an explanation of the development of the diamagnetic state can only be given when this case is regarded as actually existing; according to it the increase of the diamagnetism of a body is proportional to the inducing force acting upon the electric fluids, causing them to move without resistance in definite circular paths around the molecules, and *accelerating* the velocity of their movement in these paths. The diamagnetism of bismuth, for example, is explained by the assumption that the molecules of bismuth contain within them definite paths or canals, in which the electric fluids move without resistance, while in all other

paths these fluids can only be set in motion by first overcoming a resistance proportional to their velocity. The generation of pure diamagnetism (unmixed with magnetism) would further necessitate the assumption, that the molecules which contain the above paths or canals are not capable of being rotated; for were the contrary the case, rotatory molecular currents might be generated, of such a strength that a portion of their intensity during the rotation might be regarded as *constant*, and hence, according to Ampère, would produce the magnetic state as a consequence. Conformably to this assumption, the diamagnetism or electro-diamagnetism of a body can be determined from the magnetizing or electro-magnetizing force exerted upon it.

Determination of the Diamagnetism or Electro-diamagnetism of a Body from the Magnetizing or Electro-magnetizing Force exerted upon it.

The magnetizing or electro-magnetizing force expressed by X* exerts upon a circle of the radius r, electromotive forces whose integral value, for the time during which this circle is moved out of a position perpendicular to the direction of the magnetizing force into a position parallel with it, according to

* Every magnetizing force may be compared with terrestrial magnetism, and reduced to the same unit of measure. The electro-magnetizing force of a cylindrical spiral, through which a current passes of the intensity i, in accordance with the fundamental laws of electro-magnetism, is expressed by $\frac{2\pi ni}{\sqrt{aa+rr}}$, where n signifies the number of coils, r the radius, and a the length of the axis. This value is true, in the first place, for the magnetizing force in the middle of the cylinder, and approximates to that due to every other point of the interior space of the cylinder, excepting those which lie near its end, the more closely as the spiral increases in length and diminishes in radius. When, therefore, a bar of bismuth is situated in the centre of such a spiral, nearly equal electro-magnetizing forces are exerted upon all its particles; and hence it can be moved to or fro between certain limits within the spiral without any perceptible change of these forces. Hence such a spiral is particularly suited to experiments in which it is required that the diamagnetism shall remain unchanged. The above expression gives the electro-magnetizing force referred to the same unit as that of magnetizing forces (namely, to the absolute unit used in the determination of terrestrial magnetism), where i denotes the strength of the bar-magnetism, the action of which is equal to the action of the current circulating round a unit of surface.

Art. 10 of the Measurements of Resistance in my Electro-dynamic Determinations, is

$$=\pi r r X.$$

This integral value is the sum of the products of the electromotive force, reduced to an absolute unit in the Art. 10 aforesaid, into the element of time during which the force acts with this intensity. The expression of this integral value remains unchanged, if instead of moving the circle through an arc of 90°, the magnetizing force X *disappears*. If, on the contrary, this magnetizing force increases from $X=0$ to $X=X$ (by closing the circuit), the expression of this integral value is

$$-\pi r r X,$$

where the negative sign intimates that the induced circular current has such a direction that the poles of an equivalent molecular magnet are directed in an opposite manner to those of a compass-needle under the influence of the force X.

This determination of the integral value of the electromotive force refers to the unit deduced from the absolute measure of magnetism, as established in the place above cited, p. 338 and 339; it must be multiplied by $\sqrt{\frac{1}{2}}$ to render it true for the pure electrodynamic unit of electromotive forces given in Art. 26 of the work cited; hence

$$-\frac{\pi}{\sqrt{2}} . r r X.$$

This expression, multiplied by $\frac{4}{c}$ (where c denotes that constant value of the relative velocity at which *two electric masses exert no influence whatever upon each other*), gives the electromotive force in terms of the absolute unit of measure for all forces, established generally in mechanics (see Art. 27 of the work cited); hence

$$-\frac{2\sqrt{2}}{c} . \pi r r X.$$

This is the value of the electromotive force for the length of the entire circular path, under the assumption that in every unit of length of this path the unit of electric fluid exists. Dividing by the circumference of the circle $2\pi r$, we find the electromotive force exerted upon each unit of electric fluid to be

$$= -\frac{\sqrt{2}}{c} \cdot rX.$$

This, according to the principles of mechanics, expresses *the increase of velocity which would be imparted to each ponderable unit of mass if it were connected with the unit of electricity*, in the time during which the magnetizing force increases from $X=0$ to $X=X$. Let ϵ denote the unknown small fraction of the mass of the ponderable unit which the unit of electricity forms, then the above value divided by ϵ gives the velocity u of the current originated by the given increase of the magnetizing force. If this velocity u be multiplied by $\frac{4}{c}e$, where e denotes the quantity of electric fluid, referred to the electric unit of measure, which exists in each unit of length of the circular path, we obtain the intensity of the induced circular current according to the pure electro-dynamic unit of measure; and when multiplied by $\sqrt{2}$, we obtain it in terms of that unit according to which a current of the intensity 1, while passing round the element of surface, is equivalent to the unit of magnetism, namely,

$$-\frac{8e}{cc\epsilon} \cdot rX.$$

The *electro-magnetic moment* of this induced circular current (molecular current) is found by multiplying the intensity of the current by the area enclosed by the circular path, and is

$$= -\frac{8e}{cc\epsilon} \cdot \pi r^3 X.$$

We have here assumed that the normal to the plane of the circular path is parallel to the direction of the magnetizing force, which can only be the case for *all* circular paths by *one* particular arrangement of the molecules. In the case of bismuth we do not assume such an arrangement, but simply, in accordance with the idea of homogeneity, that the normals to the planes of the circular paths have no paramount direction. According to this, the number of circular paths whose normals make an angle ϕ with the direction of the magnetizing force, must be proportional to $\sin\phi$. The intensity of the current will then be proportional to $\cos\phi$, and the component of the moment parallel to the magnetizing force, to $\cos\phi^2$. If, therefore, we multiply the above value by $\sin\phi\cos\phi^2$, we obtain an expression proportional to

the contribution of all circular currents (molecular currents) the normals of which form an angle ϕ with the direction of the magnetizing force to the *electro-diamagnetic moment* of the bismuth, namely,

$$-\frac{8e}{cc\epsilon}\cdot\pi r^3 X\cdot\sin\phi\cos\phi^2.$$

Multiplying this by $d\phi$, and then, further, the integral taken between the limits $\phi=0$ and $\phi=\frac{1}{2}\pi$ by the number of molecular currents, we obtain the total electro-diamagnetic moment of the bismuth mass m, when μm denotes the number of molecular currents in the mass,

$$=-\frac{8\pi}{3cc\epsilon}\cdot\mu r^3 e\cdot mX.$$

The electro-diamagnetic moment of a mass of bismuth is therefore proportional to the magnetizing force X and to the mass of the bismuth m, and is found by multiplication with a constant factor $\frac{8\pi}{3cc\epsilon}$, taken from the general theory of electricity, and with a factor $\mu r^3 e$ dependent on the nature of the bismuth itself. This last factor we may call the *diamagnetic constant* of bismuth.

In this determination of the electro-diamagnetic moment, the molecular currents induced in the circular paths have been regarded *singly*, as if on each molecule the electromotive force calculated from the force of magnetization X had alone acted. Strictly speaking, however, this is not the case. In each circular path, on the contrary, electromotive forces, resulting from the action of the molecules upon each other, come into play; just as the particle of an iron bar is not affected by the external magnetizing force, for example, the magnetism of the earth, alone, but also by such forces as result from the reciprocal actions among the iron particles themselves. Although this mutual action of the diamagnetic molecules is so small as scarcely to exert a sensible influence, still a remarkable antithesis between the mutual action of magnetic and diamagnetic molecules deserves consideration here.

Comparison of the Mutual Actions of Diamagnetic and Magnetic Molecules.

When two particles of iron are situated in a line parallel to the direction of a magnetizing force X acting upon them, calling

the magnetic moment produced by the magnetizing force in each molecule regarded singly m, there results for each particle an additional magnetizing force, due to the action of the other, by which the magnetic moment m is augmented. This new magnetizing force, resulting from the reciprocal action of the particles, is expressed according to known laws by $\frac{2m}{r^3}$, where r denotes the distance between the particles. The *total* magnetizing force $\left(X+\frac{2m}{r^3}\right)$ produces, therefore, in the particle under consideration, the increased moment $\left(1+\frac{2m}{Xr^3}\right)m$. When, on the contrary, two bismuth particles are situated in a line parallel to the direction of the magnetizing force X, calling the diamagnetic moment corresponding to this force of magnetization $-\mu$ (the negative sign signifies that for similarly directed magnetizing forces the diamagnetic moment is opposed to the magnetic one), then for each particle there results, from the action of the other, a new force $-\frac{2\mu}{r^3}$, where r denotes the distance between the particles; consequently to the *total* force of magnetization $\left(X-\frac{2\mu}{r^3}\right)$ corresponds the *diminished* diamagnetic moment $-\left(1-\frac{2\mu}{Xr^3}\right)\mu$. Hence the antithesis, *that the magnetism of the two iron particles in the line of magnetization is increased by their reciprocal action*; but *that, on the contrary, the diamagnetism of the two bismuth particles lying in this direction is diminished by their reciprocal action.*

The result is the reverse when the iron and bismuth particles lie in a line perpendicular to the direction of the magnetizing force; here the magnetism of the particles of iron is weakened by their reciprocal action; the diamagnetism of the bismuth particles, on the contrary, is strengthened through the same cause. We find, in fact, the weakened magnetism of the iron particle

$$= +\left(1-\frac{m}{Xr^3}\right)m,$$

and the strengthened diamagnetism of the bismuth particle

$$= -\left(1 + \frac{\mu}{X r^3}\right)\mu.$$

From this it follows, that while to impart by a given magnetizing force the strongest magnetism to a given mass of iron, we must convert it into the form of a long thin bar, and set its length parallel to the direction of magnetization; in order to impart the maximum diamagnetism to a given mass of bismuth, we convert it into the thinnest plate possible, and set its thickness parallel to the direction of the magnetizing force. The further development of these laws of the reciprocal action of diamagnetic molecules, compared with that of magnetic molecules, leads finally to a simple distinction of magnetic and diamagnetic substances, which is worthy of more particular examination.

Distinction between Magnetic and Diamagnetic Bodies, through the Positive and Negative Values of a Constant.

For the sake of unity, let us limit ourselves to the consideration of an ellipsoid of revolution of iron or of bismuth, the principal axis of which is parallel to the magnetizing force X; for the case of iron, Neumann has proved that the magnetic moment of the ellipsoid is

$$= \frac{\kappa v X}{1 + 4\pi\kappa S},$$

where v denotes the volume and S a quantity derived from the ratio of the axes of the ellipsoid, namely,

$$S = \sigma(\sigma\sigma - 1)\left\{\frac{1}{2}\log\frac{\sigma+1}{\sigma-1} - \frac{1}{\sigma}\right\}$$

$$= \sqrt{\left(1 - \frac{rr}{\lambda\lambda}\right)},$$

r and $\sqrt{(rr - \lambda\lambda)}$ the axes of the ellipsoid. κ is supposed here to possess a constant value for iron, and to it Neumann has given the name of the *magnetic constant* of iron; this constant quantity in the case of iron, as of all other magnetic bodies, is necessarily *positive*.

The quantity κ serves, therefore, by the different *positive* values which it assumes, as a mark of distinction of the various

magnetic substances; but the use of the quantity κ as a means of distinction may be rendered more general by applying it to all bodies, and permitting it to assume negative values, the physical explanation being attached, that *a body which gives a negative value for κ is a diamagnetic body*. [The name *anti-magnetic* or *negative-magnetic* would, therefore, be more suitable to these bodies.] The negative value of κ found for a diamagnetic body may be called the *magnetic constant of the diamagnetic body*, or we may call the positive value obtained by changing the sign, the *diamagnetic constant* of the body. Denoting this *always-positive* diamagnetic constant by h, to distinguish it from the likewise *always-positive* magnetic constant κ, we obtain, in the same manner as Neumann has determined the magnetic moment of a magnetic ellipsoid, the diamagnetic moment of a diamagnetic ellipsoid,

$$= -\frac{hv\mathrm{X}}{1-4\pi h\mathrm{S}}.$$

Now for an infinitely elongated ellipsoid, for a sphere, and for an infinitely flattened ellipsoid, we obtain successively

$$\mathrm{S}=0, \quad \mathrm{S}=\frac{1}{3}, \quad \mathrm{S}=1;$$

hence the corresponding magnetic moments are, successively,

$$+\kappa v\mathrm{X}, \quad +\frac{\kappa v\mathrm{X}}{1+\frac{4}{3}\pi\kappa}, \quad +\frac{\kappa v\mathrm{X}}{1+4\pi\kappa};$$

the corresponding diamagnetic moments, on the contrary, are

$$-hv\mathrm{X}, \quad -\frac{hv\mathrm{X}}{1-\frac{4}{3}\pi h}, \quad -\frac{hv\mathrm{X}}{1-4\pi h}.$$

The most lengthened form corresponds, therefore, to the weakest, the most flattened form to the strongest diamagnetism; exactly the reverse of what is true for magnetism, as above proved. As, however, the diamagnetic constant h possesses in all known diamagnetic bodies a value which almost vanishes in comparison with the unit, the diamagnetic moment of all these bodies may, without sensible error, be regarded as independent of their form; it may be set

$$= -hv\mathrm{X};$$

and this expression may be compared with that already obtained

for the diamagnetic moment, where the reciprocal actions of the molecules were disregarded. Setting

$$v = \frac{m}{\rho},$$

where m denotes the mass and ρ the density of the body, we obtain for the diamagnetic moment the expression

$$-\frac{h}{\rho} . mX,$$

instead of the expression found above,

$$-\frac{8\pi}{3cc\epsilon} . \mu r^3 e . mX.$$

In both methods the diamagnetic moment is represented as the product of the mass m into the magnetizing force X, multiplied with a *constant coefficient*, which in the last expression consists of two factors, namely, the factor $\frac{8\pi}{3cc\epsilon}$, to be taken from the general theory of electricity, and the factor $\mu r^3 e$ dependent on the nature of the diamagnetic body; this has been already named the *diamagnetic constant* of the body. These two factors are not separated in $\frac{h}{\rho}$; $\frac{h}{\rho}$, indeed, is nothing else than the product of the above two factors.

The quantity κ is here assumed as constant (that is, independent of the strength of the magnetizing force X), because Neumann has proved from the theory of separable magnetic fluids that it must be constant (that is, independent of the strength of the separating force X). The results above stated are, however, independent of this assumption, and retain their validity even should closer examination prove κ to be a function of the magnetizing force X. From this examination, however, it will follow of itself, that even if κ changes with X, h will nevertheless possess *a constant value* for every diamagnetic body.

By the theory of diamagnetism here developed, it is easy to show that the disputed question, *whether magnetic fluids actually exist*, can be decided.

On the Existence of Magnetic Fluids.

When a certain class of actions of one body upon other bodies is so characterized that these actions may be explained

by reference to an ideal distribution of magnetic fluids upon the surface; then, for the true interior condition of the body, four different possibilities may be thought of, and thus four different cases distinguished, which have been above stated and discussed. Two of these cases rested on the assumption that two magnetic fluids exist, either in the rotatory molecules of the body, immoveable, but in *constant* separation; or in non-rotatory molecules, moveable, and in *variable* separation. The two other cases, on the contrary, rested on the assumption that the two electric fluids existed either in a definite circuit round each rotatory molecule of the body in *constant* motion, or round each non-rotatory molecule in *variable* motion. These four cases do not by any means reciprocally exclude each other; for it is easy to see that a portion of the magnetic fluids may remain constantly separated in rotatory molecules, while the separation of another portion is variable; and in the same way a portion of the electric flux in given circular paths round rotatory molecules may be constant, while another portion in circular paths round non-rotatory molecules varies in intensity. In the latter respect, indeed, when we consider the numerous electromotive forces present, the existence of a constant flux without a variable portion is perfectly inconceivable; for the electric fluids, if *free to move* in definite paths, as the existence of constant currents proves, must necessarily obey the impulsion of the electromotive forces decomposed in the direction of these paths. Nevertheless the above four cases may be combined pair-wise to two principal cases, each of which, if actually established, would leave the other in the position of a quite superfluous hypothesis, namely,—1st, that magnetic fluids exist, which with the molecules, or in them, are capable of motion; 2ndly, that the electric fluids, which, according to the doctrine of electricity, are everywhere present, move without resistance in definite circular paths around the molecules.

For each of these two principal cases a theory may be developed, and each of these theories may be divided into two portions, in one of which the results of both theories coincide, and in the other of which they contradict each other; for these theories are similarly circumstanced to the theories of *emission* and of *undulation* in optics, which likewise in many respects

were coincident, until the discovery of the phænomena of interference led to the more accurate investigation of those points in which the theories contradicted each other. Now, although the two theories resulting from the assumptions of magnetic fluids and of molecular currents have heretofore exhibited a surprising coincidence in their results, it might, nevertheless, be expected that here, as in optics, *the discovery of a new class of phænomena* would lead to a closer discussion of the points wherein both theories differ. Both theories, indeed, coincide—1st, in all phænomena which relate to *permanent magnets*; 2ndly, in the circumstance that each permits of a division of variable magnets into two classes, namely, into such as owe their magnetism to the mere arrangement of already existing rotatory molecules (molecular magnets or molecular currents), and into such as owe their magnetism to the excitation of the motion of imponderable fluids in motionless molecules (the separation of magnetic fluids in the molecules, or the excitation of electric currents in definite circular paths around the molecules); 3rdly, both theories agree in their results with regard to the first class of variable magnets. The theories, however, contradict each other in the results which have reference to the second class of variable magnets, for their conclusions *regarding the positions of the poles* are opposed to each other. In accordance with the one, the positions of the poles, in the second class of variable magnets, must be the same as those in the first class; in accordance with the other, the positions of the poles in the second class must be the reverse of those in the first. So long, therefore, as variable magnets in which the positions of the poles (for similarly directed magnetizing forces) were identical were the only ones known, both theories might be applied; but as soon as variable magnets (diamagnets) were discovered, in which the positions of the poles (for similarly directed magnetizing forces) were opposed to each other, no further choice remained between both theories, for the second alone can render an account of the generation of two classes of magnets with poles oppositely situated, the directions of the magnetizing forces being the same.

The diamagnetic phænomena discovered by Faraday serve, therefore, to decide the alternative between both theories, just

as the phænomena of interference served to decide the alternative between the theories of emission and of undulation; and this is the most essential and important character that can be ascribed to the discovery of Faraday. Through the discovery of diamagnetism the hypothesis of electric molecular currents in the interior of bodies is corroborated; and the hypothesis of magnetic fluids in the interior of bodies is refuted,—a result which also finds a corroboration in the closer and more direct examination of variable magnetism, namely, in the law according to which the strength of the variable magnetism is determined from the magnitude of the magnetic or electro-magnetic force; this, however, deserves a closer discussion here.

Dependence of the Variable Magnetism upon the Magnitude of the Magnetic or Electro-magnetic separating Force.

According to the foregoing theory of diamagnetism, the diamagnetic moment of a diamagnet is proportional to the magnitude of the magnetic or electro-magnetic separating force. According to the notion heretofore entertained regarding the moveable magnetic fluids within the molecules of iron, the same proportionality holds good for the magnetic moment of a variable magnet. If, however, this notion, together with the hypothesis of magnetic fluids in the interior of bodies, be rejected, and instead of it Ampère's notion, that the molecules of iron are the ponderable bearers of permanent molecular currents, be assumed, from it will follow a different law of dependence between the variable magnetism and the magnitude of the magnetic or electro-magnetic separating force.

In Plate I., fig. 1, let NS be the axis of an unchangeable molecular current, which is capable of rotation around its centre C; when the magnetizing force X is $=0$, let the position of equilibrium for this axis be parallel to ND. The fact that the magnetism excited in soft iron by the magnetizing force disappears again of itself, as soon as the magnetizing force ceases to act, proves that the molecular current, to the rotation of which the excited magnetism is due, recedes of itself to its original position parallel with ND. This force of recession, which is to be referred to the reciprocal actions of the iron particles, must

increase with the deflection AND, and may be represented by

$$D \sin \phi,$$

where D is a constant quantity which may be named the *molecular force of direction.* If, now, besides this molecular force of direction, the magnetizing force X act upon the molecular current in the direction NX, which encloses the angle XND $= u$ with the line of the directive force, the molecular current will by this be drawn or deflected through the angle AND $= \phi$, and for the determination of the new position of equilibrium we have the following equation,

$$X \sin u \cos \phi = (D + X \cos u) \sin \phi,$$

or

$$\tan \phi = \frac{X \sin u}{D + X \cos u}.$$

From the deflection ϕ the increase of the magnetic moment of the molecular current, decomposed in the direction of the force X, may be determined. If the total unchangeable magnetic moment of the molecular current be denoted by μ, then before the deflection that decomposed in the direction of the force X was

$$= \mu \cos u;$$

after the deflection,

$$= u \cos (u - \phi);$$

hence the required increase x,

$$x = \mu \left(\cos (u - \phi) - \cos u\right).$$

Substituting here for ϕ the value of it as given by the above equation, $\tan \phi = \frac{X \sin u}{D + X \cos u}$, we obtain

$$x = \mu \left\{ \frac{X + D \cos u}{\sqrt{(XX + DD + 2XD \cos u)}} - \cos u \right\}.$$

For a system of molecular currents whose magnetic axes, in their original positions of equilibrium, point in all directions without distinction, the number of molecular currents whose axes form an angle u with the direction NX of the force X are to be set proportional to $\sin u$. Let it be required to determine the magnetic moment y which results from the rotation of all molecular currents of the system, by the force X.

To this end let the value of x above found be multiplied by

sin udu, and let the integral be taken within the limits $u=0$ and $u=\pi$. This integral value, multiplied by the number n of the molecular currents and divided by $\int_0^\pi \sin udu = 2$, gives the required moment y,

$$y=\frac{n}{2}\int_0^\pi x \sin udu.$$

By carrying out the integration we obtain for y the following expression,

$$y=n\mu\frac{X}{\sqrt{(XX+DD)}}\cdot\frac{X^4+\frac{7}{8}XXDD+\frac{2}{5}D^4}{X^4+XXDD+D^4}.$$

The force which acted upon the iron, and by which the moment y was generated, was $=X$. Let n denote the number of molecular currents in the *unit of volume*, then the ratio of the moment y to the force X, in the theory of molecular currents, has the same meaning as the magnetic constant which Neumann has denoted by k, in the theory of separable magnetic fluids. Substituting, therefore, for k in the formula of Neumann given above, $\frac{kvX}{1+4\pi kS}$, the variable value just found $\frac{y}{X}$, we obtain the sought magnetic moment of a variable magnet of the form of an ellipsoid of revolution, to which the formula of Neumann refers,

$$=\frac{vy}{1+4\pi S\frac{y}{X}},$$

where S denotes the factor already determined from the ratio of the axes of the ellipsoid.

This result, referring to the dependence of the variable magnetism on the strength of the magnetizing or electro-magnetizing force derived from the view of Ampère in contradistinction to that usually assumed, is actually corroborated by the experiments described by Müller in Poggendorff's *Annalen*, 1851, vol. lxxxii. p. 181.

II. *Experiments.*

Having in the foregoing pages, for the sake of obtaining a simpler general view, stated, under the title of a theory, the results obtained with regard to the connexion of diamagnetism with magnetism and electricity, we shall, in the present section,

give a brief description of the experiments by which the theory is established.

1. *Electro-diamagnetism, and Measurement of the Moment of an Electro-diamagnet.*

The most convenient arrangement of an electro-diamagnetic apparatus of measurement, for the observation of diamagnetic polarity, consists in a galvanic spiral which is set vertical and symmetrical between the two poles of a magnetic needle bent into the horseshoe form. A (see Plate I. fig. 2) represents the transverse section of the spiral, which lies symmetrically between the poles N and S of the bent magnetic needle NBS. This magnetic needle is held by the clamp DE, in the centre of which, C, the suspending fibre is fixed. Figs. 3 and 4 represent two side views of the instrument. It is of advantage to give the spiral a considerable length, say from 400 to 500 millimetres; this renders it easier to regulate the suspension of the needle so that it shall swing in the horizontal plane which bisects the length of the needle (spiral? J. T.), no moment of rotation being here exerted upon the latter when the current passes through the spiral. If, however, a small moment of rotation should exist, this is easily compensated by a multiplier M (fig. 4) consisting of a few coils, through which the same current is conducted and brought within a suitable distance of the magnetic needle. For the observation of the latter it is necessary to furnish it with a reflector P (fig. 3), and to observe therein, by means of a telescope, the image of a distant scale. The magnetic needle is further surrounded by a damper QQ (fig. 3). The bismuth bar *aa* (figs. 3 and 4) is suspended vertically from a fibre within the spiral; it can be raised or sunk, so that either its under or its upper end shall lie between the poles of the magnetic needle, as represented in figs. 3 and 4. The observations are made most conveniently by means of an arrangement of pulleys or levers, which permits the observer himself, while standing beside his telescope, to raise or sink the bar of bismuth. When the circuit is established and the magnetic needle perfectly at rest, the bismuth bar is raised and the consequent small motion of the needle is observed. As soon as the needle has attained its maximum elongation the bismuth

bar is again suffered to descend; the magnetic needle then moves back with increased velocity. When the maximum elongation on this side has been attained, the bismuth bar is again raised, and so on. Between every two elongations let the position of the bismuth during the intervening time be noted. If the bismuth bar be exchanged for an iron bar of equal length, but very thin, the experimenter can convince himself that when the positions of the bars are the same, the deflections of the needle produced by the iron and the bismuth are opposite in direction.

M. Leyser of Leipzig has constructed this instrument in the simplest and most convenient manner (for 25 dollars without the telescope); it deserves to be particularly recommended for its applicability to this fundamental experiment on diamagnetic polarity. In connexion with the results of certain experiments made by him and Prof. Hankel, M. Leyser communicates to me as follows:—

"A current of four elements of Grove was made use of, and by means of a multiplier the magnet was retained in its former position. The bismuth was chemically pure, and was so suspended that it could be moved up and down by means of a string, without shaking the magnet in the slightest degree.

Observations of Leyser:—

Position of magnet without current		492·0.
With current,	bismuth in the middle . .	493·5
„	bismuth above	490·8
„	bismuth below	499·8
„	bismuth above	491·1
„	bismuth in the middle . .	493·8

We here observe, in coincidence with all other experiments executed in a similar manner, that in *drawing up* the bismuth (from the middle to the top) the position of rest for the magnet moved to *smaller* numbers, and by permitting it to descend (from the middle to the bottom) the position of rest moved to *higher* numbers. The difference between top and bottom amounts to

+ 8·9 divisions of the scale.

Without current,	bismuth in the middle .	492·0
„	bismuth above . . .	497·2
„	bismuth below . . .	490·2
„	bismuth above . . .	498·2
„	bismuth in the middle .	490·0

We here observe that *without a current* the action is opposed to that exhibited when a current is present; the difference between top and bottom being

$$-7{\cdot}5$$ divisions of the scale.

Observations of Prof. Hankel:—

Position of magnet without current 496·5.

With current,	bismuth above	492·1
„	bismuth below	500·7
„	bismuth above	491·6
„	bismuth in the middle . .	497·7

The difference between top and bottom amounts therefore to

$$+8{\cdot}9.$$

Without current,	bismuth in the middle .	497·5
„	bismuth above . . .	503·5
„	bismuth below . . .	498·0
„	bismuth above . . .	502·6
„	bismuth in the middle .	494·8

The difference between top and bottom amounts here to

$$-5{\cdot}0.$$

The bismuth bar was then reversed and the same action was exhibited, namely,—

Position of magnet without current 500·0.

With current,	bismuth above	497·3
„	bismuth below	507·1
„	bismuth above	498·0

The difference between top and bottom being

$$+9{\cdot}4.$$

When, instead of determining the position of rest, the elongations

were observed, while the arc of oscillation was multiplied by the alternate elevation and descent of the bar of bismuth, the following results were obtained:—

		Elongation.	Arc of oscillation.
With current,	bismuth in the middle	500·0	
„	bismuth above	497·0	3·0
„	bismuth below	513·0	16·0
„	bismuth above	481·5	31·5
„	bismuth below	515·5	34·0
„	bismuth above	476·5	39·0
„	bismuth below	520·3	43·8
„	bismuth above	473·0	47·3
„	bismuth below	522·0	49·0
„	bismuth above	471·0	51·0
„	bismuth below	526·0	55·0
„	bismuth above	468·5	57·5

A little bar of iron, suspended instead of the bismuth, when situated above, caused the magnet to move to *higher* numbers, and when below caused it to move to *smaller* numbers; the same was observed when the little bar was reversed. The stand of the instrument must be very heavy; a serpentine stone was found very suitable: the bismuth must be capable of moving freely, and the copper wire pure. In all observations it was found, that applying four of Grove's elements, when the bismuth was *drawn up*, the impulsion amounted to from 8·9 to 9·4 divisions of the scale, the direction being towards *smaller* numbers, while a fine iron wire under the same conditions caused the magnet to move towards *higher* numbers; further, that the bismuth *without a current* acted as an iron wire, and caused a motion through 5 to 7·5 divisions of the scale. When the latter action is taken into account, *the mean diamagnetic impulsion of the bismuth* by the application of four of Grove's elements is found =15·4 divisions of the scale. By multiplication, the arc of oscillation could be increased to 57·5 divisions, and retained at this magnitude, inasmuch as the action of the copper damper which surrounded the magnet held the diamagnetic action in equilibrium."

The following series of experiments was made with an apparatus somewhat different from that just described; a particular

description is however unnecessary, as the difference exercises no particular influence.

Experiments with Bismuth.

No. of oscillation.	Position of bismuth during the oscillation.	Position of needle at the commencement or end of each oscillation.	Arc of oscillation of the needle.	Reduced arc of oscillation.	Mean value.
		500·0			
1	above		−40·0	−63·4	
		467·0			
2	below		−50·4	−66·6	
		513·9			
3	above		−56·3	−67·1	−61·8
		459·9			
4	below		−58·5	−65·5	
		518·5			
5	above		−55·2	−59·4	
		460·0			
6	below		−46·5	−48·8	
		512·0			
7	above		+29·7		
		471·1			
8			+ 7·0		
		489·7			
9	below		− 8·9		
		494·2			
10	above		−15·6		
		480·9			
11	below		−30·0	−47·5	
		498·9			
12	above		−50·4	−66·6	
		457·0			−59·8
13	below		−57·8	−68·5	
		516·0			
14	above		−50·9	−56·8	
		459·3			
15	below		+35·6		
		504·4			
16			+12·4		
		478·3			
17	above		−14·7		
		476·9			
18	below		−36·6		
		504·9			
19	above		−42·6	−67·5	
		459·6			
20	below		−39·6	−52·3	
		499·4			
21	above		−46·6	−55·5	
		460·1			−56·1
22	below		−51·7	−57·9	
		513·9			
23	above		−45·9	−49·4	
		464·2			
24	below		−50·6	−53·1	
		506·2			
25	above		−55·2	−57·0	
		446·9			
26			+44·5		
		498·0			
27	below		+15·5		
		460·0			
28	above		−16·8		
		453·1			
29	below		−29·8		
		479·8			
30	above		−40·3	−63·9	
		446·9			
31	below		−46·0	−60·2	
		494·6			−55·8
32	above		−42·2	−50·0	
		450·4			
33	below		−44·0	−49·3	
		490·5			
34	above				
		442·6			

In the *third* column of this table the positions of the needle observed at the beginning and end of each oscillation are noted; in the fourth column the corresponding arcs of oscillation (the mean of every two successive ones). A *positive* sign before the arc of oscillation denotes that when the position of the bismuth was *above*, the needle proceeded from smaller to larger numbers, or when the bismuth was *below*, from larger to smaller numbers; the reverse applies to the *negative* sign. After the position of

the bismuth had been several times regularly changed at the end of each oscillation, and the limit of the arc of oscillation nearly attained, an interruption was effected by permitting the bismuth to remain unmoved during two oscillations, and then again regularly changed as before. The *negative* arc of oscillation was thereby suddenly converted into a *positive* one, which, however, soon diminished to zero, and very soon afterwards passed over into a *negative*; the deflection caused by the bismuth (in its upper and lower positions) was here most clearly exhibited. When the arcs of oscillation are counted from that which is nearest to zero, the arc nearest to the limit may, by means of the known *decrementum logarithmicum*, be easily reduced to the *limit-value*, and thus a more accurate mean value for the latter may be found. For this purpose we have only to divide the observed value of the nth arc of oscillation in the above experiments, where the *decrementum logarithmicum* was nearly $= \log.\frac{3}{2}$ by $\left(1-\left(\frac{2}{3}\right)^n\right)$. We thus obtain the reduced values exhibited in the *fifth* column, and the mean values derived from the latter in the *sixth* column. From all the observations taken together, we find the limit-value to be

$$x = -58{\cdot}4.$$

From this limit-value of the arc of oscillation, the deflection E corresponding to the needle's position of equilibrium can be deduced, namely $E = -5{\cdot}93$; or taking the mean of several series of experiments made by different observers,

$$E = -5{\cdot}17,$$

while for a bar of iron of 59200 times less weight, the same value, determined by similar experiments, was found to be

$$E' = +128{\cdot}4.$$

From this we learn, by reduction to the same weight, that the diamagnetism of the bismuth is 1470000 times less than the magnetism of the iron. The result, however, is only true for a particular form of the iron bar and for a definite strength of the magnetizing force, namely $X = 629{\cdot}9$, a number obtained from the measured strength of current and from the coils of the electro-magnetic spiral.

Diamagneto-electricity,—Measurement of Electric Currents Diamagnetically Induced.

The apparatus for diamagnetic induction to be first described is so arranged that the induction is excited solely by *the motion of the diamagnetic body*, while the spiral *remains at rest*, and the diamagnetism of the body *remains unchanged*; by this means the formation of galvanic currents in the bismuth *as a conductor* is avoided, and with it a secondary inductive action, easy to be mistaken for the diamagnetic induction. The practical construction of such an apparatus consists in the application of a galvanic spiral, through whose electro-magnetizing force a bar of bismuth placed in its centre may, as remarked above, be uniformly diamagnetized and moved to and fro within certain limits, without suffering any change as to the strength of its diamagnetism.

The Electro-diamagnet made use of for the Diamagnetic Induction.

The electro-diamagnet made use of for diamagnetic induction consisted of a bar of bismuth, a long wire spiral, A*cccc*, fig 5, Plate I., through which a current from eight of Bunsen's couples was conducted. The bismuth bar was 186 millims. in length, and weighed 339300 milligrms. The spiral consisted of copper wire covered with wool, and afterwards coated with gutta percha; the pure copper wire was 2·3 millims. thick, and formed eight layers one above the other, each of which was composed of 120 coils. The whole spiral was 383 millims. in length, and had an interior diameter of 23·9 millims., and an exterior diameter of 70 millims.

The Induction-spiral.

The induction-spiral, A*bbbb*, fig. 5, Plate I., is that in which a current is to be induced *by the motion of the electro-diamagnet*. This spiral must be carefully insulated from that which belongs to the electro-diamagnet, through which the current passes from the galvanic battery, and connected with a multiplier, by which the induced current may be observed. This spiral consisted of

copper wire overspun with silk, and 1 millim. thick, forming three layers, each of which contained 294 coils; the length of the spiral was 383 millims., the interior diameter was 19, and the exterior 23 millims. Surrounded with gutta percha for the sake of better insulation, it was inclosed in the wider tube of the spiral of the electro-diamagnet, or rather the latter spiral was coiled round it.

The most essential point relating to this spiral is, that its length must be divided into two perfectly symmetrical and symmetrically coiled halves. The wire is not continuously coiled throughout the entire length in the same direction, but the spiral divides itself into two equal portions, which are coiled in opposite directions. This is necessary, if through the motion of a diamagnetic bar of bismuth, or a magnetic one of iron, a current is to be induced in the spiral; for when this inducing bar is placed in the centre of the spiral and moved there, the force of induction exerted by its north end in one half of the spiral is exactly opposed to that exerted by its south end in the other half, and the action of both would destroy each other if the two halves of the spiral were coiled in the same direction. Through the opposed coiling, the inductive forces, instead of destroying each other, are caused to exert a twofold action.

This necessary arrangement, for the purposes of induction, presents an important advantage as regards the practical carrying out of the experiments. It is manifest that the current of the galvanic battery, as long as it is *constant* in the spiral of the electro-diamagnet, can exert no inductive action on the induction-spiral; the slightest alteration of its intensity, however, would be sufficient to induce in the spiral a current much stronger than that diamagnetically induced, and which therefore would prevent the observation of the latter. It is, however, manifest that the same arrangement of the induction-spiral by which we have secured that the diamagnetic induction in both halves shall be a twofold power, also effects the mutual destruction of the currents excited in the two halves of the induction-spiral by the galvanic current; so that if the symmetry of the two halves be perfect, the greatest changes of intensity on the part of the current can produce no effect. To this we may

add,—1st, that it is very easy to prove if this compensation actually exists, by effecting, not small changes of intensity, but by interrupting or commutating the entire current; 2nd, that if it should appear that this compensation is not complete, it is very easily rendered so by winding an end of the induction-spiral once or oftener round the end of the spiral of the electro-diamagnet. In this way it is easy to rescue the diamagnetic induction from every foreign influence.

The remaining portions of the Induction-apparatus.

With regard to the arrangement of the remaining portions, which are more or less left to the discretion of the observer, I would make the following remarks. To move the bar of bismuth to and fro in the induction-spiral, I connect it with a crank, attached to the wheel B, fig. 5, Plate I. To cause the current excited in the induction-spiral by the forward motion of the bismuth to have the same direction through the multiplier as that excited by its backward motion, a commutator *dd* is attached to the wheel, which moves with the latter, and by which the connexion of the ends of the spiral with the wire of the multiplier is reversed at the end of every semi-revolution (at the moment when the bismuth attains the commencement or end of its path). The uniform direction of all induced currents through the multiplier would be followed by a uniform deflection of the needle towards the same side. To enable the observer to produce a deflection towards the other side, a second commutator, *ee*, fig. 5, is placed beside the telescope E, which can be changed by the observer himself, and which may be called the *subsidiary commutator*. The following must be particularly attended to,—1st, that the induction must be increased by increasing the velocity of the wheel, rather than by lengthening the path traversed by the bismuth; 2nd, that no thermo-magnetic current shall be generated at the rotating commutator; it must be so arranged that metals of the same kind only rub against each other. The influence of such currents, when very feeble, may be readily eliminated by suitably combining the observations. Finally, to obtain an approximate idea of the strength of the galvanic cur-

rent, an ordinary compass is placed at a proper distance from the spiral of the electro-diamagnet, so that the deflection produced by the current passing through the spiral may be conveniently measured. The experiments were carried out as follows:—1. The direction of the current being *normal*, the commutator was caused to rotate, and the bar of bismuth at the same time set in motion to and fro within the induction-spiral. At each elongation of the galvanometric needle, the observer changed the subsidiary commutator, until the arc of oscillation thus multiplied approached its limit-value; 2nd, the same series of experiments was then made with the direction of the current *reversed*; 3rd, with the current again in the *normal* direction; 4th, with the *reversed* current; and finally, 5th, once more with the *normal* current; 6th, the bismuth bar was exchanged for a thin *bar of iron*, and its induction measured in the same manner, with the current *normally* directed.

1. *Induction of the Bismuth Bar with a* Normal *Current.*

No. of oscillation.	Position of subsidiary commutator.	Position of needle at beginning and end of each oscillation.	Arc of oscillation of the needle.	Reduced arc of oscillation.	Mean value.	Deflection of compass.
		475·3				
1	−	472·8	+ 3·70			
2	+	477·7	+ 5·40			
3	−	471·8	+ 6·80			
4	+	479·5	+ 8·35			
5	−	470·5	+ 9·65			
6	+	480·8	+10·55			
7	−	470·0	+11·40	+13·20		
8	+	482·0	+12·25	+13·65		
9	−	469·5	+12·70	+13·75	+13·60	32° 10′ W.
10	+	482·4	+13·00	+13·80		
11	−	469·3				

2. *With a* Reversed *Current.*

		503·5				
1	+	515·9	+ 9·50			
2	−	509·3	+ 3·65			
3	+	510·0	− 1·25			
4	−	513·2	− 4·75			
5	+	506·9	− 7·35			
6	−	515·3	− 8·90			
7	+	505·9	− 9·60	−14·12		
8	−	515·7	− 9·95	−13·10	−13·08	31° 50′ E.
9	+	505·6	− 9·85	−12·02		
10	−	515·2				

No. of oscillation.	Position of subsidiary commutator.	Position of needle at beginning and end of each oscillation.	Arc of oscillation of the needle.	Reduced arc of oscillation.	Mean value.	Deflection ψ of compass.
		3. *With a* Normal *Current.*				
		480·5				
1	+	471·0	− 7·15			
2	−	475·8	− 2·80			
3	+	475·0	+ 0·85			
4	−	472·5	+ 3·80			
5	+	477·6	+ 6·25			
6	−	470·2	+ 8·05			
7	+	478·9	+ 9·25			
8	−	469·1	+10·00	+13·17		
9	+	479·3	+10·75	+13·12	+13·06	31° 48′ W.
10	−	468·0	+11·30	+13·08		
11	+	479·3	+11·30	+12·88		
12	−	468·0				
		4. *With a* Reversed *Current.*				
		501·5				
1	+	515·0	+10·15			
2	−	508·2	+ 4·30			
3	+	510·0	− 0·05			
4	−	511·9	− 3·40			
5	+	507·0	− 5·60			
6	−	513·3	− 7·25			
7	+	505·1	− 8·65			
8	−	514·2	− 9·65			
9	+	504·0	−10·10	−12·33		
10	−	514·0	−10·55	−12·21		
11	+	502·9	−11·00	−12·25	−12·16	32° 13′ E.
12	−	513·8	−11·30	−12·24		
13	+	502·1	−11·45	−12·15		
14	−	513·3	−11·25	−11·76		
15	+	502·0				
		5. *With a* Normal *Current.*				
		486·0				
1	+	461·0	−20·40			
2	−	476·8	−12·40			
3	+	467·8	− 6·15			
4	−	471·1	− 1·25			
5	+	471·9	+ 2·75			
6	−	467·2	+ 5·75			
7	+	474·0	+ 7·10			
8	−	466·6	+ 7·30			
9	+	473·8	+ 7·75			
10	−	465·5	+ 8·90	+10·86		
11	+	475·0	+ 9·70	+11·23		
12	−	465·1	+10·05	+11·20	+10·95	31° 39′ W.
13	+	475·3	+10·25	+11·10		
14	−	465·0	+10·15	+10·77		
15	+	475·0	+10·10	+10·56		
16	−	464·8				

6. *Induction of the Iron Bar with a* Normal *Current.*

No. of oscillation.	Position of subsidiary commutator.	Position of needle at beginning and end of each oscillation.	Arc of oscillation of the needle.	Reduced arc of oscillation.	Mean value.	Deflection of compass.
		461·0				
1	+	457·2	−15·30			
2	−	484·0	−33·65	•		
3	+	443·5	−45·60			
4	−	494·2	−54·95	•		
5	+	435·0	−62·20			
6	−	500·2	−67·45			
7	+	430·5	−71·50	−84·98		
8	−	503·8	−74·50	−84·60		
9	+	428·1	−76·90	−84·47		
10	−	506·2	−78·60	−84·28		
11	+	427·1	−79·90	−84·16		
12	−	507·8	−80·85	−84·04	−83 876	31° 48′ W.
13	+	426·8	−81·10	−83·50		
14	−	508·0	−81·30	−83·10		
15	+	426·6	−81·50	−82·85		
16	−	508·2	−81·75	−82·78		
17	+	426·3				

If we denote the very trifling influence exerted by the thermo-magnetic current upon the result by x, x^{I}, x^{II}, x^{III}, x^{IV}, and neglect the still smaller differences $x-x^{\mathrm{I}}$, $x^{\mathrm{I}}-x^{\mathrm{II}}$, $x^{\mathrm{II}}-x^{\mathrm{III}}$, $x^{\mathrm{III}}-x^{\mathrm{IV}}$, we obtain the following results for the limit-value corresponding to the *diamagnetic induction* alone, reduced to the *normal* direction:—

from 1	$+13{\cdot}60+x$	$+13{\cdot}34$	Mean.
2	$+13{\cdot}08-x^{\mathrm{I}}$	$+13{\cdot}07$	$+12{\cdot}644.$
3	$+13{\cdot}06+x^{\mathrm{II}}$	$+12{\cdot}61$	
4	$+12{\cdot}16-x^{\mathrm{III}}$	$+11{\cdot}555$	
5	$+10{\cdot}95+x^{\mathrm{IV}}$		

From this limit-value of the arc of oscillation produced by the uniform distribution of the inductive shocks (by the motion of the bismuth to and fro) over the entire time of oscillation, it is easy to deduce the limit-value which would correspond to a concentration of all the inductive shocks during the oscillation, into a single moment. The value of the arc of oscillation above found, $= +12{\cdot}644$, must for this purpose be multiplied by $\frac{\pi}{2}$, or more accurately, taking the influence of the damper into account, with $1{\cdot}574235$, by which we obtain the sought limit-value

$$= +19{\cdot}905.$$

For the bar of iron (where all the inductive shocks were thus concentrated), the corresponding limit-value is found to be

$$= -83{\cdot}876.$$

From a great number of similar experiments, executed by various observers, we find the ratio of the limit-value for bismuth to that for iron to be

$$+16{\cdot}956 : -83{\cdot}49.$$

Now the intensity of the currents induced by bismuth and by iron is *directly proportional* to these limit-values, and *inversely proportional* to the number of inductive shocks during a time of oscillation (that is, as

$$1 : 216{\cdot}2,$$

because in the experiments with the bismuth bar 216·2 shocks, and with the iron bar only 1 shock, took place during a time of oscillation). Hence *the currents induced by the diamagnetic bar of bismuth were opposed in direction to those induced by the magnetic bar of iron, and their intensities were in the ratio of*

$$16{\cdot}956 : 83{\cdot}49 \,.\, 216{\cdot}2 = 1 : 1064{\cdot}5,$$

although the bismuth bar weighed 339300 milligrammes, and the iron bar only 790·86. From this we find by calculation, that a bar of bismuth of the *same weight* as a bar of iron would induce a current 456700 times weaker than the latter. This result is only true however for a definite form of the bar of iron, and for a definite amount of magnetizing force, namely $X = 3012$, which is determined from the measured strength of the current, and the number of coils of the electro-magnetic spiral.

On the dependence of the Strength of the Magnetism of Changeable Magnets upon the amount of the Magnetizing or Electromagnetizing Force.

From the theory of diamagnetism, in connexion with that of magnetism, discussed in the foregoing pages, it follows, that the assumption of two magnetic fluids capable of free motion in the molecules of iron, hitherto made in the case of magnetism, (and from which the proportionality of the magnetism to the magnetizing force follows as a consequence) is not admissible, and that

hence the hypothesis of Ampère, according to which the molecules are the rotatory bearers of permanent molecular currents, and no proportionality between the magnetism and the magnetizing force exists, must be set in its place. This consequence of the theory admits of being brought to the test of experiment, and with reference to it the experiments of Müller have been already adduced. Other experiments have however been made, particularly by Buff and Zamminer, which have led to different results. Before repeating these experiments it will be first necessary to state the conditions upon which a certain decision of the point depends.

From Müller's experiments, it appeared that the divergence of the magnetism of iron from a proportionality with the magnetizing force exhibited itself at much smaller intensities of the latter in the case of *thin* bars than when *thick* bars were made use of. In the comparison of Müller's experiments with those of Buff and Zamminer, we must remember that the thinnest bar made use of by Müller was only 6 millimetres thick, while the thinnest of those used by Buff and Zamminer was 9 millimetres thick; and this difference was rendered more influential by the relation of the length to the thickness; Müller's thin bar was 330 millimetres long, while that of Buff and Zamminer was only 200 millimetres. In the following experiments a still thinner bar than that of Müller was made use of, namely, one 3·66 millimetres in thickness, 100·2 in length, and 8190 milligrammes in weight. The magnetism of such a thin bar may be measured with great exactness by the deflection which it is able to produce upon a small magnetometer, placed at a distance, and observed by means of a telescope and mirror. The only difficulty here is the proper separation of the action of the magnetized iron from that of the galvanic current. It is manifest that when the same galvanic spiral is applied to the magnetization of thick and thin bars, as has been the case with Müller, Buff and Zamminer, the above separation admits of less accuracy in the case of thin bars; for here, inasmuch as the action of the galvanic spiral remains the same, its comparative influence will be greater than when thick bars are applied. In the following experiments, therefore, a spiral was made use of which tightly embraced the thin bar; and besides this, it was arranged

that together with these narrow convolutions, each spiral formed two greater convolutions, which were traversed by the current in an opposite direction, and which embraced an area equal to that embraced by all the narrow convolutions taken together. In accordance with the known laws of electro-magnetism, we could here have no immediate action exerted by the current upon the magnetometer, a conclusion capable of easy proof. The entire action exhibited by the magnetometer was therefore due to the *magnetism* of the *iron* alone, and may be determined with equal sharpness and exactitude as that of permanent magnets, according to the directions given by Gauss in the *Intensitas*, &c. It is especially to be mentioned that the spirals made use of by Müller, Buff and Zamminer were shorter than the bars of iron which they were used to magnetize. With Müller this difference was but trifling, for the iron bar projected only 15 millimetres beyond the ends of the spiral. With Buff and Zamminer it was however much greater, for here the ends of the largest and thinnest bar projected 45 millimetres beyond the two ends of the spiral. The injurious influence of this was increased by the circumstance that the length embraced within the spiral amounted to only 110 millimetres, whereas in the case of Müller the enclosed length was 300 millimetres. This circumstance is probably the chief cause of the divergence in the results arrived at by these observers; for it is manifest that the action of the spiral upon the iron is greatest at the centre, decreasing towards the ends, and that this decrease must, beyond the limits of the spiral, be exceedingly speedy. Hence, although the action produced by the galvanic current upon the central portions of the bar may have nearly attained its limit-value, it by no means follows that this is the case with the portions without the spiral. To effect this approximation at all points of the iron bar simultaneously, in the following experiments a spiral was made use of which was considerably longer than the bar of iron, so that the force exerted by the spiral (whose diameter was very small in comparison to its length) upon the ends of the bar did not differ sensibly from that exerted upon its centre, by which precaution alone secure results could be obtained.

Without entering upon the details of these experiments, I will here limit myself to a tabular statement of the results ob-

tained in this way, and merely remark that each determination is the result of four changes of current in the spiral, the most exact coincidence being in all cases exhibited, a proof that the coercive force of the iron did not operate to the prejudice of the measurements. The reduction, according to known rules, of the magnetism of the iron to an absolute unit, requires no further explanation. The strength of the current, measured by means of a tangent galvanometer, is also reduced to an absolute unit, and the correction depending upon the ratio of the length of the needle to the diameter of the ring is taken into account. From the strength of the current thus determined, the number of convolutions and the dimensions of the spiral, the magnetizing force operating upon the iron was finally calculated, in terms of the same unit as that to which the terrestrial magnetic force is referred; it appears in the *second* column of the following table under X. The magnetism of the iron M, divided by the mass of the iron $p = 8190$, and thus reduced to the unit of mass, is exhibited in the *third* column under *m*.

No.	X.	*m*.
1	658·9	911·1
2	1381·5	1424·0
3	1792·0	1547·9
4	2151·0	1627·3
5	2432·8	1680·7
6	2757·0	1722·7
7	3090·6	1767·3
8	3186·0	1787·7
9	2645·6	1707·9
10	2232·1	1654·0
11	1918·7	1584·1
12	1551·2	1488·9
13	1133·1	1327·9
14	670·3	952·0

By these experiments the result obtained by Müller is confirmed, and it only remains to ascertain whether the change in the strength of the magnetism of the iron effected by the action of different forces of magnetization, here exhibited, coincides with the law deduced at the end of the first section, from the assumption of a *definite capacity of rotation on the part of the molecules*. If this be the case, it evidently follows that we can assume, with Ampère, that these molecules are the bearers of

molecular currents, by which assumption the *generation and changes of the magnetism of iron are rendered quite independent of the idea of magnetic fluids, and are reduced to the assumption of moveable electric fluids alone.*

According to the assumption already made (at the end of the first section) of a definite capacity of rotation on the part of the molecules, the character of every body, in magnetical respects, is determined by two distinctive marks,—1st, *by the product of the magnetic moment of a molecule* (in the direction of its magnetic axis) *into the number of molecules of the body*; 2ndly, *by the constant to which we have given the name of the molecular force of direction.* The product is denoted above by $n\mu$, the constant by D. Setting in the case of iron,

$$n\mu = 2324{\cdot}68,$$
$$D = 276{\cdot}39,$$

where the number of molecules n is referred to the *unit of mass*, we obtain from the formulæ given at the end of the first section, reference being made to the unit of mass instead of to the unit of volume, the density of iron being taken at 7·78, and the numerical factor S referring to a bar 100·2 millimetres long and 3·66 millimetres thick, that is, $S = \frac{1}{249}$—the following contemporaneous values of X and m.

X.	m.
658·9	948·4
1381·5	1387·0
1792·0	1533·0
2151·0	1623·5
2432·8	1685·0
2757·0	1742·2
3090·6	1791·2
3186·0	1803·4
2645·6	1723·6
2232·1	1644·8
1918·7	1568·9
1551·2	1452·9
1133·1	1276·8
670·3	957·5

It will be observed that these calculated values differ but little from those contained in the foregoing table, and which were derived from observation.

According to the same formulæ, we obtain for the iron bar with which, by a comparison of the *magnetic actions*, m was found to be 1470000 times greater than its value for bismuth, X being $=629{\cdot}9$,

$$m = 2134;$$

hence for bismuth, where

$$X = 629{\cdot}9, \quad m = \frac{2134}{1470000} = \frac{1}{689}.$$

For the iron bar, on the contrary, with which, by a comparison of the *inductive actions*, m was found to be 360740 greater than for bismuth, X being $=3012$, we have

$$m = 2305{\cdot}4;$$

hence for bismuth, where

$$X = 3012, \quad m = \frac{2305{\cdot}4}{360740} = \frac{1}{156{\cdot}5}.$$

According to this it appears that when the magnetizing force is increased 4·8 times, the diamagnetism of the bismuth increases 4·4 times; that is, *nearly proportional*, although one determination is founded on a comparison of the *magnetic actions*, the other upon a comparison of the *inductive actions*. We thus find the proposition confirmed, that *the relation of the inductive actions to the magnetic ones is the same in the case of diamagnetic bismuth as in that of magnetic iron.* Reducing diamagnetism to the same *absolute unit* as magnetism, we obtain finally *the strength of the diamagnetism of* 1 *milligrm. of bismuth*, operated upon by the magnetizing force $X = 1$,

$$= \frac{1}{452000},$$

and the strength *of the magnetism of* 1 *milligrm. of iron*, under the influence of the magnetizing force $X = 1$,

$$= 5{\cdot}6074,$$

that is, *the magnetism of a thin bar of iron exceeds the diamagnetism of an equal mass of bismuth, when small and equal magnetizing forces operate upon both, about* $2\frac{1}{2}$ *million times.* For thicker bars and greater magnetizing forces this number is found to be somewhat smaller.

[J. T.]

ARTICLE VI.

On the Work performed and the Heat generated in a Conductor by a Stationary Electric Current. By R. CLAUSIUS.

[From Poggendorff's *Annalen*, vol. lxxxvi. p. 415.]

THE following is a continuation of the paper contained in the last part of these memoirs (p. 1). The inquiry there instituted into the effects of a sudden discharge I have here endeavoured to extend to the case of a continuous electric current. At present, however, I will confine myself to an especial case. I assume that the current, which I suppose to be constant, is confined to a definite conductor, and regarding the latter I make the following assumptions:—1st, that the conductor suffers neither mechanical nor chemical change by the passage of the current; 2ndly, that nowhere, within the conductor, a source of electromotive power exists, which will be the case when the conductor is composed of one material, in the same condition throughout; and 3rdly, that no inducing actions be exerted between it and other conductors or magnets.

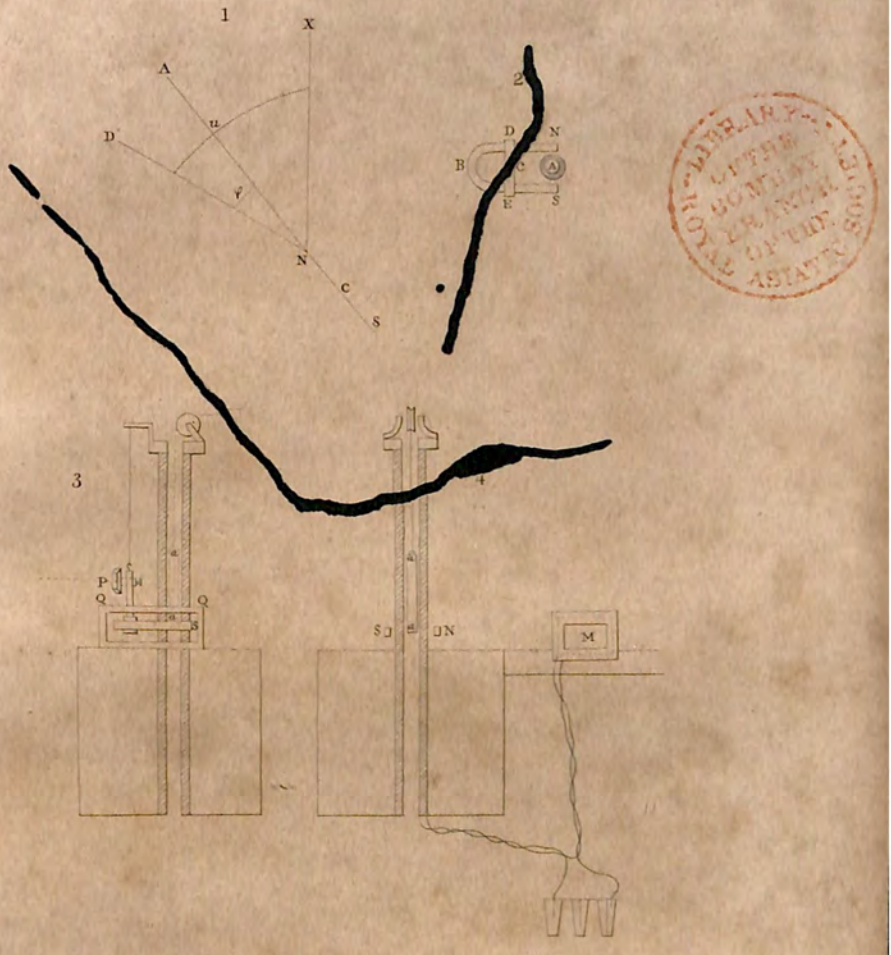
In this case the only effect produced by the electric current is the heating of the conductor. The laws which regulate the generation of heat, in the ordinary case where the conductor possesses the form of a wire, have been empirically established by Joule*, Lenz† and Becquerel‡, who have found that the heat developed in a wire in the unit of time is proportional to its resistance, and to the square of the intensity of the current. Among the theoretic investigations connected with this subject, I must particularly mention a paper by W. Thomson in the Philosophical Magazine§. The results of this paper coincide, as far as they refer to the same points, with those contained in the present memoir, for in both of them the theoretic necessity

* Phil. Mag., S. 3. vol. xix. p. 261, and S. 4. vol. iii. p. 486.

† Poggendorff's *Annalen*, vol. lxi. p. 44.

‡ *Ann. de Chim. et de Phys.*, S. 3. vol. ix. p. 21.

§ S. 4. vol. ii. p. 551.


1
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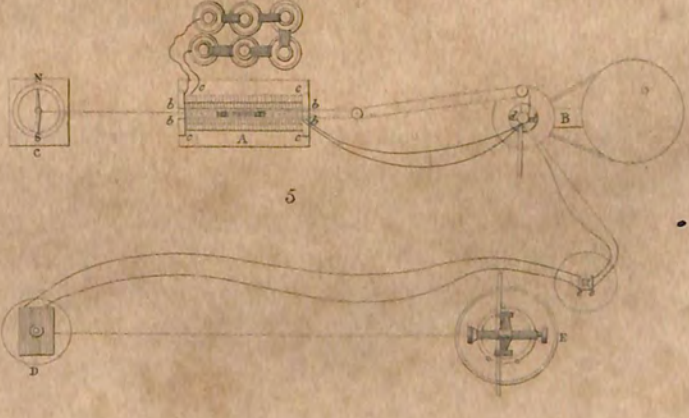
N
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of the above empirical law is established. The way, however, by which Thomson arrived at this proposition, as well as the form which he has given to it, are very different from mine. He proceeds from the laws of electro-magnetic induction, and makes use of the law of Ohm, while I make use of the latter only in the form which Kirchhoff has given to it. His attention is further exclusively confined to a lineal conductor, while my results are independent of the form of the conductor, and hence comprehend the lineal ones as a special case. This reason alone would have been a sufficient inducement to me to publish my inquiry, although his has preceded it; but I have a further inducement in the fact, that the principle established in the following pages derives additional interest from its great similarity with that already proved in the case of machine electricity*.

The law of Ohm, so far as it refers to the process within a homogeneous conductor, may be expressed quite generally in the following manner. Let dw be any element of surface within the conductor, N the normal upon it, and idw the quantity of electricity which passes through it during the unit of time, where i is to be regarded as positive or negative, according as the electricity, in reference to the normal N, passes from the negative to the positive side of the element, or in a direction contrary to this; we have then the equation

$$i = k\frac{dV}{dN}, \quad \ldots\ldots\ldots \quad (1)$$

where k represents the conductibility of the body, and V is a function which, as soon as the stationary condition has commenced, is dependent solely on the coordinates of space.

For in every point of the traversed conductor a force must act sufficient to retain the electricity in motion, notwithstanding the resistance continually opposed to it, and the differential coefficient $\frac{dV}{dN}$ evidently represents the component of this force in the direction of the normal. Nevertheless, the physical signification of this function V was formerly far from certain. Ohm called the quantity represented by it, the *electroscopic force*, and defined it as the *density of the electricity* at any

* Pogg. *Ann.* vol. lxxxvi. p. 315.

particular point of the conductor*. Against this view, however, Kirchhoff† justly urged that it is in direct contradiction to a known electro-statical theorem; for according to Ohm's view, the electricity in a conductor would remain at rest if it were distributed equally throughout the whole volume of the conductor, whereas it is sufficiently well known that the free electricity of a body—which of course can alone here enter into consideration, as it alone exercises any force—is, in a condition of rest, distributed *only over the surface* of the body.

This objection might, perhaps, create some mistrust as to the general admissibility of Ohm's law; nevertheless, Kirchhoff himself has proved that the law can be made to harmonize very well with the fundamental laws of electro-statics, and has likewise shown what meaning must be attached to the function V in order that this may be the case.

As already stated, $\frac{dV}{dN}$ represents the component of the accelerating force at any particular point, in the direction of the normal N; and similarly, the components of the same force in the directions of the three coordinate axes will be represented by $\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}$. This denotes that the force is caused by attractions and repulsions proceeding from fixed points, both of which are dependent, as far as their intensity is concerned, *solely on the distance*, and not on the position of the acting point, the law of this dependence remaining for the present arbitrary. This, however, can be inferred from other grounds, namely, in our case it is evident that such attractions and repulsions can only be exercised by the electricity itself, and its attractions and repulsions are subject to the law of *the inverse square of the distance*. Hence it follows that the function V is to be considered simply as the potential function of the whole free electricity‡.

* Die galvanische Kette, mathematisch bearbeitet von Dr. G. S. Ohm, p. 95, and Scientific Memoirs, 1st Series, vol. ii. p. 401.

† Pogg. *Ann.* vol. lxxviii. p. 506; and Phil. Mag. S. 3. vol. xxxvii. p. 463.

‡ I have chosen the letter V to represent this function instead of the letter U, which Ohm and Kirchhoff have employed, because in my former memoir this letter was used for this function; and to make the agreement with my former memoir more complete, I have also changed its sign.

The above-mentioned contradiction therefore disappears; for according to this signification of the function V, the equation V=const., which in (1) denotes that no current exists, is the same as that which in electro-statics is known as the conditional equation for a state of equilibrium.

Further, this signification of V being adopted, it is easy to determine, as Kirchhoff has shown, where the free electricity is distributed in the conductor during a stationary current. For, in order that a current may be stationary, the quantity of electricity contained in each element of space must be constant, hence the quantities of electricity entering and issuing from any element must be equal. Let $dx\,dy\,dz$ be such an element, situated at the point (x, y, z), then, according to equation (1), the quantity of electricity entering the element through the first of the two surfaces $dy\,dz$, in the unit of time

$$= k \,.\, dy\,dz \frac{dV}{dx},$$

and the quantity issuing from the element through the opposite surface

$$= k \,.\, dy\,dz \left(\frac{dV}{dx} + \frac{d^2V}{dx^2}\,dx\right);$$

hence the excess of the latter over the former

$$= k\,dx\,dy\,dz \frac{d^2V}{dx^2}.$$

Similarly, for the pair of surfaces $dx\,dz$ we obtain the excess

$$k\,dx\,dy\,dz \frac{d^2V}{dy^2},$$

and for the pair of surfaces $dx\,dy$,

$$k\,dx\,dy\,dz \frac{d^2V}{dz^2}.$$

The sum of these three last expressions gives the excess of the total quantity of electricity issuing from the element over the quantity entering it in the unit of time, and as this excess must be null, we have

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0. \quad \ldots\ldots \quad (2)$$

From a well-known theorem of the potential function, however, it follows from this equation, that the point (x, y, z) must be situated *without* the mass, of which V is the potential func-

tion; and as the same conclusion applies to every point of the conductor, it follows further, that there can be no free electricity within it, and hence, that during a stationary current, just as in a state of equilibrium, the free electricity is distributed solely over the surface of the conductor.

According as the hypothesis of *two*, or only of one electricity is admitted, the circumstance, that the electricity passing along the interior of a conductor exercises no attraction or repulsion, must receive a different interpretation. By the first hypothesis, we must assume that each element of space within the conductor contains an equal amount of both electricities, which flow with equal strength of current in opposite directions. The second hypothesis involves the supposition, that an element of space, when it contains a certain normal quantity of electricity, exercises no action on another particle of electricity, inasmuch as the electrical repulsion is compensated by some other force; and further, that an actual attraction or repulsion *then* only occurs when the element of space contains too much or too little electricity; by this hypothesis, therefore, we must assume, that during a stationary current each element of space within the conductor contains the normal quantity of electricity.

This is not the place to discuss the probability of either the one or the other hypothesis, for as far as the validity of the following conclusions are concerned, it is a matter of perfect indifference to which of the two we attach ourselves; our choice, therefore, may be governed entirely by convenience. In my former memoir on machine electricity, I adopted, for convenience of illustration, the first hypothesis; for a similar reason I will here employ the second. In order to translate all conclusions hereafter to be given so as to be in accordance with the first hypothesis, we have merely, wherever one current conveying the quantity of electricity Q in one direction occurs, to substitute two currents conveying the quantities of electricity $\frac{1}{2}Q$ and $-\frac{1}{2}Q$ in opposite directions, and then to apply to both currents the same conclusions which here have reference to one current only.

We proceed now to determine the quantity of work produced by the force acting within the conductor during the motion of the electricity.

For this purpose, let us consider any element of electricity

dq during its motion along the path s. The component of the *accelerating* force in the direction of its motion, will for every point of its path be represented by $\frac{dV}{ds}$, and hence the component of the *moving* force acting on dq by $dq\frac{dV}{ds}$. The work produced by the force during the motion along the element ds will therefore be

$$= dq \cdot \frac{dV}{ds} \cdot ds, \quad \ldots\ldots\ldots \quad (3)$$

and consequently, the work produced along the way from s_0 to s_1 will be

$$= dq \int_{s_0}^{s_1} \frac{dV}{ds}\, ds = (V_1 - V_0)dq; \quad \ldots\ldots \quad (4)$$

wherein V_0 and V_1 denote the values of V corresponding to s_0 and s_1.

From this it is at once evident, that this quantity of work is completely defined by the values of the potential function at the extreme points of the path described, without its being necessary to know the path followed between these points. Further, the product $V.dq$ is the *potential** of the free electricity upon the element dq, so that the foregoing expression represents the increase of the potential between s_0 and s_1; and as a like expression applies to every other element of electricity, and can therefore be extended to a finite quantity of electricity, we can deduce the following theorem:—

The quantity of work produced by a force in the conductor during any definite motion of a quantity of electricity, is equal to the corresponding increase of the potential of this quantity of electricity, and of the free electricity upon each other.

In this development we have conceived the motion of the electricity to be such, that one and the same quantity of electricity traverses the whole path under consideration; the actual motion of the electricity, however, may be of a quite different character. For instance, let us assume each particle of mass to

* I here make the distinction between the terms *potential* and *potential function*, introduced in my former memoir (Pogg. *Ann.* vol. lxxxvi., pp. 163 and 342. Scien. Mem. 2nd Ser. vol. i. part 1).

be provided with a certain quantity of electricity, and conceive a number of such particles, 1, 2, 3, 4, &c., situated in a row; the motion of electricity may take place in such a manner, that a small quantity will pass over from 1 to 2, an equal but different quantity from 2 to 3; again, an equal but still a different quantity from 3 to 4, and so forth. For the validity of the foregoing theorem, however, it is of no importance which of these two kinds of motion we assume, for the theorem merely requires that all parts of the entire path be traversed by an *equal*, but not by the *same* quantity of electricity.

By this theorem, it is now easy to determine the work produced within any portion of a conductor traversed by a stationary current during the unit of time.

Let a closed surface be given, bounding a part of the space filled by the conductor, then we have merely to determine the increase of the potential for every particle of electricity traversing this enclosed space during the unit of time, or, in other words, to multiply the element of electricity by the values of the potential function corresponding to the points of entrance and exit, and then to take the difference of these two products. The sum of all these differences, which gives the required quantity of work, can be conveniently represented in the following manner. Let dw be an element of the surface of the space enclosed, and idw the quantity of electricity passing through the same in the unit of time, which must be taken as positive or negative, according as it is leaving or entering the space in question; then, if W represent the work produced within the space,

$$W = \int V i dw, \quad \ldots\ldots\ldots \quad (I)$$

where the integration is to be extended over the entire surface. If herein we set, according to equation (1),

$$i = k \frac{dV}{dN},$$

whereby the external direction of the normal is to be considered as positive, then equation (I) can be also written thus:

$$W = k \int V \frac{dV}{dN} dw. \quad \ldots\ldots\ldots \quad (I\alpha)$$

Immediately connected with these equations are those which express the quantity of *heat generated* within the enclosed surface.

The work produced within the same must be accompanied by just as great an increase of *vis viva*. In our case, inasmuch as we have agreed to disregard all extraneous actions, such as, for instance, electrolytic action, the work produced is fully expressed by the equation (I) or (I*a*). Strictly speaking, in considering the *vis viva*, we should regard not only the material mass of the conductor, but the electricity also. In other words, the particles of electricity may be accelerated or retarded on their way through the space in question; for although the condition of being stationary implies that the velocity at any particular place must be constant, yet it does not follow that in different places it must be equal. For instance, if the current pass through a conductor with different transverse sections, the electricity may move more quickly through the narrow than through the wider places; just as the water of a river moves with greater velocity through places where the river-bed is contracted than through others. Nevertheless, as it is not yet certain whether electricity possesses inertia or not, and hence whether or not we may ascribe *vis viva* to the moved electricity, and as, even if this were assumed, we have here but to consider the changes in *vis viva*, and not its total value, we may, for the present at any rate, disregard this possibility*. We have therefore to consider the *vis viva* of the material mass alone; and as by hypothesis no perceptible exterior motion is produced, the increase or decrease of the quantity of heat alone remains. This may be briefly expressed thus:—The whole work is employed in overcoming the resistance opposed by the conductor, and this, just as when friction is overcome, necessarily engenders a quantity of heat equivalent to the work consumed.

Let A be the equivalent of heat for the unit of work, and H the heat generated in the enclosed space during the unit of

* I may here remark, that, in assuming the correctness of Ohm's law, the same is tacitly admitted. For if the equation (1), $i=k\frac{dV}{dN}$, be true, the magnitude and direction of the velocity of the electricity at any given point depends only on the acting force at that point; and hence the inertia of the electricity must either be null, or at any rate so small, that the force necessary to produce such changes of velocity as occur in the conductor may be neglected, in presence of the force necessary to overcome the resistance opposed by the conductor.

time, we shall then have

$$H = A.W, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and hence, according to (I) and (I*a*),

$$H = A.\int V.idw \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

$$H = A.k.\int V.\frac{dV}{dN}dw. \quad . \quad . \quad . \quad (II a)$$

The integrals contained in equations (I), (I*a*), (II), and (II*a*) are, in most cases which occur in practice, capable of great simplification.

If a part of the surface enclosing the space under consideration form at the same time the surface of the conductor, and if, in comparison to the whole quantity of electricity passing through the conductor, we neglect the small quantity lost in the surrounding air during the current, then in the integration we may entirely neglect this portion of the surface. When, for example, as is usually the case, the conductor is an elongated body traversed longitudinally by the electric current, and when we consider a portion of it situated between two transverse sections, it will only be necessary to effect the integration for these two transverse sections.

If, further, the form of the conductor at one of the transverse sections be approximately that of a prism or cylinder, so that we may assume that here all particles of electricity move parallel to each other and to the axis, then the force urging them must also have this direction. Let us now place our rectangular system of coordinate axes, so that the x axis may be parallel to the axis of the conductor, then $\frac{dV}{dx}$ represents the whole urging force, and $\frac{dV}{dy}$ and $\frac{dV}{dz}$ vanish. From this position it also follows, that when the transverse section is taken perpendicular to the axis, V must be constant throughout the same, and we may write,

$$\int V\,idw = V\int idw.$$

The integral $\int idw$,—which must be taken negative or positive, according as the transverse section is the first or second in reference to the direction of the current,—represents the whole

quantity of electricity passing through the transverse section in the unit of time. It is usually termed the *intensity* of the current, and may here therefore be represented by J, whereby the foregoing expression becomes

$$\mp V.J.$$

Let us now assume that at the other transverse section the same conditions are fulfilled, and let the values of V corresponding to the first and second transverse sections be denoted by V_0 and V_1, then the work accomplished within the entire portion will be

$$W=(V_1-V_0).J, \quad \ldots \quad (6)$$

and the heat produced

$$H=A\,(V_1-V_0).J, \quad \ldots \quad (7)$$

But, according to Ohm's law,

$$J=\frac{V_1-V_0}{l}, \quad \ldots \quad (8)$$

in which l denotes the resistance to conduction of the portion occurring between the two transverse sections, and consequently the two preceding equations pass into

$$W=l.J^2 \quad \ldots \quad (9)$$

$$H=A.l.J^2 \quad \ldots \quad (10)$$

The last of these equations contains the two laws discovered by Joule, and subsequently confirmed by Lenz and Becquerel.

The reader is requested to correct the following Errata in the author's memoir "On the Mechanical Equivalent of an Electric Discharge."

Page 16, line 9 from bottom, for *his* read *this*.

„ 19 „ 17 from bottom, for $\left(\frac{2^2}{13}\right)$ read $\left(\frac{2}{13}\right)^2$

„ 21 „ 9 from bottom, for $-\frac{Q^2}{S}$. const. read $\frac{Q^2}{S}$. const.

„ 22 „ 2 from top, for *incomplete* read *complete*.

„ 22 „ 4 from bottom, for *quantity of work* read *capacity of discharge*.

„ 23 „ 7 from bottom, for $\frac{\frac{n's'}{k'}}{\frac{ns}{k}+\frac{n's'}{k}}$ read $\frac{\frac{n's'}{k'}}{\frac{ns}{k}+\frac{n's'}{k'}}$ Q.

Article VII.

On the Deviation of Projectiles; and on a remarkable Phænomenon of Rotating Bodies. By G. Magnus.

[From the Memoirs of the Royal Academy, Berlin, 1852.]

I. *On the Deviation of Spherical Projectiles.*

It is now a well-known experimental fact, that in firing a ball whose centre of gravity does not coincide with its geometrical centre, a deviation takes place; and it has been observed, that when the centre of gravity in the gun-barrel is on the right side, the deviation is also to the right; and similarly, when it is on the left side, a deviation to the left occurs; when it is above the axis, the range is longer; when below, shorter. This deviation sometimes becomes a very material one, and can be made to amount to one-sixth or even to one-fourth of the whole range, therefore to several hundred yards. The artillery have learnt to provide for this deviation; and now that attention is paid to the position of the centre of gravity when loading, it has become possible to strike an object with far greater certainty than before.

An explanation of this deviation has however not yet been given. It has been observed merely, that in shooting with these excentric balls through targets placed at small distances from the cannon, the lateral deviation increases in a greater ratio than the distance. This deviation, therefore, cannot be produced by a force which acted only during the motion of the projectile in the cannon; as, for instance, by friction against the interior surface, or by impinging against this or the outer edge of the cannon; on the contrary, the deviating force must have been active during the whole time of motion.

Robins, who first attempted, in his 'Principles of Gunnery,' to account for this deviation, thought that the deflecting force was generated by the rotation of the projectile; and at present this opinion is generally accepted.

Nevertheless, although since Robins many attempts have been made to explain how, by such a rotation, a deviation of the projectile could be caused, all endeavours, even those of Euler and Poisson, have been fruitless. The latter more particularly treated the subject at great length in several memoirs read before the Paris Academy, which were printed in the *Journal de l'Ecole Polytechnique*, and which appeared also in a separate treatise, entitled *Recherches sur le mouvement des projectiles dans l'air*. Therein he considered, first, the influence of the earth's revolution on the course of the projectile; afterwards, that exercised by the friction of the air on its translatory as well as on its rotatory motion; and lastly, that caused by the imperfect sphericity and homogeneity of the mass. From his calculations he concluded, that although a deviation is certainly caused by the rotation of the projectile, yet the same is so small that the observed deflections cannot be produced by a friction of the surface against the circumjacent stratum of air*.

Since then various controversies on the cause of this deviation have taken place, to examine which, however, is not the purpose of the present memoir. The insufficiency of all former explanations is thus plainly asserted by M. von Heim, a major in the Würtemberg Artillery, in his *Beitragen zur Ballistik in besonderer Beziehung auf die Umdrehung der Artilleriegeschosse*. At page 13 he says, "To give an explanation of the above-named paradoxical phænomena which shall be satisfactory to the demands of science, is consequently a problem by no means yet solved."

An investigation on the motion of fluids, made by me some time ago, suggested the present explanation of this interesting phænomenon, which, although differing materially from those heretofore given, appears to me to be the correct one.

To establish the same, it appeared desirable to examine the subject more closely, and more especially to investigate, by experiments on a small scale, the pressure which the air exerts on the several parts of the projectile.

In these experiments I have assumed throughout that all relations of pressure are identical, whether the sphere move through the air, or remaining in the same place, the air move against it,

* *Recherches sur le mouvement des projectiles*, p. 77.

provided only that in both cases the relative velocity is the same. This granted, the phænomena which take place during the motion of a projectile through the air, can be observed by allowing the projectile to remain stationary, and directing a current of air against it, with a velocity equal to that which the projectile would have had; in this manner it is possible to observe the relations of pressure on a rotating as well as on a non-rotating projectile.

If we consider, first, what takes place when air is made to move against a stationary sphere which does not rotate, it is evident that around the diameter parallel to the direction of the current the motion of the air is everywhere the same. It is otherwise, however, when the sphere rotates.

For if a body, of as perfect a spherical form as possible, be made to rotate around a fixed axis in calm air, a rotatory motion will be also imparted to the latter, which is particularly strong when the sphere rotates excentrically, but is also very plainly observable when the centre is situated in the axis of rotation. This motion of the air is the same around the axis of rotation; so that if we consider the surface enveloped by the rotating sphere, whether its centre be in the axis of rotation or not, the force with which the air is moved is the same at all points of the same parallel.

We pass next to the consideration of the combined actions of rotation and translation, the axis of rotation being perpendicular to the direction of the current of air. On one side the sphere, the motion of air generated by rotation has the same direction as the translatory motion; on the opposite hemisphere, however, the two currents have opposite directions.

Hence on the side where both currents have a like direction, the air moves with a greater velocity than on the opposite one. Now, as by this motion of the air a pressure occurs directed normally against the spherical surface, and as this pressure increases when the force with which the air moves increases, one might imagine that the side of the projectile rotating in the same direction as that of the resistance of the air, would suffer a stronger pressure towards the centre than the opposite one, and consequently that a deviation of the projectile towards this opposite side must ensue. Experience however proves

that the deviation occurs towards the side on which the direction of rotation is opposite to that of the projectile's progressive motion.

On this account it became necessary to investigate, experimentally, the existing pressure. To effect this, a current of air was directed against a sphere made to rotate around a fixed axis. Afterwards, however, in place of a sphere, a cylinder was adopted; inasmuch as with the latter a greater certainty of observation was attainable. By means of a simple contrivance, the cylinder could be so fixed that its axis coincided as near as possible with the axis of rotation; or it could be slightly displaced, so that at will a concentric or excentric rotation could be imparted.

The current of air, directed perpendicularly against the axis of the cylinder, was of such a breadth that during its most excentric rotation the cylinder remained always within the current. It was further arranged, that throughout its whole width the motion of the air in the current was uniform. To generate the latter, a small centrifugal fan was usually employed, a view of which is given in fig. 1, Plate II. In the drum F of the same (of 6 in. diameter), a shaft, on which six paddle-shaped fans were fixed, revolved; and thus throughout the whole breadth (5 inches) of the drum the air was uniformly impelled, and forced out from the orifice *m n*, of equal breadth.

Small vanes, which were very moveable, served to detect the changes of pressure which took place in the current of air during the rotation of the cylinder. Two such, *a* and *b*. fig. 1, Plate II., were so placed on each side of the cylinder, that the pivots on which they turned were equally distant from the orifice *m n*, and at equal distances also from the plane passing through the cylinder's axis of rotation and the middle of the current. When the cylinder did not rotate, the two vanes assumed the direction of the current. As soon as the cylinder, however, began to rotate, the vane placed on that side of it which moved in the same direction as the current, approached; whereas the vane on the opposite side, where the motions of cylinder and current were opposed to each other, receded from the cylinder. Consequently, on the former side a less, and on the latter a

greater atmospheric pressure existed than when the cylinder was at rest.

These differences of atmospheric pressure were most observable at the points where the direction of the current was tangential to the cylinder.

When the velocity of the air in the current was very great in proportion to that generated by rotation, the vanes were but very little deflected from the positions which the current caused them to assume before the cylinder rotated. But when the motion of the air produced by rotation was not much less than that in the current, the vane on the side where both motions had a like direction was driven strongly towards the cylinder; and then also the flag on the opposite side was strongly repelled.

In passing to an explanation of these phænomena, it may be remarked, that they are intimately connected with those described and explained by me in a memoir 'On the Motion of Fluids*.'

If a fluid issues through an orifice with a certain velocity into a similar fluid mass, the pressure which exists, perpendicular to the direction in which the mass moves, is smaller than that which would be present at the same place in a condition of rest.

We can most easily convince ourselves of this decrease of pressure by blowing air through a tube, and bringing the flame of a candle near the end from which the air issues. As soon as the velocity of the air is sufficiently great, the flame is seen to move towards the current, and at last, by increasing the velocity, can be made to place itself quite at right angles to the direction of the same. A similar decrease of pressure shows itself when a current of air is directed against a fixed wall; for a flame held before a wall near the current does not move from the wall, but towards it.

Exactly the same occurs on the surface of a fixed cylinder, if a current of air be directed against it. If the direction of this current be perpendicular to the cylinder's axis, and at equal distances from its surface, at opposite sides, two flames or two vanes be placed, both move towards the cylinder, at least

* Abhandlungen der Akademie der Wissenschaften zu Berlin für 1848, p. 135. Pogg. *Ann.* lxxx. p. 1; and Phil. Mag. for January 1851.

if the breadth of the current be less than the cylinder's diameter. The motion of the air along the cylinder's surface, therefore, does not produce an increase of pressure against it, but rather a decrease, and that in a direction perpendicular to the current; and further, this decrease is greater the greater the velocity of the air in the current.

How such a decrease of pressure is caused, is explained in the above-mentioned essay, § 35 *.

* I will here give the explanation there furnished, in a somewhat different form.

If a liquid or gaseous body issues through an orifice into a space filled with a mass of similar nature, but in a state of rest, the moving mass expands itself, so that its transverse sections have a greater area the greater their distance from the orifice. If the influx proceed with a constant velocity, then, after some time, the velocity of the moving mass at any particular place is constant, at the same time, however, the velocity in different transverse sections decreases the greater the distance of the latter from the orifice. If we suppose that all particles of the fluid, passing simultaneously through the same transverse section, whose area may be P, have an equal velocity C, then, considering the fluid's density equal to unity, the mass which passes through this transverse section in the unit of time will be PC; and if P_1 be the area of a more distant, and consequently greater transverse section, and C_1 the velocity present in the same, then the mass which passes through this section in the unit of time will be P_1C_1. The moving force, or quantity of motion in any transverse section, is represented, however, by the product of the mass which moves through the same in the unit of time, into the velocity, *i. e.* PC^2 and $P_1C_1^2$.

These values must be equal to each other, for there is no reason why the quantity of motion should vary. In each following transverse section, it is true, the motion will be extended to a greater number of particles of the fluid than in the preceding, and on that account the velocity of the same will be less; nevertheless, as all particles are perfectly moveable, the quantity of motion, disregarding the influence of friction, will suffer no essential diminution. Hence must

$$P_1C_1^2 = PC^2,$$

but as $C_1 < C$; so also

$$P_1C_1 > PC,$$

i. e. the mass which passes through a more distant transverse section in the unit of time is greater than that which in the same time passes through a transverse section nearer to the orifice.

To render this possible, a part of the fluid mass at rest on each side must enter the moving mass. The above-mentioned memoir contains (§ 33) the experimental proof, that in liquids the mass at rest on each side permeates even to the centre of the moving mass; without doubt this takes place in gaseous bodies also, in an exactly similar manner, and thence arises, in both, a decrease of pressure perpendicular to the direction in which the original motion was propagated.

When the cylinder does not rotate, the decrease of atmospheric pressure is equal on both sides. On the contrary, when it rotates, the velocity, and consequently the decrease of pressure is greater on the side which moves in the same direction as the current than on the other, where these motions have opposite directions. The experiment showed, that an increase, and not a decrease of pressure occurred on the side which rotated against the current, so that here the pressure perpendicular to the direction of the current is greater than it would have been if no motion had taken place.

From Savart's investigations, it is known that when two jets of liquid, issuing from circular orifices of equal diameters, meet each other with equal velocities, and so that their axes are in the same right line, all motion is not thereby annihilated, but the liquid moves laterally, and forms a circular disc perpendicular to the direction of the jets. In general, when two fluid masses thus strike against each other, instead of a mutual annihilation of motion, as in non-elastic solid bodies, a lateral motion is produced, whose direction and velocity depend upon the ratio of the masses which strike against each other, as well as upon their respective velocities.

In the experiment with the cylinder, the air by rotation

Hence the decrease of lateral pressure essentially depends upon the expansion of the mass in motion, or upon the fact, that through each more distant transverse section a greater number of particles of the fluid move.

That the decrease of lateral pressure depends, as is here stated, upon the expansion of the fluid mass, can be easily confirmed by causing air or a liquid to move with a certain velocity through a horizontal tube of uniform diameter. If the fluid enters the same through an orifice whose transverse section is equal to that of the tube, so that the stream does not expand itself, then also no decrease of lateral pressure occurs. For if from the horizontal tube another narrow one descend vertically into a vessel of water, the water does not ascend in the latter during the motion in the horizontal tube. If, however, the orifice through which the fluid enters the horizontal tube be less than the transverse section of the same, and the vertical tube reaches inwardly to where the fluid expands itself, the water is seen to ascend. If the fluid issues into the horizontal tube from a wide vessel under considerable pressure, a decrease of lateral pressure is observable close to the orifice, even when the diameter of the jet there is equal to that of the horizontal tube. This, however, depends also upon an expansion, that is to say, the fluid entering the horizontal tube contracts itself, and afterwards, beyond its point of greatest contraction, again expands.

moves against that in the approaching current, and thus a lateral motion of the same ensues, repelling the vane placed on this side. When the velocities of both masses of air have a certain ratio to each other, this repulsion attains a maximum, and then its direction is perpendicular to that of the currents. Hence, as above mentioned, the vanes were pressed most in a lateral direction when the velocity of the air generated by rotation had a certain definite ratio to that of the approaching current.

The application of these experiments to the deviation of spherical projectiles is self-evident.

If we suppose that a sphere, during its progressive motion, rotates so that the axis of rotation is situated always in the principal normal of the trajectory, and if we call the motion of such a sphere a right rotatory one when the front part of it turns from left to right, in reference to an observer behind the gun, and, on the contrary, a left rotatory motion when the front part of the sphere turns from right to left, then it follows immediately, from the described experiments, that a decrease of atmospheric pressure takes place against the right side of a right rotating sphere, and an increase on its left side; hence such a projectile deviates towards the right. Similarly, a left rotating sphere deviates to the left.

When the axis of rotation is perpendicular to the plane of the trajectory, or if this be a curve of double curvature, perpendicular to its osculating plane, then, although no lateral deviation occurs, the pressures on the upper and lower hemispheres are different. If the upper one move in the direction of the progressive motion, the pressure from above on the sphere is greater than that from below; hence the projectile descends, and its range is less than it would be if upper and under pressures were equal. If the lower hemisphere rotates in the direction of the progressive motion, the pressure from below on the projectile is greater, and it ascends, making its range a longer one. In one position only, that is, when the axis of rotation is tangential to the trajectory, no deviation is caused by the sphere's rotation. In every other position of the axis, either a lateral or longitudinal, or some combination of both deviations must ensue.

This explanation of the deviation of spherical projectiles, suf-

ficiently established, as I think, by the described experiments, is, it is true, contrary to the hitherto accepted notions regarding the resistance of the air, but is not on that account the less correct. In order to free it from every objection, however, it has yet to be proved that the difference of atmospheric pressures against the opposite hemispheres is sufficiently great to produce a deviation of the projectile itself. As we cannot yet measure the magnitude of this difference, I have endeavoured to produce in miniature a lateral motion in order thus to demonstrate that the difference of atmospheric pressures is sufficiently great to deflect the projectile from its course.

To produce such a deflection, a light, hollow brass cylinder, 3 in. high and 2 in. in diameter, *ab*, fig. 2, Plate II., was used; very easily moveable around its axis, which turned on two pivots, fixed in a metal ring. Cylinder and ring were fixed to the end of a light wooden beam, *yz*, 4 feet long, suspended in its centre from a fine wire *vw*, 8 feet long, and thus forming a kind of torsion balance. To keep the beam horizontal, it carried at its other extremity a moveable counter weight *p*, and it was fastened to the vertical wire by two strings, *rt* and *ru*, from the points *t* and *u*, equidistant from the centre *v*. The object of this connection was to avoid, as much as possible, all oscillatory motion. For the same purpose a silk thread *vc* descended from the centre *v* of the beam, and was fastened exactly vertically under the wire *w*, so that the position of the suspension-point *v* of the beam could not vary more than the elasticities of the wire *wv* and silk thread *vc* permitted.

To set the cylinder in rotation, a small pulley, *e*, was fixed on the production of its axis, and a fine silk thread was wrapped round the same. A sharp pull at this string gave the cylinder a rotation, which continued for two or three minutes. During its rotation a current of air was directed against it. To generate the latter, the small centrifugal fan EF, described above (page 213), was used, and so placed that the air moved in the direction of the beam *yz*, from the point of suspension *v* towards the extremity *z*, to which the cylinder was fixed; the latter, in its ring, whose plane was perpendicular to the current of air, was placed vertically, at a distance of 2 inches, before the orifice *mn* of the centrifugal fan. As soon as the cylinder moved to one side, it

would have left the field of the current, had it not been arranged that the centrifugal fan could so follow it that the current always acted in the direction of the beam *yz*. For this purpose the fan was placed on a board AB, capable of turning on a vertical axis *c*, and carrying on its other extremity a counter weight G.

When the cylinder rotated without the current of air being directed against it, the former remained stationary. It was also stationary when the current was directed against it during its non-rotation. If the current, however, struck against the rotating cylinder, the latter, together with the beam *yz*, moved towards the side on which the air, by rotation, and that in the current, had a like direction, as shown by the arrows in fig. 3, Plate III.; *a* indicates the direction of the current of air; *b* that of the cylinder's rotation; and *c* that of the lateral motion.

If, by turning the board AB, the centrifugal fan was made to follow the cylinder, the latter moved laterally as long as its rotation continued to be pretty strong, often describing a complete circle. When made to rotate in an opposite direction, it moved towards the opposite side. When the current was interrupted, the motion of the beam *yz*, to which the cylinder was fixed, owing to its rather high moment of inertia, did not immediately cease. If, during the rotation of the cylinder and its lateral motion, together with the beam *yz*, the latter received a push towards the opposite side—the centrifugal fan, however, made to follow it—the motion caused by the push soon ceased, and the primitive one was re-established. It cannot, therefore, be doubted that the lateral motion of the cylinder was caused by its rotation during the action of the current upon it.

It is true that the current of air exercised also a pressure against the surface of the ring in which the cylinder rotated, and when this surface was not perpendicular to the direction of the current, cylinder and ring moved towards one side, even when the former did not rotate. That the observed deviation of the rotating cylinder, however, was not caused by such a position of the ring, is evident from the fact, that even when the plane of the same, during the rotation of the cylinder, made an angle of 45 degrees with the current, a lateral deviation towards the right or left always ensued, according as the rotation of the cylinder was to the right or left; hence the lateral pressure against the

surface of the ring—0·25 inch only in breadth—was but very small, indeed almost vanished, in comparison to the force with which the cylinder, by its rotation, was moved sidewards.

If we take into consideration the great velocity of the periphery of a rotating projectile in comparison to that of the cylinder, it cannot be doubted that the difference of the atmospheric pressures against the opposite sides of such a projectile is sufficient to cause the observed lateral deviation.

II. *On the Deviation of Elongated Projectiles.*

The artillery of different countries have lately made experiments with elongated hollow projectiles discharged from rifled cannon. Where such projectiles have been used, they have generally had a cylindrical shape, with conical or ogival point, the other end or base being either a plane, or having a hemispherical form. The spiral imparts to them a rotation around their axes, *i. e.* around the line from the cone's apex to the centre of the base, or at least around a line in the immediate vicinity of this. At all places where such experiments with rifled cannon have been instituted, and the results published, a similar lateral deviation of the projectiles has been observed*, that is, a deviation always towards the right of an observer behind the gun, and never to the left. In reference to this constant deviation, which is far less than that with spherical projectiles discharged from smooth barrelled guns, the question arises—can it be explained in a manner similar to the latter?

That elongated projectiles always deviate towards the same side, is, without doubt, caused by the rifle of the cannon having always the same direction, that is, so that when an observer behind the gun faces the same, and follows the direction in which a point in the rifle recedes from him, he sees it move in the upper part of the barrel from left to right, in the lower from right to left; or to define it more concisely, as the hands of a clock.

This description of rifle I will call right-handed; the other description, progressing in a contrary direction, left-handed.

As far as I know, no observations have yet been made as to

* A similar deviation has lately been noticed when pointed projectiles were fired from rifles.

the direction of the deviation in left-handed rifles: but the constant deviation to the right in right-handed, leaves no room for doubt, that the direction of this deviation is conditioned by the condition of the rifle; and that in left-handed also, a constant deviation, but to the left, would occur.

It were certainly desirable, however, to have this confirmed by actual experiment.

By firing such elongated projectiles at a target, it has been found that the axis of the projectile coincides nearly with the tangent of the trajectory. If the axis and tangent continually and exactly coincided, the resistance of the air, whose direction coincides always with that of the tangent, would act always parallel to the axis. The motion of the air all around the axis would then be the same, and hence resistance could produce no lateral deviation. But the experiments, shortly to be described, show that the axis is not exactly tangential to the trajectory, but, on the contrary, makes always a small angle with it; and hence, also, at this angle the resistance of the air must meet the projectile.

It might at first be thought that this small angle was sufficient to produce a decrease of pressure on the one side, and increase on the other, as in spherical projectiles; but a closer examination shows that if this were the case with a right-rotating projectile, there would be a decrease of pressure on the left side of the above-mentioned observer, and consequently a deviation of the whole projectile towards this side; whereas we have seen that it is ever deflected towards the opposite side, *i. e.* to the right.

The cause of the deviation in question, that is, of elongated projectiles, cannot therefore be the same as that of spherical ones.

To discover this cause, it appeared first necessary to observe more closely the direction of the projectile's axis during its motion.

The members of the Royal Artillery Commission, entrusted with the execution of the Commission's experiments, have obligingly made several for the above purpose. Such elongated projectiles were discharged with so small a charge of powder, that it was possible for the eye to follow their motion, and to observe the position of the axis during the motion. In these

experiments, which were executed in my presence, the members of the Commission were kind enough to undertake the observation of the several projectiles. The results were as follow:—

During the motion of the projectile, the observers placed on both sides of the trajectory all agreed that the axis of the projectile coincided almost with the tangent of the trajectory. In the descending branch, however, it was observable that the point was situated somewhat higher than it would have been had the axis been exactly tangential.

At the same time in all the shots it was evident, as well from the projectile's motion through the air as from the shape of the furrow it ploughed in the earth, that the point, at the moment of striking the ground, *was deflected somewhat to the right.* This is approximately represented by fig. 4, Plate III., where AB shows the original direction of the shot, CD the direction of the furrow produced, and *a b* the direction of the axis at the moment the projectile struck the earth.

When a projectile falls on the ground in the direction named, and its point first touches, it not unfrequently happens, as it pierces deeper, that the hinder part is turned to the front. This was actually the case in most experiments; for the projectiles which remained fixed in the earth, had in general, with respect to the range, the position shown in fig. 5, Plate III., where AB represents the range, and *a b* the position of the projectile's axis in the earth; whereby, however, the point *b* must be imagined as being situated much deeper than the hinder part *a*.

These observations prove, not only that the axis of the projectile, during its translatory motion, does not exactly coincide with the tangent of the trajectory, but also that it makes an angle with the vertical plane through the tangent, and in such a manner that the projectile's point is always to the right of the observer behind the gun.

If we suppose that the projectile's axis along the whole length of the trajectory has the above-named inclination towards the vertical plane through the tangent, then it is evident that the projectile will deviate also towards the right, owing to the resistance of the air acting on it then as a force against an inclined plane. If, therefore, the presence of a force, acting on the projectile's

axis, could be proved, whose effect was to give the same an inclination towards the vertical plane through the tangent, the cause of the projectile's lateral deviation would be thereby also shown.

If on the axis of a body, generated by the rotation of any curve around this axis, a force acts in a direction not passing through the centre of gravity, such a force, when the body does not rotate, would move the axis in the plane passing through the same, and the direction of the force. If the body rotate, however, with great velocity, the axis will not move in this plane, but towards one side, describing a cone if the centre of gravity be considered as a fixed point.

The explanations given of the precession of the equinoxes and nutation rest on this theorem. Bohnenberger* has, with great ingenuity, invented a small apparatus to illustrate this by experiment; and Poisson has treated the theorem at length in a separate memoir in the *Journal de l'Ecole Polytechnique*, Cahier xvi. p. 247.

Bohnenberger's apparatus consists of an oblate spheroid, capable of rotating with facility around its axis, and so suspended within three rings that any required direction can be given to the axis.

I have had a similar apparatus constructed, which, however, instead of a spheroid, contains a miniature representation, in brass, of one of the above-described elongated projectiles. Its cylindrical part has a diameter of 2 inches, and an equal height. The height of the whole body, including its rectangular conical apex, is 3·4 inches. Fixed to the axis at the apex, is a small pulley, around which a fine silken thread can be wrapped. By unwinding this quickly with a sharp pull, the body is made to rotate, and continues so to do for some time with almost unchanged velocity. The whole apparatus is shown in fig. 6, Plate III. The steel axis *ab* of the body L turns on two pivots, fixed in the inner ring EF. By means of this suspension within the rings, it is, of course, possible to give the axis any required position; in order, however, that it may retain the same, it is necessary that the body's centre of gravity, and that of the ring also, coincide with the centre of the latter. On that

* Gilbert's *Ann.* lx. 60. Tübinger Blätter für Naturwissenschaften und Arzneikunde von v. Autenrieth und Bohnenberger. Bd. III. Heft I.

account, the body L, though apparently solid, is in fact partly hollow.

The execution of such an apparatus requires great delicacy and correctness of workmanship, inasmuch as it is essential that the centre of gravity of the body L, in every position, coincide with the common centre of all the rings. Here too it is necessary that the two axes of rotation of the inner rings, AB and EF, as well as the axis *a b* of the body L, intersect all in *one* point, and that in this point the centre of gravity be also situated. To secure the last, the points *a* and *b* can be moved backwards and forwards by means of screws.

By this arrangement, as long as no rotation takes place, every ring can be moved with the greatest facility. If the body is made to rotate, however, it requires a very sensible force, in order to change, even slightly, the position of any one of the rings. Consequently the axis *a b* of the body, during its rotation, retains its direction unchanged.

If now, during the body's rotation, a force, not passing through the centre of gravity, is applied to the axis; for example, if, when the axis of the body is inclined to the horizon, a vertical force act on it, produced by a weight applied to the ring EF near to the extremity of the axis *b*; then the latter will not be moved by it in a vertical plane, but will describe a cone, in that it begins thereby to move very slowly and horizontally towards one side. If the force act in a horizontal instead of a vertical direction, the axis describes still a cone, but commences moving, very slowly and vertically, upwards or downwards. This motion, at the commencement, has always a direction perpendicular, or approximately so, to the plane passing through the direction of the force and axis of rotation. For the following considerations, it is of great importance towards which side of this plane the motion occurs.

If we suppose that an observer, placed in the production of the axis of rotation through the base, sees the body rotate in the same manner as the hands of a clock, or from left to right, and that the force acts from below upwards on the extremity of the axis of the body L turned from him, *i. e.* on the apex, then this apex will move towards the right of the observer; if, on the other hand, the force acts from above downwards, the apex will

move to his left. If the force be applied to the extremity nearest the observer, *i. e.* near to the point *b* of the base, then the extremity turned from him, or the apex, moves to his left, when the force acts from below upwards; to his right, however, when it acts from above downwards. If the rotation of the body were in an opposite direction, *i. e.* contrary to the hands of a clock, or from right to left, then the extremity of the axis turned from the observer, would in each of the above cases move in an opposite direction to the one mentioned.

If, instead of the suspended body, an elongated projectile, of the description already named, be imagined, which during its progressive motion rotates around its axis, and the action of the resistance of the air on it be considered, it will be readily seen that the resultant of pressure produced by this resistance passes certainly through the axis, but in general, owing to the presupposed form of the body, not through its centre of gravity. The resistance of the air, therefore, gives rise to a force which must cause the projectile's point to move laterally, and the axis to describe a cone. Whether this motion of the apex be to the right or left, depends, as we have seen, upon the direction of the resultant of resistance, and whether it acts against the axis above or below the centre of gravity—supposing in both cases the direction of the body's rotation to be the same.

The above-mentioned observations of projectiles discharged with a small charge of powder, show that the axis has, approximately, the same direction as the tangent; that, therefore, during the progressive motion, the apex, in reference to the centre of gravity, sinks. It appears to follow from this, that the resultant of resistance passes through the part of the axis under or behind the centre of gravity. Were this the case, however, the apex of a right-rotating projectile would deviate to the left of an observer behind the same; and hence the centre of gravity would be pressed towards this side; therefore a lateral deviation would be produced exactly contrary in direction to the one actually observed.

I must confess that this conclusion, contradictory to experiment, caused me for some time to seek another explanation of the deviation of elongated projectiles; until, indeed, the experiments which follow showed me that the contradiction was but

apparent, and that the hypothesis, that the resultant of resistance must cut the axis of the projectile below the centre of gravity, was incorrect. For the apex of the projectile may be made to sink, even when the resultant of resistance acts from below upwards on the part of the axis above the centre of gravity. It is true that in this case the apex will be at first raised, but with a sufficient velocity of rotation this elevation is scarcely perceptible, and instead of it, a motion of the apex towards the right is produced.

As soon as this motion has commenced, however, the plane passing through the resultant of resistance, *i. e.* the tangent of the trajectory, and through the projectile's axis, is no longer vertical, and the more this deviation of the apex increases, the inclination of this plane towards the horizon becomes continually greater. But, as we have seen, the apex moves always perpendicularly to this plane, or approximately so; hence from the known direction of this motion, it follows that the apex must be depressed.

This depression of the apex can extend even so far, that it becomes situated below the tangent through the centre of gravity. Then the resistance of the air, parallel to the tangent, acts from above downwards on the apex—above and below having reference to the before-mentioned observer—and thereby a motion of the apex ensues in a direction opposite to that just mentioned. During the projectile's flight, however, owing to its short duration, so great a depression of the apex is not to be expected; nevertheless, for the following experiments it is worthy of notice, that when such a depression does take place, the apex must move towards the contrary side.

In order to discover whether the resultant of resistance passes through the part of the axis before or behind the centre of gravity, the following experiments were made. Against the above-described body L, fig. 6, Plate III., suspended within its three rings, a current of air was directed, equally strong in every part of its transverse section, which was of such a magnitude, that the body, whatever position it might assume, remained always entirely within the current.

To produce this current, a strong pair of bellows was employed, by means of which the air was first compressed into a so-called

wind-chest. This was simply a box, 1 foot high, and of equal length and breadth, in which was a four-cornered orifice 3 inches square. This large orifice was necessary, in order that the body in every position might remain within the current. As, however, the amount of air furnished by the bellows was not sufficient to sustain a constant and sufficiently strong stream of such a transverse section, a plate was placed before the orifice, containing 484 fine apertures, all equally distant from each other, and all having an equal diameter of 1 millim. Thereby, it is true, not *one*, but 484, thin parallel currents of air, of equal intensity, were produced, which, however, on account of their close proximity to each other, might be considered as equivalent to one current, at least as far as regards their action on the body L.

When the apparatus, fig. 6, Plate III., was so placed that the horizontal current of air acted against the body L when this did not rotate, but had its axis situated in the vertical plane through the middle of the current, and at a small inclination towards the horizon, then the apex, if above the horizontal plane through the centre of gravity, was elevated; but it was depressed when situated below this plane.

From this, it is evident that the resultant of pressure which the current exercised against the body, passed through the part of the axis between the apex and centre of gravity.

The action of the air's resistance against an elongated projectile is similar to that of this current against the body. In the former also, when, as in the body L, the centre of gravity is situated in the middle of the length, the resultant of resistance must pass through the part of the axis between the apex and the centre of gravity; and without doubt this is also the case when the centre of gravity is not at too great a distance from the middle of the axis.

The elevation of the apex of the body L, and hence that of elongated projectiles also, is dependent alone upon the form of the pointed end; for when the body L was taken out of the rings, and in its place a cylinder, with equal diameter and height, substituted, which at each extremity was bounded by a plane perpendicular to its axis; when this cylinder was exposed to the action of the current of air, as the pointed body had been,

its axis being equally inclined to the horizontal direction of the current, the axis assumed always a horizontal position. That part of the cylinder, therefore, facing the approaching current, was depressed, whereas in the pointed body this part was elevated; from which we may conclude that the elevation of the latter is due to its pointed form.

The pointed body was next exposed to the action of the horizontal current while rotating within its ring. If, as before, the axis was in the vertical plane through the middle of the current, and its apex somewhat above the horizontal plane through the centre of gravity, the latter moved slowly sidewards, at first in an almost horizontal direction; soon, however, its direction became inclined. When the body rotated to the right, the lateral motion of the apex was towards the right; and when it rotated to the left, the lateral motion was also to the left. If the rotation continued until the apex sank below the horizontal plane, then its lateral motion again gradually decreased. It moved, therefore, exactly as was above predicted when the resultant of pressure passed through the part of the axis above the centre of gravity, *i. e.* the part situated between the centre of gravity and the apex.

From these experiments, we may conclude that the deviation of elongated projectiles is caused by the resistance of the air seeking to elevate the apex. The elevation hereby produced is, however, scarcely perceptible, for during rotation the forces acting on the mass of the projectile so combine themselves, that the apex, instead of being elevated, is moved sideways, and indeed, towards the right when the projectile rotates to the right. In consequence of this motion to the right, the resistance of the air presses the projectile's centre of gravity towards the same side, and thus produces the deviation. At the same time the apex sinks, and thus it appears as if the pressure of the air against the hinder part of the projectile was greater than that against the fore part, whereas, in fact, this pressure is greatest on that part of the axis which is placed between the centre of gravity and the apex.

I believe the cause of the deviation of elongated projectiles is thus fully explained, and with it that of the whole phænomena of deviation, as well of spherical as of elongated projectiles,

the causes of which have for so long a time been completely unknown.

On a remarkable Phænomenon of Rotating Bodies.

In the foregoing described experiments I had an opportunity of observing a phænomenon which, although following immediately from the above theorems on the motion of rotating bodies, deserves here to be separately described, partly because it is, I believe, but little known, and partly because it illustrates, particularly well, the phænomena which are observable when forces act on a rotating body, the resultant of which passes through the axis, but not through the centre of gravity.

If the body L, in the apparatus shown by fig. 6, Plate III., be made to rotate around its axis, it is well known that a somewhat considerable force is requisite to change the position of this axis, or that of either of the two inner rings. If, however, the middle ring AB, in which the pivots E and F holding the inner ring are fixed, be held fast with the hand, then the axis *ab* of the rotating body, and with it the ring EF, can be easily moved.

If such an apparatus be constructed with only two rings, the outer one being fixed, then the axis, which can only move in a plane, will assume no fixed position. The smallest force will be sufficient to change its direction.

The explanation generally given of this unchangeability in the direction of the body's axis is, that a considerable force is requisite to move every single point of the body from the plane in which it rotates. The above-mentioned experiments, however, show that this explanation is insufficient.

In order to illustrate this remarkable phænomenon in another manner, I had the apparatus shown in fig. 7, Plate III., constructed.

It consists of an iron axis 9 inches long, around each extremity of which a circular brass disc, 4 inches in diameter and 0·25 inch thick, can rotate. Each of these discs, E and F, is provided with a small pulley, around which a silken thread can be wound. By unwinding this quickly, each disc separately can be made to rotate. Through the centre of gravity of the whole instrument, and perpendicular to the axis, a wire, CD,

is fixed, which turns in two holes, C and D, of the hoop CDG. AB is moveable in this hoop like the beam of a common balance. The hoop itself is suspended in G from a string, around which the whole apparatus easily revolves. And hence it is possible to give the axis AB any required position.

If one of the two discs, E or F, be set in rotation, the axis AB remains in the position first given to it, and a somewhat considerable force is requisite to change the same. This difficulty is much increased when both discs rotate in the same direction. If the hoop CDG, however, be held firm with the hand, the axis is immediately and with facility moveable; it is indeed surprising how this moveability changes, according as the hoop is held firm or left free. These phænomena are exactly similar to those described in the apparatus with the three rings.

If, however, one of the discs rotate to the right, the other to the left—these directions having reference to an observer at one extremity of the axis AB—then, even if the hoop CDG be left free, the axis AB is easily moveable, at least as long as both discs rotate with equal velocity.

If they have unequal velocities it is more difficult to move the axis, and the greater the difference of these velocities the greater this difficulty becomes.

If on a solid of revolution, whose axis is freely moveable in all directions, a force acts during its rotation with great velocity around its axis, and the direction of this force pass through the axis but not through the body's centre of gravity, a lateral motion will, as we know, take place; but its velocity is always very small, not only in comparison to the velocity of rotation, but also to the motion which the same force would have imparted to the axis had not the body rotated. On this account the axis appears to retain its position unchanged.

The action of every force passing through the axis, but not through the centre of gravity, causes a motion of the former, which motion, however, is exceedingly small when the body rotates quickly, and on that account the axis is apparently fixed.

The above-described experiment, with the apparatus shown in fig. 6. Plate III., particularly illustrates the connection between this fixity of the axis and its lateral motion: for it shows that

if two equal masses rotate around the same axis with equal velocity, but in opposite directions, then by the action of a force which does not pass through the centre of gravity of the apparatus, the axis is moved with equal force in two opposite directions, therefore no lateral motion can occur, and the axis deports itself exactly as if by some mechanical contrivance its motion were restricted to one plane, in which it could only turn on its centre of gravity. If, however, the two masses rotate with unequal velocities, the opposite lateral motions are also unequal, and the axis moves to one side. The fixity of its position, however, decreases with the decrease of the difference between the two lateral motions.

The above-mentioned experiments are the more surprising, on account of the apparent contradiction, that the axis of a rotating body, when completely free, should appear fixed; and on the other hand, that when its motion is restricted to a plane, it should be moveable with great facility. These experiments, at the same time, illustrate the influence which a free axis exercises not only upon elongated projectiles, but on rotating bodies in general, whether celestial bodies or those on the earth, having simultaneously both a rotatory and a translatory motion.

Article VIII.

On the Motion of Liquids in a closed Galvanic Circuit.

By G. Wiedemann.

[From Poggendorff's *Annalen*, No. 11, 1852.]

§ 1.

In examining the action of a galvanic current, so far as its activity is confined to the path traversed by it, the attention of physicists has been directed principally to two of its properties; on the one hand to its capability of generating heat, on the other to its chemical action. In reference to the first of these properties, the observations of Joule in particular have led to general laws, which are all the more important on account of their intimate connexion in many respects with those established by the excellent experiments of Riess, for the heating of the conducting wire in electrical discharges. Faraday's observation, that one and the same current always separates from different binary combinations equivalent quantities of their elements, has by later investigations, among which it may suffice to mention Daniell's, been extended to the decomposition of compound bodies in general. Although these subjects are still far from being exhausted, a path at least has been opened towards obtaining further results.

A third action of the galvanic current, which has been less studied than its thermic or chemical actions, consists in a peculiar mechanical power possessed by it, by means of which it is capable of producing, in one way or another, a motion of the material particles through which it passes. At one time this activity shows itself in the capability which a powerful galvanic current possesses of tearing asunder certain materials introduced into its circuit, as was shown by Grove, when investigating the deportment of molten platinum and lead wires in the galvanic circuit. This phænomenon is analogous to the dissipation of wires observed by Riess in powerful battery discharges.

Then again the mechanical action of the galvanic current manifests itself in the transference of material particles at places where, for a short distance, the circuit is interrupted by a non-conductor; in the galvanic sparks and arcs of light, which are always accompanied by a motion of the incandescent particles of the conductor, at the place of interruption. Many attempts have been made to establish a connexion between this phænomenon and the direction of the galvanic current producing the arc; in particular, it has been thought that a motion of the particles from the positive towards the negative conductor was perceptible; nevertheless, later experiments—for instance, those of Van Breda and Matteucci—have proved that these phænomena can always be explained by the different intensity of heat, and hence different state of fusion, of the ends of the two poles: this is the more probable, inasmuch as these phænomena are seen only in gases which, at high temperatures, exert a chemical influence on the substance of the polar wires. A transference of matter in the voltaic arc, according to a fixed direction, is not therefore established with certainty; and perhaps the action of the galvanic current here may be analogous to that of frictional electricity; for by means of the sparks produced by the latter, the material particles are moved in one direction as well as in the other.

Nevertheless, just as in frictional electricity, a certain difference in mechanical action manifests itself, depending on the direction of the two opposite electricities (no matter whether this be observable in the different facility with which a brush of light may be produced at points by means of positive or negative electricity, or whether it be in the different Lichtenberg's figures which positive and negative electricity separately produce), so also in a galvanic current a similar difference can with certainty be proved. When liquids are introduced into the circuit, this difference shows itself by a motion of the liquid particles in the direction from the positive towards the negative pole. This phænomenon, which under certain circumstances can be rendered visible, was observed a long time ago, but has never yet been made the subject of a separate and closer examination*.

* The few experiments which bear upon the subject considered in the present memoir, were collected in a note by Poggendorff at the time of Daniell's

The author hopes, that, by the following experiments, he may have succeeded in approaching somewhat nearer than has hitherto been done, to the true nature of this phænomenon, so important to the whole science of the galvanic current.

§ 2.

In order partly to repeat former experiments, partly also to investigate the general phænomena connected with the transporting action of the galvanic current, the author employed an apparatus of very similar construction to the one generally used for the decomposition of water.

Two bottle-shaped glass vessels $a\,a_1$ (Plate IV. fig. 1), with double necks $c\,c_1$ and $d\,d_1$, communicate with each other by means of a wider aperture at $b\,b$, provided with a thick glass margin. The

investigation of the electrolysis of secondary combinations (Pogg. *Ann.* Ergänzungs, Bd. i. s. 569). From the facts there cited, it may be seen how little the real cause of the phænomenon has been approached. In order to complete the literature on this subject, already cited by Poggendorff, the following may be added. The fact of the transference of water through strata of sand or bladder by means of a galvanic current, was discovered by Porrett in the year 1816, or perhaps much earlier, in 1809, by Reuss, in Moscow (Seyffer's *Geschichtliche Darstellung des Galvanismus*, 1848, s. 542). In giving to it the name electrical endosmose, it was immediately referred to an improper cause. The subject received but little extension from the investigations of Becquerel and Daniell, who, it must be admitted, did not submit the matter to direct investigation. With the exception of the interesting observation made by Becquerel, that in electrical endosmose particles of the porous diaphragm were sometimes also transported with the current of liquid, the experiments of these two physicists confine themselves almost entirely to the proof that dilute sulphuric acid does not follow the current, and that saline solutions in general manifest the phænomena in a higher degree than pure water (?). Lately the subject has been further treated by Napier (Phil. Mag. July 1846). Napier distinguishes visible and invisible electrical endosmose, inasmuch as the current may transport liquids as well as the salts they contain in solution. He considers more particularly the latter phænomenon, which may be said to have a close connexion with the secondary electrolytic phænomena observed by Daniell and others. To Napier, the visible endosmose was always most perceptible when, in the saline solution, the salt was most decomposed; it was also very visible with distilled water. Lastly, it is possible that the interesting observations of Armstrong (Phil. Mag. S. 3. vol. xxiii. p. 94, and Pogg. *Ann.* Bd. lx. s. 352), where, with a strong hydro-electrifying machine, a motion of distilled water towards the part of the apparatus charged with negative electricity could also be perceived, may be intimately connected with the phænomena of the transference of liquids in the galvanic circuit.

glass margins at b are ground on to each other, and held firmly together by means of the board $e\,e_I$ and the three screws $f_I\,f_{II}\,f_{III}$. The glass margin, b, of one of the vessels is ground internally, so that in the hollow thereby produced, plates of different substances can be cemented, and thus introduced between the two glass vessels. In the necks $d\,d_I$, two vertical glass tubes, $g\,g_I$, are fixed by corks. The necks, $c\,c_I$, are also closed with corks, through which two wires, $h\,h_I$, pass into the interior of the glass vessels. To the interior extremities of these wires, circular metallic plates, $i\,i_I$, are screwed; these at first were of platinum foil. After cementing a porous plate of clay between the vessels at b, the latter were filled with water. When the wire h was connected with the positive, the wire h_I with the negative pole of a galvanic battery, the water ascended in the tube g_I and sunk in the tube g; at the same time oxygen was liberated at i, hydrogen at i_I; both of which escaped through the two tubes g, g_I.

The quantity of gas thus liberated is so small in proportion to the quantity of water raised, that we are by no means justified in ascribing this ascent to the liberation of gas alone. [In one case the total volume of liberated gas in the apparatus amounted only to about one-third of the volume of water elevated into the tube g_I.]

Another experiment showed this more plainly. Instead of the platinum plate, a plate of copper was substituted; and in place of water, a solution of sulphate of copper was used. Here no liberation of gas took place; but the solution nevertheless ascended in the tube g_I, and sank in the tube g.

This phænomenon must therefore have a different cause, and can only be explained by assuming that a transference of the liquid particles takes place from the metallic plate i in connexion with the positive pole, through the porous diaphragm to the plate i_I in connexion with the negative pole. A peculiar moving action of the galvanic current is the only cause to which we can attribute this transference. Before endosmose could cause it, the liquid on both sides of the porous clay must have become heterogeneous; hence, when pure water alone was employed, the phænomenon should not at all have shown itself. Further, the motion of the liquids by the galvanic current is much more active than that which endosmose could produce;

for the latter could only be called into action by the gradual decomposition of the liquids by the current.

When, instead of *water* and the solution of *sulphate of copper*, other liquids were introduced into the apparatus, the same phænomenon was observable.

A solution of *sulphate of zinc* behaved exactly as a solution of sulphate of copper.

Solutions of *sulphate of potash, sulphate of soda*, &c., manifested a weaker action than when, with equal intensity of current, pure water was employed. With these, however, the rapid liberation of gas during the decomposition of the solution always more or less impeded an exact observation.

Dilute alcohol, as well as *absolute alcohol*, manifested the action more powerfully than pure water.

Dilute sulphuric acid showed no action whatever (as Becquerel before observed).

If, in place of the porous clay above described, a plate of gypsum, or a piece of bladder stretched over a glass frame, were introduced, a transference of the liquids again occurred, in a greater or less degree, the motion of the liquids having always the same direction, *i. e.* from the positive to the negative electrode.

As yet the author has not succeeded in replacing the porous diaphragms by single capillary tubes, or a system of such. The least difference of hydrostatic pressure on each side of these tubes is so much stronger than the action of the current, that when the liquid in the capillary tubes was moved by the current towards the negative electrode, the ascent of the liquid on the negative side was sufficient immediately to annihilate the action of the current, and cause the liquid to flow back. Even when the level of the liquid's surface on each side the capillary tubes was kept constant, by means of a contrivance admitting of an overflow on the one side and a supply on the other, no such overflow could be produced.

The reason of the failure of this and other experiments instituted for the purpose, probably was, that by employing one or a few capillary tubes it is not possible to conduct even a pretty strong galvanic current through the liquid contained in them; by employing many capillary tubes, or tubes with wider aper-

tures, it is very possible that the liquid, by flowing back through parts of the apertures, may annihilate the result of transference through other parts.

At all events, from the failure of these experiments it must not be concluded that the phænomenon of the motion of liquids, by means of a galvanic current, is necessarily dependent upon the presence of a porous partition. On the contrary, the experiments yet to be recorded will show that the phænomenon depends solely upon the several constants of the galvanic current, and that the porous partition serves merely to render visible this motion of the liquids, which it effects by destroying causes that would otherwise act against and modify the effect of the galvanic current.

§ 3.

After establishing by the experiments above described, the fact of a motion of liquids from the positive to the negative pole of a closed circuit, an attempt was next made to discover some quantitative relation between this phænomenon and others already established.

Of the several instruments which were successively constructed, for this purpose, the following appeared to be the simplest and most convenient.

On the top of the porous cylinder *a* (Pl. IV. fig. 2.), which was closed underneath, a small bell-shaped glass, *c*, was cemented, having an aperture into which a perpendicular tube *d*, with lateral efflux tube *e*, was fitted. Within the porous one stood a cylinder *g* of platinum foil. This was connected by means of a wire, *f*, with one pole of the galvanic battery. This wire, *f*, was cemented into an air-tight glass tube let into the upper part of the bell-shaped glass *c*. Externally, the porous cylinder was surrounded by a second cylinder of platinum foil, which by the wire *k* was connected with the other pole of the battery. The whole apparatus stood in a wider glass cylinder, *n*, which, together with the porous cylinder, was filled with water or some other liquid, and protected from dust by means of the glass-plate cover, *n*. Before the tube *e*, a small weighed vessel, *l*, was placed to collect the liquid issuing from *e*. Previous to experiment, the porous cylinder made use of was boiled for

some time in dilute hydrochloric acid, and subsequently washed with distilled water until all soluble materials were removed. By this means the cylinders were preserved from any change which the solution of extraneous matter in their pores might have effected.

When the whole apparatus was filled with distilled water, so that within the porous cylinder the water stood at the level of the tube *e*, or rather exactly filled it, then in consequence of capillary attraction no water issued from *e*. Further, when the outer cylinder, *h*, was also filled to the brim with water, the difference of pressure within and without the tube *d* was so small that during many hours no perceptible change in the level of the water in the tube *d* took place. We may therefore assume that the action of the difference of pressure, though extended over many hours, is equal to zero.

If the end of the wire, *k*, be connected with the positive pole, the end of the wire, *f*, with the negative pole of the galvanic battery, the water soon ascends in the tube *d*, above the level of its junction with the tube *e*. The hydrostatic pressure thus produced overcomes the capillary attraction at the mouth of *e*, and hence the water flows out into the vessel, *l*, placed to receive it. The principal cause of this phænomenon is, as we have already said, the galvanic current, by means of which the water is forced from the outer cylinder, *h*, into the clay cylinder. The simplicity of this action is nevertheless disturbed by many other circumstances.

The hydrogen gas, liberated in consequence of the chemical action of the current, collects on the inner surface of the clay cylinder on the platinum plate *g*, and on the bell-shaped glass *c*, and thus displaces a certain quantity of water. Nevertheless, as soon as a maximum amount of gas has thus collected, it afterwards escapes through the tube *d*, and causes no further ascent of the water in this tube unless the current has in the mean time exceeded a certain intensity. In order to measure the pure transferring action of the galvanic current, it is necessary therefore to wait until this maximum amount of gas has collected. Moreover, with distilled water the quantity of gas liberated is so small in comparison to the quantity of water transported, that the gas bubbles from time to

time ascending the tube *d* have no essential influence on the ultimate result. By using copper instead of platinum plates, and a solution of sulphate of copper instead of water, this disturbing influence of course entirely disappears.

If a galvanic current of constant intensity be conducted through a new apparatus for some time, it is observed that the quantities of water issuing from the tube *e*, in equal times gradually increase a little, especially if the surface of the clay cylinder be but a small one. At the same time the water in the cylinder becomes cloudy. The cause of both of these phænomena is, that the water streaming forcibly through the pores of the clay cylinder carries particles of clay along with it (as Becquerel has already observed). The capillary apertures of the cylinder become thus widened and offer less frictional resistance to the water passing through them. Consequently, after longer use of the same apparatus more constant results can be obtained than at first. As before mentioned, the same effect is obtained by boiling the clay cylinder for some time in hydrochloric acid.

Lastly, the results of the first two or three observations made with a new apparatus are in general too small, because the atmospheric air present in the pores of the cylinder requires first to be displaced by the water moved by the galvanic current.

§ 4.

The object of the first experiments made with the above-described apparatus was, to investigate whether currents of different intensity forced different quantities of liquid through the clay cylinder, and if so, whether any simple relation existed between these quantities and the corresponding intensities.

The galvanic battery employed in these experiments consisted of Daniell's elements. A tangent compass was used to measure the intensities, or if the current was weak, a galvanometer was employed, which had previously been compared with the tangent compass, and whose needles deviated somewhat from the astatic position.

After proving by many experiments, that by repeated employment of the same current equal quantities of liquid were always pushed into the clay cylinder in equal times, and thus made to issue from the tube *e* into the vessel *z*, currents of different

intensities were next experimented with. The following tables show the results thus obtained. Beside the intensity i, is placed the corresponding quantity of water m, in grammes, which passed in equal times (a quarter of an hour) through the tube e into the vessel z.

I. *Distilled Water.*

i.	m.	$\frac{m}{i}$.
144	17·77 grms.	1·23
108	13·26 „	1·23
83	10·59 „	1·27
60	7·46 „	1·24
48	5·89 „	1·23
36	4·47 „	1·24
29	3·38 „	1·17
	Mean . . .	1·23

By employing another cylinder the following numbers were obtained: —

i.	m.	$\frac{m}{i}$.
110	18·20 grms.	1·65
91	14·60 „	1·60
82	13·72 „	1·67
50	8·26 „	1·65
37	5·92 „	1·60
30	4·56 „	1·52
23	3·46 „	1·51
	Mean . . .	1·60

II. *Solution of Sulphate of Copper* (20 parts $\dot{Cu}\,\ddot{S}+5$ aq to 100 parts of solution).

i.	m.	$\frac{m}{i}$.
148	3·30 grms.	2·23
138	3·01 „	2·18
118	2·48 „	2·10
64·5	1·36 „	2·11
	Mean . . .	2·16

III. *Solution of Sulphate of Copper* (19·1 parts $\dot{Cu}\,\ddot{S}+5$ aq to 100 parts of solution).

i.	m.	$\frac{m}{i}$.
106	2·48 grms.	2·34
101	2·32 „	2·30
98	2·26 „	2·31
93	2·11 „	2·26
65	1·49 „	2·29
53·5	1·25 „	2·33
	Mean	2·30

IV. *Solution of Sulphate of Copper* (7·27 parts $\dot{Cu}\,\ddot{S}+5$ aq to 100 parts of solution).

i.	m.	$\frac{m}{i}$.
817	4·145 grms.	5·07
687	3·510 „	5·11
660	3·245 „	4·92
636	3·175 „	5·00
442	2·190 „	4·96
	Mean . . .	5·01

These tables prove that, whilst the intensities of the currents vary in the proportion 1 to 6, the quotients $\frac{m}{i}$, or the ratios of the intensity of current to the quantity of liquid transported, differ only about $\frac{1}{16}$ from their mean value. Hence we may safely assert that *the quantities of liquid transferred by the galvanic current into the clay cylinder in equal times, are directly proportional to the intensities of the current.*

§ 5.

A second question to be submitted to experiment is, what relation exists between the quantities of liquid transferred by the galvanic current, and the surface and thickness of the porous diaphragms employed.

In order to investigate this, various portions of the clay-cylinder in the above-described apparatus were successively over-

laid with a non-conducting stratum of wax or shell-lac varnish, impervious to the liquid employed, and then applied as before.

The following are the mean results of many experiments. In each case the relation between the intensity i of the galvanic current and the corresponding quantity m of transported liquid is given.

I. *Water.*

	$\frac{m}{i}$.
With the whole surface of the clay cylinder . .	1·23
„ $\frac{3}{4}$ of the „ „ „ . .	1·22
„ $\frac{3}{8}$ „ „ „ „ . .	1·24
„ $\frac{3}{16}$ „ „ „ „ . .	1·10
„ $\frac{1}{16}$ „ „ „ „ . .	1·11

II. *Solution of Sulphate of Copper* (19·1 parts $\dot{Cu}\,\ddot{\dot{S}}+5$ aq to 100 parts of solution).

	$\frac{m}{i}$.
With the whole surface of the clay cylinder . .	2·30
„ $\frac{2}{3}$ of the „ „ „ . .	2·31
„ $\frac{1}{3}$ „ „ „ „ . .	2·35
„ $\frac{1}{6}$ „ „ „ „ . .	2·28
„ $\frac{1}{12}$ „ „ „ „ . .	2·31

From a comparison of the quotients $\frac{m}{i}$, it is evident that they remain almost constant even when the permeable surface of the clay cylinder is materially diminished, so that to the above law, expressing the dependence of the quantity of transported liquid on the intensity of the acting current, may be added,—

The quantities of liquid transported by a galvanic current in equal times through different porous partitions are independent of the magnitude of permeable surface which these partitions present.

The effects of a galvanic current remain equal, under equal conditions, throughout its whole length. If, therefore, in a liquid placed between the electrodes, two equal porous (clay)

plates be immersed at two different places, through each clay partition exactly as much liquid will be transported as if it alone were immersed in the liquid, provided always that in both cases the same intensity of current be employed. Consequently, through the side of the clay partition turned towards the negative pole, exactly the same amount of liquid will pass when the intensity of the current remains constant, no matter whether it alone intersect the liquid, or whether between it and the positive pole another equal plate be introduced. This will also be the case if both plates touch each other, or, in other words, if instead of a certain plate another of double its thickness be immersed in the liquid. Hence, *with the same intensity of current, the quantities of liquid transported in equal times through plates of different thicknesses are equal.*

Experiment confirms this conclusion. In the apparatus before described (§ 3), a clay cylinder was placed whose upper denser surface had been previously scraped off; the apparatus was then filled with water, and used as before. It was found that the mean ratio between the quantities of liquid transported by currents of different intensities, to the corresponding intensities i themselves, was

$$\frac{m}{i} = 1{\cdot}60.$$

After making in a similar manner the sides of the cylinder still thinner, the apparatus was again used as before; the above ratio had then the value

$$\frac{m}{i} = 1{\cdot}64,$$

or nearly the same as when it possessed its original thickness.

Other experiments gave like results. For example, into the apparatus described in § 2 a clay plate 4·3 millims. in thickness was introduced; the apparatus was then filled with a solution of sulphate of copper, and the tube g, replaced by a ⊢ formed tube, such as that described in § 3; a vessel was placed before the efflux tube, so that the quantities of liquid transported by currents of different intensities into the vessel a_1 could be measured. The mean ratio of the corresponding intensity of the current to the quantity of liquid transported was

$$\frac{m}{i} = 1{\cdot}87.$$

When the thickness of the plate was diminished to 3·9 millims., this ratio had the value

$$\frac{m}{i} = 1{\cdot}92;$$

for a still thinner plate 2·8 millims. in thickness, it became

$$\frac{m}{i} = 1{\cdot}91.$$

In another case the thickness of the clay plate was about 8 millims., and the quotient $\frac{m}{i} = 1{\cdot}72$; when the thickness of the plate was reduced to 1·7 millim., even then the quotient had nearly the same value as before, viz. $\frac{m}{i} = 1{\cdot}78$.

Stronger burnt and denser clay plates allow less liquid to pass through with equal intensity of current, and in equal times, than less dense ones. The reason of this must be, that the fluids encounter different degrees of friction when passing through clay plates with wider or narrower pores.

§ 6.

Another problem to examine was the connexion between the observed transference, and the nature of the liquids used in the experiments.

By the investigations of Hagen, Poiseuille and others, on the efflux of liquids from narrow apertures, it has been shown how essentially the quantities of liquid issuing in equal times, under equal hydrostatic pressures, depend upon the friction of the different liquids in passing through these narrow apertures; hence in causing liquids, by means of a galvanic current, to pass through porous partitions, a simple relation between the other properties that such liquids may possess, and the quantities of them transported by equal galvanic currents, is scarcely to be expected. In general, however, it can be shown with great certainty that the quantities of different liquids transported by the same intensity of current are greater the greater the resistance which these liquids oppose to the conduction of a galvanic current through them.

The following table gives examples of this; in it $\frac{m}{i}$ always

represents, as before, the relation between the quantity of liquid and intensity of current; $\frac{0}{0}$ denotes the per-centage of crystalline salt contained in the solution employed, and the column r contains the relative resistances to conduction which the several liquids exert.

$\frac{0}{0}$ of $\dot{C}u\ddot{S}+5aq.$	$r.$	$\frac{m}{i}.$	$\frac{m}{r.i}$
$20\frac{0}{0}$	26·3	217	825
19·1	26·9	230	855
18	27·3	262	960
13·5	31·4	310	987
10	37·2	385	1035
7·3	(45)	501	1113

In determining the value $\frac{m}{i}$ in each case, the mean of several experiments, with currents of different intensities i, was taken. The resistances r are taken from those furnished by Becker (Liebig's *Annalen*, lxxiii. s. 1).

According to this, the relation $\frac{m}{i}$ increases quicker than the increase of resistance.

As additional examples of the increase of quantity with the resistance, distilled water and alcohol may be mentioned; a galvanic current, which produces no visible action upon a solution of sulphate of copper, transports them very visibly. Another proof is furnished by sulphuric acid; it is a good conductor, and it is impossible to effect its transference. If we seek to accomplish this by employing stronger galvanic currents, the violent liberation of gas opposes great difficulties to correct observation*.

The true connexion between the electrical motion of liquids and their peculiar composition, cannot be recognised in the above experiments; those yet to be described will first establish it.

* According to this, the increase of the liquid on the platinum plate of a Grove's battery cannot be explained by the galvanic transference of the liquids; inasmuch as all the liquids employed in that battery are comparatively good conductors. The cause of the phænomenon must rather be sought in the chemical processes which take place in the battery.

§ 7.

The disturbing influence which the different friction of the liquids in the pores of the porous diaphragms exerted on the results last given, may also have effected in some measure the laws previously expressed. It has already been mentioned, that, by employing clay plates of different densities, other conditions being equal, different quantities of liquid were transported by the same current in the same time; in the pores of the clay plates, owing to their different densities, a different friction was encountered, and the generality of the results would be thus impaired. In order to eliminate such disturbance, or to be able to draw conclusions independent of the friction of the liquids in the pores of the diaphragm, the former experiments were somewhat modified; so that in place of the quantities of liquid transported in equal times through the clay cylinder of the apparatus described in § 3, the force by means of which the galvanic current transported the liquid was measured directly. To this end a certain hydrostatic counter-pressure was applied within the clay cylinder. Its effect alone would have been to force the liquid out of the clay cylinder; but an attempt was made so to arrange its intensity, that it and the transporting action of the galvanic current exactly equilibrated each other.

The above-described apparatus was accordingly thus modified. The tube *a* (fig. 3.) attached to the clay cylinder was provided at its upper end with a brass cap, which could be closed air-tight by a screw. The efflux tube *e*, attached to the tube *a* by means of the blow-pipe, was then connected, by a strong caoutchouc tube, with a manometer, *p m*, partly filled with mercury; the height of which could be read off on a suitable scale. When the clay cylinder and surrounding cylinder, *h*, were filled with a liquid, the screw at the upper end of *d* closed air-tight, and the galvanic current set in action; the liquid entering the clay cylinder pressed down the mercury in the shorter leg *p* of the manometer, and thereby caused it to ascend in the longer leg *m*. The mercury continued to ascend until its difference of level in the two legs *p* and *m* exerted on the liquid within the clay cylinder a hydrostatic pressure sufficient to force in a given time through the pores of the latter into the wider cylinder *h*, as much

liquid as the galvanic current would transport into it. If, in this manner, the height attained by the mercury in the manometer be observed in the several experiments, the results thus obtained will be independent of the influence of the friction of the liquids in the pores of the clay cylinder, inasmuch as now, in place of motion, a purely statical condition is produced.

In such experiments it is necessary to avoid every kind of liberation of gas within the clay cylinder; for were such to occur, the level of the mercury in the manometer would be thereby affected. This inconvenience was avoided by employing copper electrodes and differently strong solutions of sulphate of copper. Thus also a greater constancy of galvanic current was ensured, inasmuch as the phænomena of polarization were avoided.

§ 8.

Repeating *the same* experiment under equal conditions until a satisfactory agreement in the results had manifested itself, the relation between the intensity of the galvanic current and the heights attained by the mercury in the manometer under otherwise equal circumstances, was ascertained. The results of some of the observations may be seen in the following tables. i denotes, as before, the intensity of the current; h the corresponding height of mercury in the manometer; $\frac{h}{i}$ the ratio of the two.

I. *Solution of Sulphate of Copper* (19 parts $\dot{Cu}\,\ddot{S}+5$ aq to 100 parts of solution).

i.	h.	$\frac{h}{i}$.
128	176·5 millims.	1·38
109	147·5 „	1·35
97	132·5 „	1·37
73	100·5 „	1·38
65·3	89·0 „	1·36
58·3	80·5 „	1·38
45	61·0 „	1·36
26·5	37·5 „	1·41
14·3	19·5 „	1·36
	Mean . . .	1·37

II. *Solution of Sulphate of Copper* (9·3 parts $\dot{C}u\dddot{S}+5$ aq to 100 parts of solution).

i.	h.	$\frac{h}{i}$.
67	132·0 millims.	1·97
60	122·0 „	2·03
39	78·0 „	2·00
32	62·5 „	1·95
15·5	30·0 „	1·94
	Mean . . .	1·98

III. *Solution of Sulphate of Copper* (3·4 parts $\dot{C}u\dddot{S}+5$ aq to 100 parts of solution).

i.	h.	$\frac{h}{i}$.
44	161 millims.	3·66
40	148 „	3·70
28	112 „	4·00
24	89·5 „	3·73
20	77·0 „	3·85
	Mean . . .	3·80

The numbers furnished by these tables, as well as those obtained by other experiments, give for each series of observations almost a constant value for $\frac{h}{i}$; we may therefore conclude that *the heights to which liquids are caused to ascend by the galvanic current are directly proportional to the intensities of the current.*

This result agrees perfectly with the law before expressed, according to which the quantities of liquid transported by the galvanic current into the clay cylinder in equal times are directly proportional to the intensities of the current. The beautiful experiments of Hagen* and Poiseuille† prove, that the quantities of liquid issuing in equal times from very narrow capillary tubes are directly proportional to the charges under

* Pogg. *Ann.* Bd. xlvi. s. 483.; see also *Repertorium der Physik*, Bd. vii. s. 138.

† *Ann. de Chemie et de Physique*, S. 3. vol. vii. p. 50.

which such efflux takes place. The same law applies, without doubt, to the passage of liquids through porous clay plates, which represent in some measure an aggregate of very narrow tubes. Similarly, however, the quantities of liquid transported by a galvanic current through a clay partition, are also in direct proportion to the current's intensity. If therefore the action of hydrostatic pressure be equivalent to the elevation of the liquid by a galvanic current of a certain intensity, then by increasing the intensity of the current this hydrostatic pressure must increase in the same proportion in order to neutralize the increased action. In this manner, therefore, the transporting action of a galvanic current may be represented by a hydrostatic pressure.

§ 9.

By changing the magnitude of permeable surface presented by the clay cylinder, it was found that *the heights to which the liquids ascend under the influence of the same galvanic current are, under otherwise equal circumstances,* INVERSELY *proportional to the permeable surface presented by the clay cylinder.*

The following table furnishes an example of this. In it the magnitude $\frac{h}{i}$ denotes the mean ratio of the height to the corresponding intensities of current obtained in each case from several experiments; o represents the corresponding permeable and conducting surfaces presented by the clay cylinder; these were gradually diminished by the application of layers of water-tight and non-conducting shell-lac.

o.	$\frac{h}{i}$.	$\frac{h}{i}o$.
With the whole surface . . .	1·37	1·37
„ $\frac{7}{10}$ of the „ . . .	1·80	1·26
„ $\frac{4}{10}$ „ „ . . .	3·42	1·37
„ $\frac{2}{10}$ „ „ . . .	6·00	1·20

The small deviations in the values of $\frac{h}{i}o$, which should all in fact be exactly equal, may be easily accounted for by the different thicknesses of the clay cylinder at different places, as well as by

the difficulty of covering exactly a certain fraction of the surface with shell-lac.

The result expressed in this section also agrees with the law before established, according to which the intensity of the galvanic current being the same, the quantities of liquid transported were independent of the magnitude of permeable surface presented by the clay cylinder. For if we assume that the capillary apertures of the clay cylinder are equally distributed over its surface, then the quantity of liquid driven through a given fraction of that surface by means of a certain hydrostatic pressure will be directly proportional, first to the magnitude of surface, and secondly to the pressure itself. But the quantity of liquid transported by a galvanic current of constant intensity through a surface, for example, ten times smaller, is exactly the same as that through the whole undiminished surface. In order, however, that the same amount of liquid may be pressed through two surfaces, one ten times smaller than the other, by hydrostatic pressure, the pressure on the former case must be ten times greater than on the latter. Hence to equilibrate the action of the galvanic current, the hydrostatic pressure must increase in the same proportion as the permeable surface of the clay cylinder decreases.

Exactly similar considerations apply to the case where clay plates of different thicknesses are used. With capillary tubes of different lengths, and doubtless also with clay plates of different thicknesses, the quantities of liquid forced through are inversely proportional to the lengths of the capillary tubes or the thicknesses of the clay partitions. In order therefore that the action of a galvanic current, which with constant intensity transports equal quantities of liquid in equal times through clay partitions of different thicknesses, may be always equilibrated by hydrostatic pressure, the latter must increase in direct proportion to the thickness of the partitions. Hence *the heights to which liquids ascend under the influence of the same galvanic current are, under otherwise equal circumstances, directly proportional to the thicknesses of the clay partitions.*

This law, which follows necessarily from the foregoing, the author also sought to corroborate by experiment. As it was difficult however to obtain clay partitions of different thick-

nesses, which at the same time were perfectly homogeneous, great exactitude in the results could not be calculated upon, and the more so, as it was necessary, in order to obtain plates of the greatest possible homogeneity, to employ none above 6 or 8 millims. thick: in consequence of this the variations in thickness were limited to a few millimetres. The experiments were conducted much as before. After fastening a ⊢-formed tube to the apparatus (§ 2) exactly in the manner described in § 5, the tube was provided with a metal cap which could be closed air-tight, and the lateral efflux tube was connected with the manometer (as described in § 7, when using the apparatus before employed in § 3). Afterwards clay partitions of different thicknesses were introduced between the two vessels, the apparatus itself was filled with a solution of sulphate of copper, the brass screw of the metal cap on the ⊢-formed tube was closed, and after conducting the galvanic current through the apparatus for some time, the height of the mercury in the manometer was read off.

In one particular case the thickness of the clay plate was $d = 4{\cdot}3$ millims., and the mean ratio between the intensity i, of the current and the corresponding height h, attained by the mercury in the manometer, was $\frac{h}{i} = 1{\cdot}73$, which made $\frac{h}{id} = 0{\cdot}402$. When the thickness of the plate was decreased to 3·8 millims., we had $\frac{h}{i} = 1{\cdot}60$, hence $\frac{h}{id} = 0{\cdot}421$. The thickness of the plate was still further decreased to 2·8 millims., which made $\frac{h}{i} = 1{\cdot}21$ and $\frac{h}{id} = 0{\cdot}432$. Here, therefore, the ratios $\frac{h}{i}$ decrease in the same proportion as d.

Another plate of the thickness $d = 8$ millims. gave $\frac{h}{i} = 3{\cdot}30$ and $\frac{h}{id} = 0{\cdot}41$. When the thickness of this plate was diminished to 4 millims., then $\frac{h}{i}$ was decreased to 1·62, making $\frac{h}{id} = 0{\cdot}40$. Lastly, the thickness of the plate was still further diminished until it became from 1·7 millim. to 2 millims. thick: we obtained

$\frac{h}{i}=0{\cdot}73$, giving a value of $\frac{h}{id}$ between 0·43 and 0·36, in fact exactly the same as before.

§ 10.

Lastly, the author attempted to investigate yet more accurately the relation between the transferring power of the galvanic current and the nature of the several liquids transferred. On account of the different degrees of friction in the pores of the clay partition, a satisfactory answer to questions connected with the above relation could not be expected from mere measurements of the quantities of the different liquids which equal currents could transport in equal times. The method last employed, however, promised more satisfactory results, inasmuch as the influence of friction is thereby eliminated. As it was before proved that the conductibility of the liquids had an essential influence upon their transference, the chief object in the following experiments was to examine whether or not a more accurate relation could be established between the heights of the mercury in the manometer of the apparatus described in § 7, caused by currents of equal intensity passing through different liquids, and the relative resistances which these liquids opposed to the conduction of the current.

In the absence of sufficient data showing the galvanic conductibility of liquids, particularly of such where liquids of different resisting capabilities are compared with each other, it was necessary to make a few preliminary experiments, in order to determine, if not very accurately, at least with an accuracy sufficient for the object in hand, the relative resistances opposed by several solutions of copper of different degrees of concentration.

The method according to which these determinations were made, was throughout the same as the one adopted by Becker and described in his memoir on the subject*. The liquids were placed in a rectangular glass trough about 45 millims. long, 45 millims. broad, and 80 millims. high, whose two end surfaces were covered in the interior by copper plates cemented thereon; the trough was always filled to the brim with the liquid to be

* Liebig's *Annalen*, Bd. lxxiii. s. 1.

tested, and then previous to every experiment covered with a ground glass plate. The resistances themselves were measured by a rheostat of Jacobi's construction. The coils were of German silver wire 0·62 millim. in thickness; the length of each coil, which is assumed as the unit of resistance to conduction, amounted to 220·5 millims. The temperature was from 18° to 20° C. After determining the resistances, the liquids were poured into the apparatus described in § 7, and the heights of the mercury in the manometer were noted for currents of different intensities. The following table shows the results thus obtained. In it the numbers in the column $\frac{\delta}{0}$ denote the percentage of crystalline salt ($\dot{C}u\,\ddot{S}+5aq$) in the solutions; the numbers under r are the observed resistances of the solutions; the numbers under $\frac{h}{i}$ are the mean ratios, each found from several experiments, of the height attained by each solution to the intensity of the current.

$\frac{\delta}{0}$.	r.	$\frac{h}{i}$.	$\frac{h}{ir}$.
16·25	18·0	1·35	7·50
9·22	27·0	1·98	7·33
6·6	32·5	2·44	7·50
3·4	55·5*	3·79	6·83
1·8	100·0*	6·80	6·80
		Mean . . .	7·19

From this table it is evident, that, whilst the resistances increased from 1 to 6, the ratios of the heights attained by the liquids with currents of equal intensity to the corresponding resistances of the liquids differed only $\frac{1}{15}$ from their mean value. If, therefore, a general conclusion may be drawn from the present experiments, it must be thus expressed:—

The heights to which the several liquids are raised by the galvanic current are, under otherwise equal circumstances, directly proportional to the specific resistances of these liquids.

* The resistances of the very dilute solutions are perhaps here somewhat too great, inasmuch as on the copper electrodes immersed in them, a thin stratum of oxide was deposited during the process of determination.

§ 11.

The results of observation given in the foregoing paragraphs satisfactorily prove how nearly the mechanical action of a galvanic current conforms to all conditions which modify its general deportment.

Let us next represent by m the quantity of liquid transported by a galvanic current of the intensity i, through a porous partition of the thickness d and surface o; then, according to § 4–6,

$$m = k.i,$$

where k is a constant magnitude for every liquid, whose value changes, however, according to the nature of the different liquids employed (the friction in the pores of the porous partitions, and their resistances to galvanic conduction), and is also dependent upon the different densities of the partitions. Hence *the quantity of a certain liquid, transported by a galvanic current of given intensity through a porous partition in the unit of time, is directly proportional to the intensity of that current, and independent of the thickness and transverse sectional area or surface of the porous partition.*

Further, let us represent by h the height attained when employing the manometer; by r, the specific resistance of the liquid used, then

$$h = c\,\frac{i.r.d}{o},$$

where c is a constant.

The force with which a galvanic current forces any liquid introduced into its circuit from the positive towards the negative electrode, through a partition presenting a given surface, may be measured by a pressure directly proportional to the intensity of the current, to the specific resistance of the liquid and to the thickness of the partition, but inversely proportional to the permeable surface presented by that partition.

Although these laws include the sum of all the observed phænomena, and in each special case allow comparisons to be instituted between the results obtained in moving liquids by the galvanic current, and the other manifestations of the current's activity, such as for instance its chemical or calorific action, yet they do not furnish any correct insight into the inner nature of

such motion. The intensity of a galvanic current represents an idea composed of many elements, and the moving force of the current must be referred to the same elements. To this end it is necessary to consider the matter more closely.

Let a liquid at any place be divided into two compartments, by means of a porous clay partition. Let both sides of the clay partition be covered by metal plates, which exert an electric tension (electromotive power) on each other. Let, for example, one of these be a copper, the other a zinc plate, both being connected, without the liquid, by a wire whose resistance to conduction is small enough to be neglected. The tension (E) generates a galvanic current in the liquid through the porous partition, whose intensity (i) is directly proportional to the tension (E), and to the surface (o) of the partition, but inversely proportional to the specific resistance (r) of the liquid used, and to the thickness (d) of the partition,

$$\left(i = \text{const.}\,\frac{Eo}{rd}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By means of this galvanic current the liquid will be transported from the positive to the negative side of the porous partition, with a force equivalent to a certain hydrostatic pressure (h). This pressure (h) is directly proportional to the intensity (i) of the galvanic current to the thickness (d) of the partition, and to the relative resistance (r) of the liquid, but inversely proportional to the surface (o) of the partition. Hence

$$\left(h = \text{const.}\,\frac{ir.d}{o}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By combining the equations (1 and 2),

$$h = \text{const. const.}\,\frac{Eo}{rd}\cdot\frac{rd}{o} = \text{C.E},$$

it follows immediately *that an electric tension present on both sides of a porous partition, immersed in any liquid, transports that liquid from the positive to the negative side, with a force equivalent to a hydrostatic pressure directly proportional to that tension**.

* Without doubt the law here established under the supposition that porous diaphragms are employed, is also immediately applicable, no matter what kind of a transverse section be laid through the liquid traversed by the galvanic current.

By this law a simple measure is given for the electric tension (or electromotive force) and its mechanical action, expressed in atmospheric pressures, or in the unit of the force of gravity; this measure too is completely independent of all concurrent circumstances.

We have seen that the transportation of liquids by the galvanic current is a manifestation solely of the electromotive force, which may thus in a certain measure be viewed as a moving force.

Let us imagine an electromotive power, acting at the surface of contact of two liquids, forming a circuit; and let us suppose that in each liquid a porous partition is immersed whose sectional area shall be equal to that of the liquid, and whose thickness shall be unity. Then, by means of the generated galvanic current, the liquids will move through the clay partitions with forces which can be measured by hydrostatic pressures. According to the above law these *pressures* are *directly* proportional to the resistances of the liquids, and *inversely* proportional to the surfaces of the clay partitions (the intensity of the current and thickness of the clay partitions being in this case equal in both liquids). The descents of the liquids measured by the differences of pressure on both sides of each clay partition of the unit of thickness *correspond* therefore in like manner *to the same magnitudes.*

But according to Ohm's mode of viewing the subject, the distribution of electric tension has exactly the same connexion with these same magnitudes. Here also the *falls of electricity* (the difference of two ordinates, representing the electric tensions at two points separated by the unit of length) are *directly* proportional to the *resistances,* and inversely as the *transverse sectional areas* of the liquids.

The foregoing experiments, therefore, furnish an experimental proof that the mechanical actions of the galvanic current follow exactly the same laws as Ohm has established for the distribution of electricity in the galvanic circuit. This proof may have some value, as confirmatory of a former hypothesis; and, on the other hand, the intimate connexion of the newly-observed action of the current, with earlier and already investigated actions, lend an increased interest to the present investigation.

Article IX.

Researches upon the Optical Characters of Double Refraction in Isomorphous Substances. By M. H. de Senarmont.

[From the *Annales de Chimie et de Physique*, vol. xxxiii. Dec. 1851.]

THE laws of isomorphism, in the form in which they were made known by M. Mitscherlich, and in the sense in which he appears to have conceived them at the time of that great discovery, have a signification which is perfectly intelligible and clearly defined.

Elements which possess the same chemical characters ought, according to those laws, when entering into combination in the same equivalent proportions, to assume geometrical forms similar in their principal features, almost identical in their angles, and ought likewise to be capable of associating together in crystallization in indefinite proportions. They would thus form a kind of type at once chemical and geometrical, a structure almost independent of the particular nature of its constituent materials, and in which one substance may be replaced by another without modifying any of its essential properties, provided that they are capable of occupying the same place in the structure.

Isomorphism appears, moreover, according to the views of M. Mitscherlich, to be connected with a certain identity in the molecular constitutions of particular elements, and ought, except in cases of dimorphism, to attend them in all their combinations in such sort, that by an almost constant reciprocity, the isomorphism of compounds may be inferred from the isomorphism of elements, and the isomorphism of the elements themselves, from the isomorphism of their compounds.

This doctrine, supported by numerous examples in which its truth amounts almost to certainty, in which there is no circumstance which leaves room for doubt or objection, has moreover thrown an entirely new light upon the chemical constitution of what have hitherto been called mineral species; it has brought that constitution into accordance with the law of multiple pro-

portions; for it was only necessary to alter the limits of genus and species, properly so called, in order to explain how different compounds might present one and the same geometrical type; by this means it even became possible to reproduce, at least mentally, the relatively simple compounds, nearly all the true species which occur naturally only as complex mixtures.

But although the truth of these beautiful laws, for which mineralogy is indebted to M. Mitscherlich, is proved even by the comprehensiveness and simplicity of their results, as far as regards their application to the limited class of facts which are capable of direct observation, and under general conditions which do not widely differ from those in which isomorphism is obvious at first sight, it is very difficult to advance beyond those limits; and on attempting to abstract from the phænomena a precise idea of isomorphism, of its actual principles and its essential conditions, it soon becomes evident that our knowledge of the subject is not profound, but at best merely superficial.

We are, in the first place, acquainted with elements isomorphous themselves and in many of their compounds, but which cease to be so in nearly all the others. Thus, for example, potash and soda, manifestly isomorphous both geometrically and chemically when in the state of chlorides, bromides, nitrates, and alums, replacing each other moreover in variable proportions in many minerals, present a constant antagonism in the chemical and geometrical character of almost all their other homologous compounds. What then has destroyed in these compounds the chemical and geometrical isomorphism which ought invariably to accompany the elements of which they are composed?

We find, on the other hand, substances chemically very different possessing almost the same crystalline form; for example, carbonate of lime and nitrate of potash, which are not only isomorphous but isodimorphous; arragonite and bournonite, calcite and proustite, together with many other salts and minerals, which it would be useless to mention in this place. The resemblance of form in the above instances can scarcely be referred to a chemical isomorphism, which ought to be the cause or the consequence of the geometrical isomorphism, and that remarkable capability of associating in crystallization, which appears to be one of its essential characters.

It would indeed seem as if the true problem to be solved is, whether geometrical and chemical isomorphism may not in reality exist independently and separately.

Moreover, the physical conditions of isomorphism appear to be no less obscure than the chemical conditions; and if we are in the first instance led to believe that the interior resemblance is always a consequence of the exterior geometrical resemblance and the chemical analogies of composition, certain properties, which we ought to regard as closely connected with the essential constitution of substances, will at once present themselves to us as sometimes almost constant, sometimes essentially variable in the same class of compounds; indeed the curious observations of M. Pasteur have made us acquainted with substances which are not merely analogous, but positively identical in elementary composition and chemical characters, nearly identical in every part of their forms, but nevertheless incapable of associating in crystallization, on account of a molecular difference indicated solely by an optical peculiarity, and by specialities of form which were considered almost accidental.

It is therefore evident that isomorphism supposes certain conditional analogies, and is at the same time consistent with certain differences of internal organization, of which no indications are recognizable, except in those characters which are most immediately dependent upon that organization. In order to detect these delicate shades of difference, it is necessary to have recourse to every means of investigation which have been applied in any manner whatever to the study of the molecular constitution of substances. It was from such a motive that I have been induced to undertake the comparative study of the optical characters presented by isomorphous substances.

Since the time when Brewster discovered the distinctive phænomena of double biaxial refraction, and thus divided crystals into two principal classes distinguished by their optical characters, many physicists have occupied themselves in studying the phænomena of double refraction; but they have directed their attention to the fact itself, for the purpose of ascertaining its laws, or the general relations of position between the form and the optical axes. It is thus that they have established the fact, that in all crystalline substances an axis of symmetry is always an axis of ordinary elasticity for different colours, and that every axis of

optical elasticity which does not coincide with an axis of symmetry, generally has a direction a little different for different colours. They scarcely appear, however, to have been led to the comparative investigation of the phænomena of double refraction presented by crystals similar in form but different in substance. And this is the object I have had in view in the experiments I am about to describe.

Fresnel has demonstrated that the optical characters of all doubly-refracting media admit of being accounted for by the unequal elasticity of the æther in three fixed rectangular directions throughout the entire extent of that medium, which he has denominated the *axes of optical elasticity*. The elements which it is necessary to compare together for this purpose are, the position of these three axes in the crystalline form, the relative values of the elasticity in their three directions, and the crystallographic data which define the crystal; all of them dependent equally, although perhaps in a different manner, upon the molecular constitution and the internal structure of the crystal; and it is necessary to ascertain how it happens that the different effects of the same cause present themselves as functions of each other.

Before proceeding further, it will perhaps be advisable to state in a few words the definition of the optical characters which it was necessary to determine.

Let a^2, b^2, c^2 be the three coefficients of elasticity corresponding to the axes of optical elasticity A, B, C. The numerical constants a, b, c are directly proportional to the velocities, consequently inversely proportional to the indices of refraction of a ray of light perpendicular to the axis of elasticity in question, and at the same time polarized.

Let $a > b > c$, then both of the optical axes are included within the plane of the axes A and C of the maximum and minimum elasticity, and both make with the axis A an angle α determinable by the formula—

$$\tan \alpha = \pm \sqrt{\frac{b^2 - c^2}{a^2 - b^2}}.$$

The axes A and C, of maximum and minimum elasticity, therefore divide the acute and obtuse angle included between the two optical axes. The line which divides the acute angle is specially denominated the bisecting line.

When therefore α is less than 45°, the bisecting line is the

axis A of the maximum optical elasticity; if, on the contrary, α is more than 45°, the bisecting line is the axis C of minimum optical elasticity.

When one of the two quantities a and c are equal to b, the direction of both the axes of elasticity, corresponding to equal constants, becomes indeterminate in a plane perpendicular to the third axis; the doubly-refracting medium has then only one optical axis, which moreover coincides with that of maximum or minimum elasticity, according as its proper numerical constant is greater or less than the equal constants corresponding to the indeterminate axes which are perpendicular to it.

I have determined the direction of the three axes A, B, C, the actual or relative values of the coefficients a, b, c, by the ordinary methods based upon the well-known laws of double refraction and chromatic polarization. This determination is more or less complicated for the various systems of crystals. I will briefly state the course which I have followed in my experiments.

In crystals which may be referred to a right quadratic prism, to a rhombohedron or a hexagonal prism, the direction of the single axis is known beforehand; it therefore remains only to determine the two principal indices, or at least their order of magnitude.

Wherever the refraction was measurable, I have always determined the indices directly. In the first place, I cut prisms in the proper direction; and on account of the striæ, always left after polishing a salt, cemented upon their refracting surfaces either small fragments of thin sheet glass or laminæ of gypsum. Perfect juxtaposition may be effected by means of soft cement. I have, moreover, in all instances operated with at least two separate prisms, and have not considered any results sufficiently accurate unless they agreed with the calculated indices in the second decimal place. The goniometer employed for measuring the minimum deviation was that of M. Babinet, an instrument well adapted for this purpose; but as the prisms were very small and the light feeble, the opake thread at the focus of the collimator was replaced by a luminous slit; the readings were made in the yellow part of the spectrum.

When the measurement of the indices was impossible, I was contented with determining the character of the optical axis by

means of some one of the processes of compensation for which science is indebted to M. Biot and Sir D. Brewster.

The rings in convergent polarized light were observed upon a plate cut perpendicular to the axis, and I examined what modifications were effected in them by the interposition of a plate of mica in which the axis of minimum elasticity had previously been determined, and the thickness of which produced a retardation amounting to half an undulation.

Sometimes the great breadth of the rings rendered this mode of proceeding inapplicable, and I then placed a plate of gypsum giving a sensible colour, and the principal sections of which formed an angle of exactly 45° with the plane of polarization, in a bundle of parallel rays. No change takes place when the crystalline plate is interposed perpendicularly to the path of the rays issuing from this plate of gypsum, to the axis of which it is likewise perpendicular; but when it is inclined, so that the angle of incidence is parallel to one of the principal sections of the lamina of gypsum, the colour of the latter is altered in an inverse manner, according as the optical axis of the plate is that of maximum or minimum elasticity.

In order to determine the character of the optical axis in a plate parallel to that axis, it was placed upon a plate of quartz of convenient thickness, and cut parallel to its axis. This arrangement, placed perpendicularly in a bundle of parallel polarized rays, so that the axis of the quartz forms an angle of 45° with the plane of polarization, developes colours when the axes of the crystals, the one of maximum and the other of minimum elasticity, are parallel, or when both the axes of minimum elasticity are perpendicular.

It has long since been shown by Brewster, that in crystals referable to a right rhombic prism, the directions of the three axes of optical elasticity are parallel to the height of the prism and to the two diagonals of its base. When therefore the isochromatic curves are observed in convergent polarized light, it is easy to recognise at the first glance the position of the plane of the optical axes and of the bisecting line.

The perfect determination of the three indices requires operations which can rarely be executed with precision on the more or less imperfect crystals that were at my disposal; I have there-

fore in most instances been satisfied with determining the angle of the optical axes and the character of the bisecting line, and shall describe further on the details of the processes which I have adopted.

In crystals which are referable to an oblique rhombic prism, one of the axes of optical elasticity is always parallel to the horizontal diagonal of the base; it therefore remains to determine the direction of the other two axes. For this purpose I adopted two different methods.

The method which I shall call the first, consists in cementing two crystals together by a plane parallel to the horizontal diagonal of the rhombic base, taking care moreover to place them in an inverse position. An accurate adjustment may be obtained by allowing reflected light to fall upon the surfaces, which, notwithstanding the torsion, must remain parallel, and thus ascertaining with certainty that the surfaces parallel to these diagonals belong to one and the same zone in both parts of the artificial hemitropic group.

When the surfaces placed in juxtaposition are smooth, and the cement by which they are joined sufficiently soft, it is not difficult to effect an almost perfect coincidence. The cement which I employed for this purpose is Canada balsam, which hardens slowly on exposure to the air, or a thick solution of gum-arabic. This latter substance cements certain crystals very firmly, for example efflorescent salts, for which Canada balsam is less suitable.

From the hemitropic group I cut a plate with parallel surfaces perpendicular to the common horizontal diagonal of the rhombic base; and when it was to be cemented with Canada balsam, ground it with water, or with alcohol when it was to be cemented with gum. The halved plate included within its plane two axes of optical elasticity belonging to each crystal; and it is obvious that from one crystal to the other the homologous axes form an angle with each other, which is twice as great as their inclination to the plane of the artificial hemitrope.

The divided plate was then placed in an apparatus very similar to that employed by M. Biot* in studying the colours developed

* *Précis élémentaire de Physique expérimentale*, 3rd edit., vol. ii. p. 475, and pl. 8. fig. 2.

by crystalline laminæ, perpendicular to a bundle of parallel polarized rays, and the angle described by the lamina in its own plane, when the original polarization was no longer disturbed, was observed at first in one half of the plate, and then in the other.

This method appears to give tolerably accurate results when the observations are repeated with different plates. The section of the crystals may be verified by means of the goniometer; it is moreover evident, that when the surfaces in juxtaposition coincide perfectly, there must be a considerable error in the section to produce any appreciable inaccuracy in the results.

The other method, which I shall call the second, is applicable only when the plane of the optical axes is perpendicular to the horizontal diagonal of the base. It is very nearly the same as that adopted by Mr. Miller in his researches*.

I selected from among the faces of the crystal two which were parallel to each other and normal to the plane of the optical axes, through which might be seen, in convergent polarized light, at least one of the systems of rings. When such faces did not exist, I cut them artificially, taking care at the same time to leave some of the natural faces untouched, in order that the position of the former ones might be accurately determined by means of the goniometer. I then measured the apparent inclination of each optical axis to the normals of these artificial or natural faces in the following manner:—

The plate was fastened upon a Wollaston's goniometer with a little soft wax, in such a way that its plane was perpendicular to that arm of the carrier which is moveable about its axis parallel to the limb. The adjustment is known to be exact when a reflected image remains immoveable during that revolution. On bringing the plane of the optical axes parallel to the plane of the limb, the rings were observed, making their centre coincide with a very remote object seen directly, then rotating the plate in its plane 180°, and again bringing the centre of the rings to coincide with the same object. Between these two positions the limb had passed over an angle twice as great as that which the optical axis makes with the normal. In order to

* Transactions of the Cambridge Philosophical Society, vols. v. and vii.

observe in this manner the centre of the rings, it is advisable to place the plane of polarization in the first instance parallel to the plane of the optical axes, then at an angle of 45° to this plane.

If the apparent inclination of the optical axes to the normal of the plane of emergence is once measured, it is easy to calculate their true interior inclination, provided that the corresponding index of refraction, that is to say the index of a ray situated and polarized in the plane of one of the optical axes, has been previously determined.

If the true inclination of the optical axes to the normals of one or two faces of known position is known, the direction of their bisecting line in relation to these normals follows from it, and the position of the axes of optical elasticity in the interior of the crystal is thus completely determined.

This second method requires therefore a tolerably large number of measurements; each of these involves an error; and consequently the former method, which is moreover more general, appears to me susceptible of a higher degree of accuracy.

In experimenting with biaxial crystals, I have always determined the optical character of the bisecting line in artificial plates, either very nearly normal to that line or parallel to it, and at the same time normal or parallel to the plane of the optical axes.

In the former case the plate includes within its plane the axis of mean optical elasticity B, and one of the axes A or C. If the maximum elasticity is found in the direction of B, the axis of minimum elasticity can be no other than C. The bisecting line will therefore coincide with the axis of maximum elasticity A. When, on the contrary, the minimum elasticity is found in the direction of B, the axis of maximum elasticity can be no other than A, and the bisecting line will coincide with the axis of minimum elasticity C.

This bisecting line is included within the plane of the two other kinds of plates; its character may therefore be determined at once.

In order to determine with these plates the character of the axes of optical elasticity which they contain, I placed them upon laminæ of quartz, which were likewise parallel to their axis, and brought them in the way already described in a pencil of parallel polarized light. The thickness which is necessary to enable them to develope colours by compensation may always be found,

if, as M. Biot has recommended, slightly prismatic laminæ be employed; and the axis of the plate parallel to the optical axes of the quartz will always be that of the maximum optical elasticity.

I have applied the above-described methods of investigation to several groups of isomorphous substances selected from the different crystal-systems, and will now proceed to give a statement of the results of my experiments, reserving any inferences which I may draw from them for a subsequent occasion.

Crystals with One Optical Axis.

Right Prismatic System with a Square Base.

Biphosphate of potash. Binarseniate of potash.
Biphosphate of ammonia. Binarseniate of ammonia.

$$BO\,R^2\,O^5 + 2H^2\,O.$$

The biphosphate of potash was crystallized in square prisms, 001*, surmounted by an octahedron, 011, upon the edges of the base; the angle included between the two contiguous faces of that octahedron is 122° 16′. The three other salts presented a very lengthened octahedron, *oxy*, the faces of which, like those of the preceding, rest upon the edges of the base of the square prism; the marked curvature of the faces gives to the extremity of the crystals a somewhat acicular form: they are terminated by an octahedron, 011, of about 122° 10′.

All the crystals whose transverse diameter was 2 or 3 millims., could be cut into more or less perfect prisms:—

	Angle of refraction.	Minimum deviation		Indices	
		ordinary.	extraordinary.	ordinary.	extraordinary.
Biphosphate of potash	45° 37′	26° 2′	23° 58′	1·510	1·472
	39 33	21 29	19 44	1·505	1·465
Biphosphate of ammonia	47 55	27 8	25 48	1·512	1·477
	35 51	21 10	19 19	1·519	1·476
Binarseniate of potash	39 33	25 48	23 10	1·596	1·538
	39 11	25 5	22 42	1·587	1·534
Binarseniate of ammonia	50 18	34 0	30 32	1·579	1·525
	38 46	24 19	21 58	1·576	1·523

In these four salts, the isomorphism of which has been demonstrated by M. Mitscherlich, the optical axis is therefore that

* I shall in this memoir always employ the crystallographic notation proposed by Mr. Miller in his Crystallography.

of maximum elasticity. The examination of the rings led to the same result.

Double chlorides of copper and potassium—of copper and ammonium.

$$BCl, CuCl + 2H^2O.$$

These chlorides appear as octahedrons with a square base, 111, the lateral angles being truncated by the faces of a square prism, 001. The potassium double salt does not crystallize distinctly; that of ammonium presents, moreover, small octahedral faces, 021, resting upon the edges of the base of the prism; two contiguous faces of the octahedron, 111, form an angle of 118° 18′.

In these two salts the optical axis is that of maximum elasticity; the doubly refractive power is nearly equal.

The indices were measured only in the ammoniacal salt:—

Angle of refraction.	Minimum deviation		Indices	
	ordinary.	extraordinary.	ordinary.	extraordinary.
39°	28°·15	27°·20	1·744	1·724

The small number of isomorphous substances which I have as yet examined belonging to the very limited class of right prisms with a square base, possess therefore analogous optical properties associated with similar geometrical forms.

Rhombohedral or Hexagonal System.

Carbonate of Lime.	Nitrate of Soda.	Sulphate of Potash.?
$CaO\,CO^2$.	$NaO\,N^2O^5$.	$KO\,SO^3$.

Calcite crystallizes in rhombodedrons of 105°, and nitrate of soda in rhombodedrons of 106° 30′, both possessing, as is known, an optical axis, which is that of maximum elasticity, and a very great power of double refraction, which does not differ much. The sulphate of potash, the prismatic form of which approaches very closely to that of arragonite*, is probably not very remote

* J. F. L. Hausmann, Poggendorff's *Annalen*, vol. lxxxiii. p. 572.

from calcite in its hexagonal form*. I have measured its optical constants:—

Angle of refraction.	Minimum deviation		Indices	
	ordinary.	extraordinary.	ordinary.	extraordinary.
32° 13′	16° 42′	17° 0′	1·493	1·501
53 5	30 33	31 13	1·493	1·502

The prisms employed for these determinations were cut from regular but rather opalescent crystals.

The optical axis of the hexagonal sulphate of potash is therefore that of minimum elasticity; a character which was moreover verified by the observation of the rings.

Hyposulphates of lime, strontia, and oxide of lead.

$$BO\,S^2O^5 + 4\,H^2O.$$

The hyposulphate of lime was obtained in thick crystals of hexagonal form, presenting upon their entire contour a bevelment of about 120°. The hyposulphite of strontia generally forms thinner laminæ, bounded by a bevelment of the same kind, with an angle of about 119° 54′. This measurement is however somewhat uncertain, because the lateral faces are slightly striated.

Two crystallizations of hyposulphate of lead gave forms which differed slightly from each other.

The first was in thin hexagonal laminæ, with pairs of unequal faces, the lateral edges of which were replaced in pairs by a double row of facettes, forming angles of 139° 50′ and 120°

* The isomorphism of the hexagonal sulphate of potash and the rhombohedral carbonate of lime is by no means proved. Nevertheless I have obtained some hexahedral prisms of the former, the edges of which were modified by a series of facettes, *b*, inclined to each other at angles of 49° 20′, and consequently to the base P at an angle of 123° 25′ (the direct observation, less exact on account of the irregularity of the faces M and P, gave 122° 57′ and 123° 32′), by a second series, *b′*, inclined to the same base by about 142° 29′. The corners were likewise truncated by facettes *a*, forming with that base an angle of 127° 18′. (See Pl. IV. fig. 4.)

These forms may be very simply derived from each other, because in the first place *b*, *a*, *b*, and then *b′*, *a*, M belong to one and the same zone.

If the faces *a* compose a birhombohedric form, its axis will be $\frac{4}{3}$ of the axis of a rhombohedron of 105° 10′.

with the inferior and superior base. The other crystals were very thick; and all the inferior and superior lateral edges presented the same facettes, accompanied by others, forming with the same bases angles of 130° 15′ and 105° 50′.

These different systems of facettes tend to the formation either of rhombohedrons or of birhombohedric isoscelohedrons, the principal axes of which bear the same proportions to each other as the numbers 2, 1, $\frac{1}{2}$, $\frac{2}{3}$.

It is impossible to measure directly the indices of refraction of these three salts; but the circular rings may be easily observed through the natural bases. The optical axis of the hyposulphate of lime is that of maximum elasticity, as is the optical axis of the hyposulphate of strontia. The optical axis of the hyposulphite of lead is that of minimum elasticity.

Crystals with Two Optical Axes.

Right prismatic system with a rhombic base.

Sulphates of magnesia and zinc.
Chromate of magnesia.

$$BO\,RO^3 + 7\,H^2O.$$

All these salts crystallize in right rhombic prisms of about 90° 38′ with one of the faces of truncation at the acute edge of 89° 22′, parallel to an easy cleavage. They bear on their summits only the faces of an octahedron with a rhombic base 127° 22′ and 126° 48′, resting upon the horizontal edges of the prism, rarely truncating faces of the obtuse edges of this octahedron, which form with each other an angle of about 120°.

The plane of the optical axes is parallel to the rhombic base; the bisecting line is parallel to the major diagonal of this base. When, therefore, the faces of the prisms 120° and 60°, which truncate the obtuse edge of the octahedron, are placed vertically, by which means the plane of easy cleavage becomes parallel to the new base, the plane of the optical axes will be parallel to the major diagonal of this new rhombic base and to the height of the prism, the bisecting line parallel to this height.

I have measured the apparent angle of the optical axes as seen across the parallel faces of easy cleavage, and the indices necessary for deducing the true interior angle.

Measurement of the Angles included within the Axes.

	Angle of the prism.	Minimum deviation.	Corresponding indices.
Sulphate of magnesia	55° 22′	30° 0′	1·459
	49 4	25 15	1·454
Sulphate of zinc	51 6	28 27	1·483
	42 30	22 43	1·486

Measurement of the Indices.

	Apparent angle.	True interior angle.
Sulphate of magnesia	56° 56′	38° 14′
Sulphate of zinc	64 18	44 2

In the sulphates and the chromate the bisecting line is the axis of maximum elasticity.

Nitrate of Potash. $KO\,N^2O^5$.

Arragonite. Carbonate of Lead. $BO\,CO^2$.

The nitrate of potash crystallizes, according to Mr. Miller, in right rhombic prisms of 118° 50′, truncated by tangential faces at the acute and obtuse edges, and bearing at the summits several faces parallel to the minor diagonal of the base, together with the faces of an octahedron with a rhombic base of 91° 46′ and 131° 22′ resting upon the edges of that base.

The plane of the optical axes is parallel to the major diagonal of the base and to the height of the prism; the bisecting line is parallel to that height.

Mr. Miller has likewise determined the three optical constants of nitrate of potash and the angle included between the axes*. He has found that the constants *a*, *b*, *c* bear an inverse proportion to the numbers 1·333, 1·5046, and 1·5052; and that the bisecting line is the axis of maximum elasticity. This result is also verified by the usual methods of compensation.

Arragonite crystallizes in right rhombic prisms of 116° 16′, truncated by tangential faces at the acute edges, and bearing at their summits several faces parallel to the minor diagonal of the rhombic base, together with the faces of an octahedron of 93° 30′ and 129° 37′ resting upon the edges of that base. The plane of

* Poggendorff's *Annalen*, vol. l. p. 376.

the optical axes is parallel to the major diagonal of the base and to the height of the prism; the bisecting line is parallel to that height.

The three optical constants of arragonite have been determined by Rudberg*, and appear slightly variable in different specimens. The bisecting line is the axis of maximum elasticity.

I was unable to procure a specimen of carbonate of strontia sufficiently perfect for observations. This was likewise the case with the carbonate of baryta and baryto-calcite in right prisms, more than twenty crystals of which, from different specimens in beautiful pseudo-isoscelohedrons, I have had cut without finding one that did not present at least six *plages orientées*, the separating lines of which passed out upon the sides of the hexagon. Each of these *plages* is itself composed of thousands of hemitropic laminæ, which entirely destroy the regularity of the isochromatic curves. The carbonate of lead was crystallized in prisms of 117° 13′, truncated by tangential faces at the acute edges, furnished at the summits with two faces parallel to the minor diagonal of the base, together with the faces of an octahedron of 92° 19′ and 130° resting upon the edges of that base.

The plane of the optical axes is parallel to the minor diagonal of the rhombic base and to the height of the prism; the bisecting line is parallel to that height.

The exterior angle of the optical axes is, according to Sir David Brewster, 5° 15′; the bisecting line is the axis of maximum elasticity.

Calcite and carbonate of lime, which are not only isomorphous, but are capable of crystallizing together both in the prismatic† and the rhombohedral‡ forms, have, as is seen, their optical axes in rectangular planes with a bisecting line of the same optical character. Nitrate of potash approximates closely to arragonite.

* *Annales de Chimie et de Physique*, 1st Series, vol. xlviii. p. 225.

† The plumbiferous arragonite of Tarnowitz contains as much as 0·0386 carbonate of lead. Poggendorff's *Annalen*, vol. xlvii. p. 497.

‡ The crystallized plumbo-calcite, as well as the calcite occurring in rhombohedrons of 104° 53′ (Brewster), contain 0·078 of carbonate of lead. Edinburgh Journal of Science, vol. vi. p. 79.

Sulphates of baryta, strontia, and oxide of lead.
Anhydrite?

BO, SO^3.

The sulphates of baryta, strontia, and oxide of lead are manifestly isomorphous, and if the right rhombic prism determined by the planes of cleavage is adopted as the primitive form, the plane of the optical axes is parallel to the height of the prism and to the minor diagonal of the base, and the bisecting line is parallel to that minor diagonal.

I observed the rings with plates of sulphate of baryta and sulphate of strontia, which were cut in a direction tangential to the obtuse edges of the prisms of 101° 42′ and 104°. I likewise saw them very distinctly with a crystal of sulphate of lead through the faces which replace the base by a bevelment of 135° 16′ (a^4, Levy and Dufrenoy), the edge of which is parallel to the major diagonal of the base.

These crystals occurring at Leadhills are small and striated; they present only the already mentioned truncations and the prismatic faces 103° 40′.

The striæ rendered it impossible to make any measurement. Upon one of these crystals I cut faces parallel to the rhombic base of 103° 40′, and convinced myself that both in the sulphate of lead and in the sulphates of baryta and strontia the bisecting line is the axis of minimum elasticity.

These three isomorphous sulphates possess therefore the same optical character.

The forms of anhydrite generally differ from those of the preceding sulphates. Nevertheless, small quantities of sulphate of lime may be associated in crystallization with sulphate of strontia and baryta, and H. Hausmann has examined crystals of anhydrite, whose form approximates very closely to that of sulphate of strontia*.

When the faces which may be regarded as homologous are made parallel, anhydrite becomes referable to a right rhombic prism of 98° 54′ with three cleavages, one very easy and parallel to the base, a second equally easy tangential to the vertical obtuse edge, and a third less easy tangential to the vertical acute edge.

* Poggendorff's *Annalen*, vol. lxxxiii. p. 572.

This prism presents various modifications, among which are distinguished those which replace the base by a bevelment of 105° 8′, the edge of which is parallel to the major diagonal of that base (a^2 in sulphate of strontia, Levy and Dufrenoy).

The plane of the optical axes is parallel to the height of the prism and to the diagonal of its base, the bisecting line parallel to this major diagonal.

The values of the three optical constants a, b, c are, according to Mr. Miller*, inversely proportional to the numbers 1·571, 1·576 and 1·614; the interior angle of the optical axis is 40° 37′, the bisecting line is therefore the axis of maximum elasticity.

Sulphate of potash. Chromate of potash.

BO, RO^3.

These two substances were both crystallized in longitudinally striated prisms, in which the angle 73° 30′ could readily be recognized; the ends present smooth reflecting faces of an octahedron of about 92° 30′ and 112° 40′ with truncations tangential to the edges of that octahedron, which in sulphate of potash, according to Mitscherlich, form an angle of 120° 24′, and in chromate of potash of 120° 41′.

The plane of the optical axis is parallel to the rhombic base, and the bisecting line parallel to the major diagonal of that base; if, therefore, the faces of the prisms of 120° 24′ and 120° 41′, which truncate the edges of the octahedron, are made vertical, the plane of the optical axes will be parallel to the height of the prism and to the major diagonal of the new base, the bisecting line parallel to that major diagonal.

The rings may easily be observed through an artificial face tangential to the edge of 73° 30′, and the apparent angle of the optical axes is thus found to be equal for sulphate of potash, to 100° 52′, for the chromate to 92° 20′.

The dispersion of the optical axes corresponding to different colours, which is not very marked in sulphate of potash, is tolerably strong in the chromate, and the isochromatic curves are orange outside and greenish inside.

I have measured the indices of refraction of a ray polarized in the direction of the plane of the optical axes.

* Poggendorff's *Annalen*, vol. lv. p. 525.

	Angles of the prisms.	Minimum deviation.	Corresponding indices.
Sulphate of potash	57° 4′	34° 3′	1·494
Chromate of potash	52 16	46 20	1·722
	40 3	32 9	1·722

It follows from this that the interior angle of the optical axes is, in sulphate of potash 66° 54′, and in the chromate 49° 32′.

This double result is, moreover, verified by particular consequences.

The rings may easily be observed in sulphate of potash through the faces of the prisms of 73° 28′ and 112° 22′. In the former case the axes form an apparent angle of 45° 56′, and in the second an angle of 66° 52′.

These data suffice for the calculation of the interior angle of the optical axes without the necessity for determining the index of refraction*, and with the above measurements this angle is found to be = 67° 20′, almost identical with the first.

In chromate of potash, likewise, the rings may be observed through the faces of the prisms of 73° 28′. They have, at least apparently, green outside and the orange inside, but it is easy to perceive that the axes have intersected after their emergence, and the angle which they thus include between them was found equal to 4° 7′. From the above data, then, it follows that

* Let (Pl. IV. fig. 5.)

m be the semi-angle between the normals of the first pair of faces.
M the same angle for the second pair.
ϕ the interior semi-angle of the optical axes.
r and R their interior incidence upon the two kinds of faces.
i and I the corresponding angles of emergence.
θ, Θ the apparent semi-angles of the optical axes after their emergence.
l the index of refraction,

$$m=r+\phi=i+\theta,\quad M=R+\phi=I+\Theta,$$

$$\sin(m-\theta)=l\sin(m-\phi),\quad \sin(M-\Theta)=l\sin(M-\phi);\ \text{therefore}$$

$$\tan\left(\frac{M+m}{2}-\phi\right)=\tan\frac{M-m}{2}\,\frac{\tan\left[\frac{M+m}{2}-(\theta+\Theta)\right]}{\tan\left[\frac{M-m}{2}+(\theta-\Theta)\right]}.$$

In the present instance,

$$m=33°\,49',\quad M=53°\,17',\quad \theta=33°\,26',\quad \Theta=22°\,53',$$

$$\tan(43°\,33'-\phi)=\tan 9°\,44'\,\frac{\tan 15°\,5'}{\tan 15°},\quad \phi=33°\,40'.$$

the apparent optical axes must form with the normals of the planes of emergence an angle of 55° 13′, but these normals form with the bisecting line an angle of only 53° 16′, consequently the apparent axes intersect at double the difference or 3° 54′, an angle which is tolerably near to that obtained by direct observation..

The bisecting line of the optical axes is in sulphate of potash the axis of minimum elasticity, and in chromate of potash that of maximum elasticity, a result which was confirmed by observation with plates cut in various directions.

Lævo- and dextro-tartrates of soda and potash.
„ „ „ ammonia.

$$(NaO+RO)\ C^8\ H^8\ O^{10}+8H^2\ O.$$

These salts, which I was enabled to examine through the kindness of M. Pasteur, present a peculiar interest, for the complete isomorphism in almost all parts of the form of lævo- and dextro-tartrate of soda and ammonia does not only fail to prevent their crystallizing out separately from a state of common solution, but this tendency even suffices as a means of separating the lævo- and dextro-acids, notwithstanding the fact that their previous combination took place with such energy as to develope a sensible amount of heat.

These four salts crystallize, absolutely in the same manner, in rhomboidal prisms, modified by numerous facettes parallel to their height. Among the most definite and well-developed are those which tend towards a first rhombic prism of about 100° 30′, to a second rhombic prism of about 135° which truncate the obtuse edges of the former, and finally, those of a rectangular prism tangential to the obtuse and acute angles of both these prisms.

The lævo-tartrate and the dextro-tartrate of soda and of potash possess characters of optical double refraction, which are absolutely identical. I convinced myself of this fact in the following manner:—

1. Two crystals cemented upon the same plate of glass by the same planes and in a parallel position, presented exactly the same refraction when afterwards cut into prisms in any direction.

2. It is impossible to discover the slightest difference between

the two systems of isochromatic curves presented by two plates of equal thickness; every circumstance is identical, angle and dispersion of the optical axes, but this absolute identity may be demonstrated by a still more decisive proof.

It is known that the bisecting line is normal to the rhombic base of the prism of 100° 30′. If then two plates, one of lævo-tartrate, the other of dextro-tartrate, are cut without injuring that face, and then joined together by their natural bases with Canada balsam, it will be possible after a few trials to bring the crystals into perfect parallelism, and then a single system of isochromatic curves may be seen through the complex plate the same as with a simple plate of a thickness equal to the sum of the partial thicknesses. Nothing is altered, either in the singular form of the rings or in the disposition of the colours.

The angle of the optical axes in dextro-tartrate of potash and soda was found by Sir J. Herschel to be $=76°$ for the red and 56° for the violet rays; and further, the index of refraction of the red rays $=1{\cdot}4929$, and that of the green $=1{\cdot}4985$, when both lie in the plane of the optical axes and are polarized in it. The same data were likewise obtained for the lævo-tartrate. If, however, plates perpendicular to the optical axes of the intermediate ray are cut, in accordance with these data, from Seignette salt, it is easy to perceive that the inclination of the optical axes is greater than is stated by Sir J. Herschel.

I have observed the isochromatic curves through several plates of one or other salt parallel to the rhombic base, and cemented between the hypothenuses of two rectangular isosceles prisms of glass. By this means, the axes, which are too divergent to be seen directly through a plate with parallel faces, are rendered more convergent at their emergence, and I have thus convinced myself that in both salts the plane of the optical axes is parallel to the minor diagonal of the rhombic base.

The bisecting line is the axis of major elasticity*.

* As this latter result is not in accordance with the statement of Sir D. Brewster, which appears to have been confirmed by other physicists, I will describe the method by which it was obtained.

From large and perfectly pure crystals of Seignette salt, the form of which was previously determined, three large thick plates were cut, one parallel to the rhombic base, the other two parallel to the very well developed faces of the rectangular prism; all these plates were bounded laterally by the natural faces,

The optical properties are therefore perfectly identical in lævo- and dextro-tartrate of soda and ammonia on the one hand, and in lævo- and dextro-tartrate of soda and potash on the other; of this I satisfied myself by repeated trials. On the contrary, the ammonia salts differ considerably from those of potash.

In the lævo-tartrate and dextro-tartrate of soda and ammonia, the bisecting line is indeed parallel to the height of the rhombic prism of 100° 30′; but the plane of the optical axes is parallel to the major diagonal of the rhombic base, and consequently perpendicular to the diagonal plane in which the optical axes of the potash salt would be situated. The optical axes of the different colours present a dispersion which is at least equal to that observed by Sir J. Herschel; the general angular divergence is less considerable than in the previous salt, and the isochromatic curves may easily be seen through a plate normal to the bisecting line.

I have determined the mean index of a ray polarized and situated in the plane of the optical axes, and the apparent angle of these axes corresponding to the tolerably homogeneous red light which glass of that colour transmits, and to the violet transmitted by a concentrated ammoniacal solution of carbonate of copper. These latter observations are not however susceptible of any great accuracy.

Determination of the Mean Index.

	Angle of the prisms.	Minimum deviation.	Indices.
Lævo-tartrate	48° 14′	27° 22′	1·490
Dextro-tartrate.....	39 5		1·495

so that there could be no mistake as to the direction in which the determinations were to be made.

With the first plate I determined the position (*trace*) of the plane of the optical axes by observing the rings through rectangular glass prisms; I then proved by placing the plate upon a parallel quartz plate, that it developes colours in polarized light when the plane of the optical axes is perpendicular to the axis of the quartz; the other plates, both parallel to the bisecting line, develope colours under the same conditions when the bisecting line is parallel to the axis of the quartz plate.

These latter experiments were made after a great number of others, all of which led to the same result.

Determination of the Angle of the Optical Axes.

	Apparent.	Real.
Red rays	100°	62°
Violet rays	70	46

The bisecting line is the axis of major elasticity.

Oblique Prismatic System.

Dextro-tartaric acid and lævo-tartaric acid.

$C^4 H^4 O^5, H^2 O.$

These two acids possess the form described by M. Pasteur; and I shall employ the same designation for the faces as he adopts in his memoir*.

Their characters of double refraction are absolutely identical; for when two faces of cleavage are cemented upon the same plate of glass in a parallel position, and the refracting angles cut in any direction, the double refraction produced is precisely the same in both crystals.

The horizontal diagonal of the base is parallel to the bisecting line of the obtuse angle of the two optical axes, and the plane of these latter makes an angle of 20° 27′ with the normal of the face M; it follows from this that it is difficult to perceive the rings through the natural faces, and above all, that the means of measuring with accuracy the reciprocal inclination of the optical axes are wanting.

The rings may be seen when a plate, whose parallel faces are cut artificially in a direction perpendicular to the horizontal diagonal of the rhombic base, is cemented between two appropriately adjusted equilateral prisms of glass. By this means I found the obtuse angle of the optical axes to be about 120°; but it is evident that this method does not ensure any great accuracy.

The direction of the bisecting line I determined according to the first method, by making a divided plate hemitropic round the normal of the plane of cleavage; and this means of observation is more accurate the greater the refractive power. Ten corresponding observations gave me the above angle, which is inscribed on the figure. Plate IV., fig. 6.

The bisecting line is the axis of maximum elasticity.

* *Annales de Chimie et de Physique*, 3 sér. t. xxviii. p. 56.

Dextro-tartrate and lævo-tartrate of ammonia.

$$C^4 H^4 O^5 N^2 H^8 O.$$

Both salts present the form described by M. Pasteur*; and I shall designate the faces by the letters which he employs.

The optical characters are perfectly identical; this may be seen when two crystals are cemented by their planes of cleavage, or their generally very definite faces, M, upon a glass plate. When the two crystals are in a parallel position, the prisms cut from them in any manner possess exactly the same refraction. The identity may be confirmed by a still more delicate proof. One of the two optical axes is, as will be shown further on, almost normal to the easy cleavage; if therefore a plate of cleavage is separated fromboth the lævo- and dextro-tartrate, and cemented together in a parallel direction, the rings coincide completely, and the combination acts like a single plate, of a thickness equal to the sum of the partial thicknesses.

The plane of the optical axes is normal to the horizontal diagonal of the rhombic base; and I have determined the direction of the bisecting line by the second method.

Determination of the Indices of Refraction.

	Refracting angle.	Minimum deviation.	Indices.
Dextro-tartrate.....	39° 27′	22° 51′	1·534†
Lævo-tartrate	40 1	23 16	1·533

In order to determine the position of the bisecting line, I cut a plate, the faces of which were parallel to the horizontal diagonal of the rhombic base, and whose normal made an angle of 18° 22′ with the normal of P, and an angle of 79° 40′ with the normal of M, as the angle comprised between the latter two was equal to 88° 2′. Through this plate I could see both

* *Annales de Chimie et de Physique*, 3 ser. t. xxviii. p. 56.

† Mr. Miller has found another index for the same salt (Transactions of the Phil. Soc. of Cambridge, vol. v.). He measured the minimum deviation through the faces M and *b*, the angle of which he estimates at 39° 53′, and found it equal to 25° 17′. I experimented with the same faces, slightly polished, and with an angle of 39° 27′, I found the ordinary deviation to be 22° 51′, the extraordinary deviation 25° 1′. Mr. Miller appears, therefore, to have mistaken one ray for the other.

systems of rings, and thus found the apparent angles which the first and second optical axes make with the normal, are respectively equal to 27° 3′ and 32° 32′, corresponding to the exterior angles 17° 22′ and 20° 40′; the interior angle of these axes is therefore 38° 2′. It follows from these data, that the bisecting line is inclined at an angle of 16° 43′ to the normal of the face P, and at an angle of 71° 19′ to the normal of M.

Figure 7 represents an ideal section of the crystal parallel to the plane of the optical axes; it shows the direction of the normals to the faces, those of the optical axes and of their bisecting line.

This result is moreover susceptible of verification; for as each of the optical axes makes with the bisecting line an angle of 19° 1′, the former makes with the normal of P an angle of 2° 18′ in the interior, and of 3° 31′ at the exterior.

If, then, two plates, separated by cleavage in a direction parallel to P, are cemented upon a glass plate parallel to each other, but in inverse positions, the rings proper to each will be seen, and the corresponding optical axes will be found to form in the air an angle of 6° 59′, almost twice as great as in the preceding experiment.

The bisecting line is the axis of minimum elasticity.

Neutral phosphate and arseniate of soda.

$$NaO, P^2O^5 + 24H^2O.$$

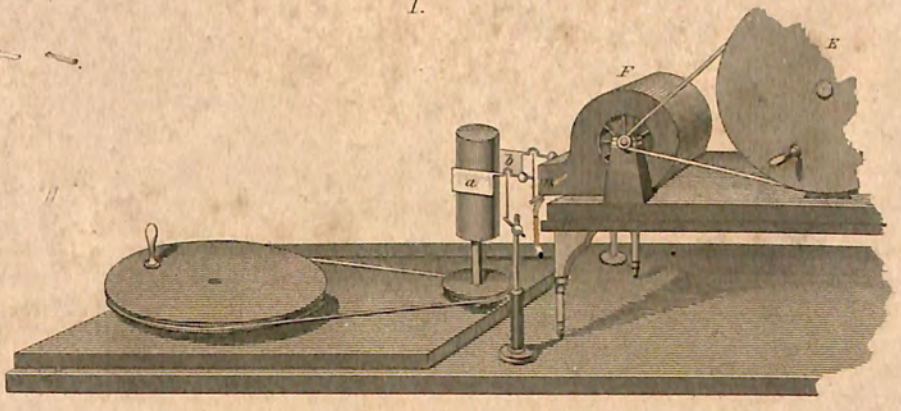
These salts, whose form has been described by M. Mitscherlich*, have their optical axes in a plane normal to the horizontal diagonal of the rhombic base, and appear to possess optical characters which are perfectly identical.

It is difficult to determine with accuracy the direction of the optical axes and the indices of refraction, on account of their tendency to effloresce; but, by cementing two crystals, one of phosphate and the other of arseniate, base to base in a parallel position, it is possible to ascertain that the refraction and the direction of the axes of optical elasticity are not appreciably different in either one or the other.

I likewise cemented together two crystals of arseniate and two

* *Annales de Chimie et de Physique*, 3rd sér. vol. xix. p. 419.

1.



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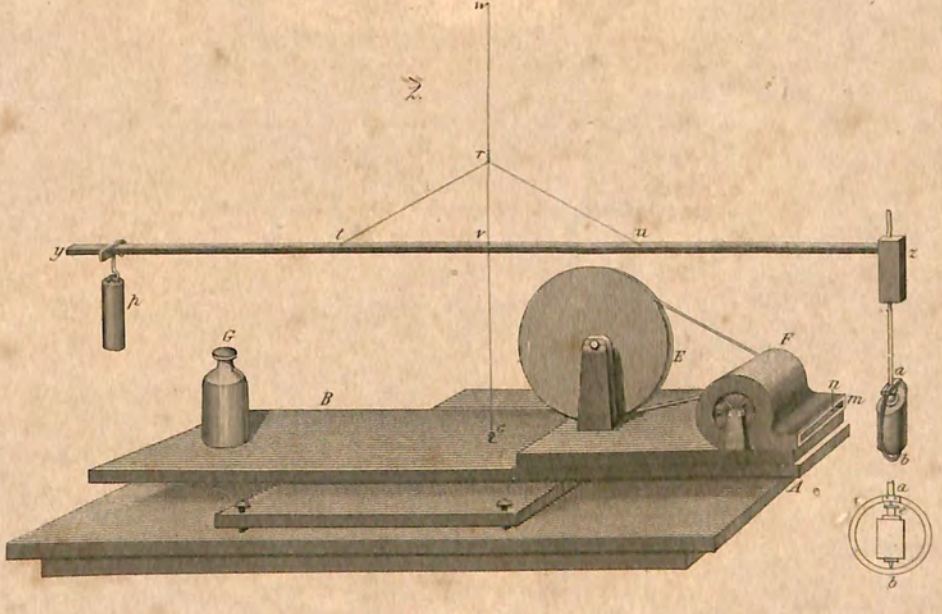

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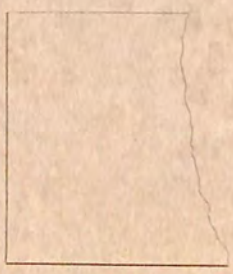

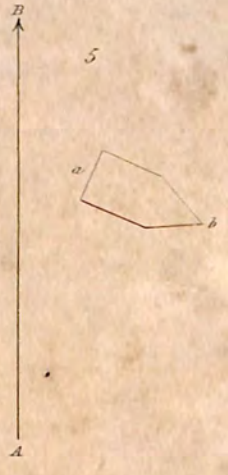

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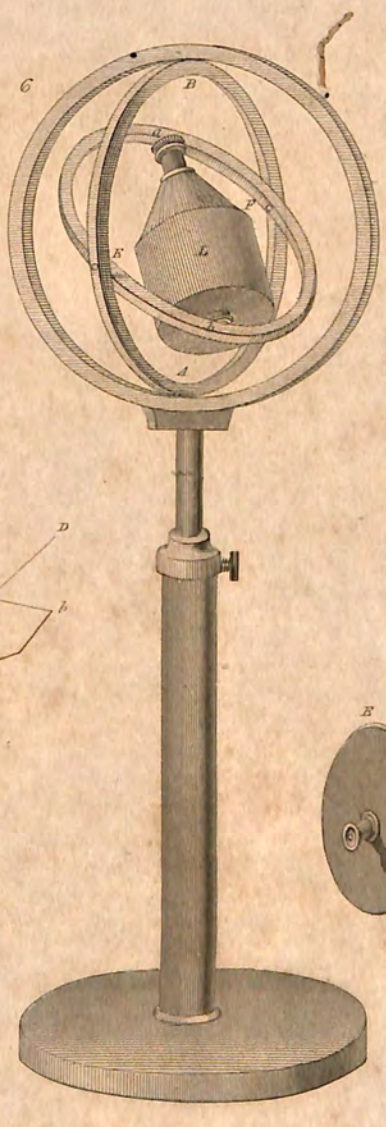

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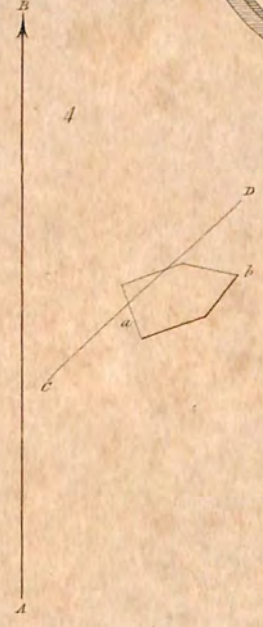

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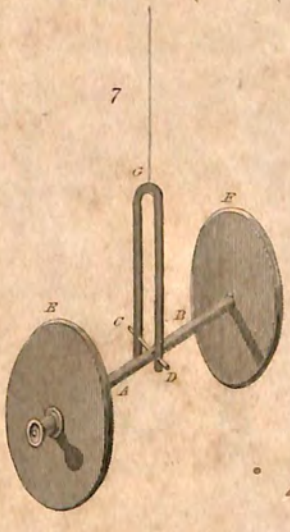

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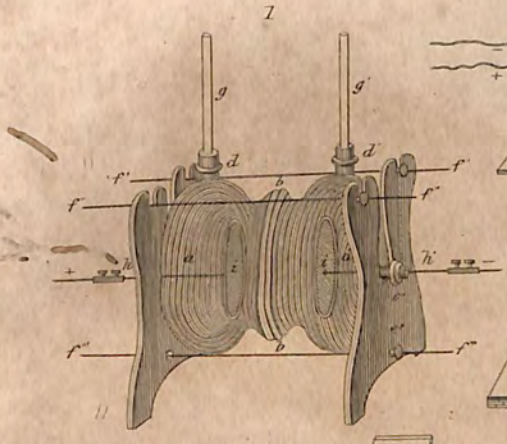
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K
P
M
7
1st opt. axis
Bisectrix
2nd opt. axis
8
E
G
H
O
9
1st inter.r axis
normal to the plate
1st exter.r axis
Bisector
2nd inter.r axis
10
N

of phosphate, base to base, in a hemitropic position, and thus found that the inclination of the bisecting line to the normal of this base P amounts to 64° 35′, to the normal of *d* 118° 7′; since the normals of P and *d* make with each other an angle of 53° 32′.

This result differs but little from that obtained by Mr. Miller; and further, from the extremely feeble double refraction, the experimental examination is not very delicate.

Double sulphates of potash or ammonia with sulphates of the magnesian series.

$$BO, SO^3, MgO, SO^3 + 6H^2O.$$

This numerous family of isomorphous salts is remarkable on account of the almost complete similitude of the crystalline forms; I took as the type of the class the sulphate of manganese and potash, which presents the greatest number of faces.

The figure (Pl. IV. fig. 8) represents an orthogonal projection of half of the faces upon a plane normal to the horizontal diagonal of the base.

The symbols of the faces are respectively—

P 001, H 100, G 010, M 110, *g*′ 120, *g* 130,
O $\bar{1}01$, K 101, T 201, E 011, *b* $\bar{1}11$, *a* 111,
c 311.

In the salts belonging to this family, the plane of the optical axes is normal to the horizontal diagonal of the base, and the axes of elasticity have very nearly the same direction, their bisecting line being always included within the obtuse angle of the normals to the faces H and P, and lying tolerably near to the former. I have determined its direction by the first or the second method, sometimes by both; but as the frequent description of the same operation would involve much repetition, I will once for all lay down a literal notation for the angles, and will limit myself to giving for each separate experiment the numerical value of these letters.

First Method.—The crystals being always cemented together by their bases P,—

α represents the obtuse angle included between the normals to the faces H and P;

θ the angle which the bisecting line makes with the second of these normals;

2ϕ the reciprocal inclination of the bisecting lines belonging to each crystal; $\theta = 90° + \phi$.

Second Method.—The artificial plate parallel to the horizontal diagonal of the rhombic base being always cut in such a manner that its normal, near to the bisecting line, falls within the obtuse angle α, if the figure (fig. 9) represents an ideal section of the crystal perpendicularly to the same diagonal of the base;

λ the angle included between the normal of the plate and the normal to the face P;

I, i the angles which the optical axes make respectively with the normal of the plate, the first directed in such a manner that it deviates from the normal of P, the second in such a manner as to approach it;

R, r the corresponding interior angles;

2ω the interior angle of the optical axes;

β the angle of their bisecting line with the normal to the plate;

m the index of refraction of a ray polarized and situated in the plane of the optical axes;

$$\sin R = \frac{1}{m}\sin I, \qquad \sin r = \frac{1}{m}\sin i, \qquad \omega = \frac{R+r}{2},$$

$$\beta = \omega - r = \frac{R-r}{2}, \qquad \theta = \lambda - \beta = \lambda - \frac{R-r}{2}.$$

Determination of the Indices m.

Double sulphates	Angles of the prisms.	Minimum deviations.	Indices.
Of potash and manganese	54° 10′	30° 42′	1·487
	50 29	28 11	1·487
Of potash and nickel	44 45	34 37	1·492
	42 49	23 1	1·489
Of potash and cobalt	38 32	19 16	1·469
	35 6	17 12	1·462
Of ammonia and magnesia.....	58 56	34 9	1·483
	39 17	20 31	1·476
Of ammonia and nickel	50 1	28 31	1·498
	46 9	25 51	1·500
Of ammonia and cobalt	54 35	31 37	1·491
	48 55	27 30	1·494
Of ammonia and zinc.....	60 19	36 39	1·491
	49 47	27 57	1·491

Determination of the Angle θ.

Double sulphates.	2φ.	λ.		i.	R.	r.	2ω.	β.	θ=90+φ.	θ=λ−β.
Of potash and manganese	18° 42′	98° 54′	37° 4′	42° 20′	24° 2′	27° 4′	51° 6′	−1° 31′	99° 21′	100° 25′
Of potash and nickel	15 4	97 50	46 17	39 7	29 1	25 3	54 2	+1 54	97 32	96 56
Of potash and cobalt	15 12	103 40	52 58	28 45	33 1	19 10	52 11	+6 55	97 36	96 45
Of potash and iron	...	...	...	...	...	...	...	...	(Miller.)	98 51
Of ammonia and magnesia	18 2	92 11	28 47	51 49	18 59	32 5	51 4	−6 33	99 1	98 41
Of ammonia and nickel ...	24 2	...	...	...	...	...	...	...	102 1	
Of ammonia and cobalt ...	24 0	...	...	...	...	...	...	...	102 0	
Of ammonia and zinc	20 16	...	...	...	...	...	...	...	100 8	

The double sulphate of potash and cobalt presents a remarkable dichroism. A plate normal to the bisecting line, observed with a Nichol's prism, appears of a wine-yellow colour (*couleur de vin paille*) or violet-red colour, according as the plane of polarization is parallel or normal to the plane of the optical axes. A plate parallel to that plane likewise appears violet-red or rose-coloured with a shade of orange, according as the plane of polarization is successively parallel or normal to the bisecting line.

The double sulphate of ammonia and cobalt is very slightly dichroic, the two salts of nickel still less so.

In all these salts the bisecting line is the axis of minimum optical elasticity.

Three kinds of facts result from the preceding experiments, and present in some sort a *résumé* of them.

In the first place, it is seen that in many groups of substances chemically and geometrically isomorphous, there is a similarity of optical characters altogether comparable with the similarity of their crystalline forms; isomorphism itself permits of slight deviations of almost the same order in the geometrical elements, and the agreement is such as should be expected to attend a specific difference of substance with an almost perfect identity of structure.

Such, for example, are the binarseniates and biphosphates of potash and ammonia; the double chlorides of copper and potassium, of copper and ammonium; the sulphates of zinc and magnesia, the chromate of magnesia; the sulphates of baryta, of strontia, oxide of lead; then the neutral phosphate and the arseniate of soda; and the numerous group of double salts formed

by the union of the sulphates of potash or ammonia with nearly all the oxides of the magnesian family.

The almost perfect identity of optical characters is especially remarkable in these latter salts, referable to a symmetrical oblique prism. For not only do the values of the three principal elasticities maintain the same proportion, but likewise the homologous axes of elasticity have almost the same direction in the crystal, and still this direction is not in this case a forced consequence of the geometrical symmetry.

However, exceptions are not wanting, for instance in the lævo- and dextro-tartrates. These salts possess an imperfect geometrical isomorphism, because they present an undisguised hemihedrism; they are no longer chemically isomorphous, because, although possessing the same elementary composition, they are capable of entering into combination in definite proportions, and of developing heat in the act of union, giving rise to a substance differing from both in its figure, composition and chemical characters; but still these non-isomorphous lævo- and dextro-tartrates present a perfectly absolute identity in their phænomena of double refraction.

Lastly, there is found a third class of compounds, geometrically isomorphous by reason of their perfect similarity of form, chemically isomorphous by reason of their elementary composition, their chemical characters and the capability of crystallizing together, but which nevertheless possess optical characters altogether opposite.

Thus we have seen, in the hyposulphates of lime, of strontia and of oxide of lead, the single axis of double refraction situated in the direction sometimes of the maximum and sometimes of the minimum optical elasticity; the axis of mean elasticity situated in the same manner in the chromate and sulphate of potash, while there is a reciprocal exchange in the directions of the axes of minimum and maximum elasticity; finally, in arragonite and carbonate of lead, the same exchange takes place between the axes of minimum and mean elasticity, whilst the axis of maximum elasticity retains a constant direction.

The double tartrates of soda and potash, of soda and ammonia, present the same differences, which are rendered still more striking by a very remarkable dispersion of the optical

axes corresponding to different colours. The latter salt, which may be called ammoniacal Seignette salt, presents all those remarkable characters which Sir John Herschel has pointed out as belonging to the true potash Seignette salt. The divergence of the optical axes, the dispersion of the axes for different colours, the contortion of the isochromatic curves are in both cases the same; but the optical axes extend in two diametric rectangular planes.

It is then sufficiently proved that the elasticity of the ætherial medium in certain isomorphous substances presents a perfect inversion of relative magnitudes in the same direction. But as this elasticity in crystals assumes, under the influence of the forces emanating from the molecular network, unequal values in dissimilar directions, the resultants acting in these directions must present the same inversion of magnitude without the forms suffering any essential alteration in their angles or in the arrangement of their faces.

If this species of mobility of the optical characters belonging to the same geometrical envelope requires any further demonstration, this would undoubtedly be found in the remarkable phænomena presented by crystalline alloys of substances endowed with opposite optical characters.

Crystals of Hyposulphates of Strontia and Oxide of Lead.

In the first instance I allowed hyposulphate of strontia, whose optical axis is that of maximum elasticity, to crystallize in various and progressively-increasing proportions with hyposulphate of lead, whose optical axis is that of minimum elasticity; or rather I combined hyposulphate of strontia, in which the ordinary ray is always more refracted than the extraordinary ray, with hyposulphate of lead, in which the contrary obtains.

The rings of colours which the mixed crystals present in convergent polarized light enlarge at first, on account of the diminution of birefractive power resulting from the combination of opposite optical characters; and they subsequently contract as soon as this power reappears with an opposite sign, so that it is not difficult to obtain, in the series of salts containing these two bases, crystals whose birefractive power possesses an almost equal absolute value, and in which the optical axis is sometimes

that of maximum and sometimes that of minimum elasticity, or in which the directions of the ordinary and extraordinary rays are mutually inverted.

But as the transition from one optical character to the opposite appears to be gradual and continuous, it must be possible to obtain mixtures in which the double refraction would be destroyed, or rather which would behave for one extremity of the spectrum, like hyposulphate of lead, by refracting the ordinary ray less than the extraordinary ray, while they would act for the other extremity of the spectrum like hyposulphate of strontia, refracting the ordinary ray more than the extraordinary ray, and which, lastly, would not present any double refraction for the intermediate part of the spectrum, the ordinary and extraordinary rays remaining united.

I have, in fact, after several attempts, obtained crystals which present under Amici's polarizing microscope a violet field with a black cross. The rings are so much expanded by the almost total obliteration of the birefractive power, that the microscope includes only the second obscure ring. But as the field of the first bright ring presents a uniform violet colour, instead of a white of the first order, even when white light is employed, it follows that all the elements of this light except the violet must be extinguished in the analyser. They have therefore retained their original polarization, and consequently have not experienced double refraction.

Other crystals presented, instead of the violet, a blue colour; as the proportion of strontia and lead salts varied, another part of the spectrum escaped double refraction.

It would be interesting to ascertain whether, in this transformation of optical characters, the dispersion of the colours does not suffer in the ordinary and extraordinary rays some inversion of direction or magnitude; but as this inquiry would necessitate the use of large and pure crystals, I am compelled to postpone it until a later period.

Crystals of Potash and Ammonia Seignette Salt.

Sulphate and chromate of potash combine together, forming crystals which are very well developed, although too small to admit of their optical characters being readily studied. This may however be easily done with crystals consisting of ammo-

niacal Seignette salt, whose optical axes are open and dispersed in a certain diametric plane, and gradually increasing quantities of potash Seignette salt*, whose optical axes are open and dispersed in the diametric rectangular plane. The biaxial double refraction and the considerable dispersion of the optical axes for different colours in this case give to the phænomena a character of marked peculiarity.

The angular distance of the optical axes in the potash salt, amounting originally to 76°, decreases at the same time as the red axes, which are at first wider apart than the violet, approximate to each other, because they approach more rapidly, so that they ultimately unite. At this moment the dispersion of the optical axes for different colours ceases, and with it the remarkable anomalies in form and coloration which Sir John Herschel has pointed out in the isochromatic curves.

The violet and red optical axes then separate again; but the latter, from always moving more rapidly than the others, are less wide apart. The dispersion is the reverse of that originally presented, although still sufficiently strong to cause a considerable contortion in the rings, which have then the red inwards and

* The complex crystals are generally smaller than the simple ones, and it is necessary to take care that the solution is kept strongly ammoniacal in order to prevent the precipitation of bitartrate of potash.

One of the most perfect of the mixed crystals, in which the two constituents must have been nearly in atomic proportions, possessed the form represented in fig. 10 Plate IV; and the normals of the planes formed the following angles:—

$$90° = \frac{P}{h} = \frac{P}{N} = \frac{P}{M} = \frac{P}{g} = \text{\&c.} \quad \frac{h}{g} = 90°,$$

$$\frac{g}{M} = 50° 19', \quad \frac{g}{N} = 67° 28',$$

$$\frac{P}{e^{I}} = 12° 20', \quad \frac{P}{e^{II}} = 23° 37', \quad \frac{P}{e^{III}} = 41° 10',$$

$$\frac{P}{m^{I}} = 12° 44', \quad \frac{P}{m^{II}} = 34° 10', \quad \frac{P}{a} = 14° 46'.$$

Two other vertical prisms are likewise found where the normals of the planes μ and ν are respectively inclined to the normal of g by angles of 16° 50′ and 31° 10′; m', n, and h are situated in the same zone, as are likewise m', e^{II} and m'_1.

The symbolic notations for all these faces are—

$h(100)$, $g(010)$, $P(001)$, $\mu(410)$, $\nu(210)$, $M(110)$, $N(120)$,
$e^{I}(102)$, $e^{II}(101)$, $e^{III}(201)$, $m'(111)$, $m''(331)$, $a(012)$, $n(121)$.

$$A : B : H = 1{\cdot}205 : 1 : 0{\cdot}527.$$

the violet outwards. There is one position in which the red axes again unite. The crystal has then only one axis for the red rays, and for other colours two axes, which diverge more and more up to violet; these latter are moreover at least 12° apart in original planes.

From this moment the red optical axes separate in their turn, but in the diagonal plane perpendicular to the first, while the violet axes continue to approach. The complex crystal then contains ammonia salt by the position of the red optical axes, potash salt by the position of the violet optical axes, and has only one axis for the intermediate colours. Lastly, inasmuch as the optical axes continue to suffer the same alteration of position in proportion as the influence of the ammoniacal salt increases, the violet axes pass after the red axes into the plane normal to that in which they are at first situated, and ultimately form in this plane an angle of 46° or 48°, while the red axes form an angle of 60°. The optical character of the potash-salt gradually disappears, and is superseded by that of the ammoniacal salt.

The gradual transformations which the dispersion of the optical axes for different colours suffers in the mixed salts do not take place without giving rise to very curious distributions of colours in the isochromatic curves, and such contortions of them that it is impossible to recognise their ordinary outlines. The apparent confusion nearly reaches its maximum when the red and violet axes situated in two rectangular planes respectively form with each other an angle of from 6° to 7°. However, the ordinary uniformity is restored in the appearance of the rings when homogeneous light is employed, such for example as that which is transmitted through red or dark green glass, or a concentrated solution of carbonate of copper and ammonia.

The contortion of the isochromatic curves is moreover accompanied by certain remarkable alterations of colour in the central region of the rings, when the analyser is rotated.

For example, when the analyser is turned from left to right, the colours successively seen in the field are red, yellow, green, blue and violet, following each other almost in the same manner as the colours of the spectrum. These phænomena present a considerable resemblance to those which result from circular polarization; it is nevertheless obvious that they are, at least to

a great extent, a consequence of the dispersion of the optical axes for different colours in different planes.

In fact, when a thread of white light, which has traversed the crystal either as an ordinary or extraordinary ray, and under a certain inclination, is observed, each of the elementary colours of which it consists is found to be polarized at its emergence in a different plane.

According to the rule laid down by M. Biot, and confirmed by the theory of Fresnel, this plane of polarization divides the angle contained between the planes which pass through the ray itself and through each of the two optical axes. But in the present case these latter have always a different direction for different colours; consequently when the analyser is rotated, it arrests or permits, according to the adjustment, the passage of different colours from the light emerging from each point; and these phænomena necessarily call to mind the optical properties of quartz, because all the elementary colours of a white ray previously polarized in a similar plane are each polarized in a different plane at their emergence from that substance.

Nevertheless, if the chromatic phænomena just mentioned are in the case of the tartrates solely the consequence of the dispersion of the optical axes, they ought to disappear entirely when the plate is illuminated normally by a bundle of perfectly parallel rays polarized in any one of the rectangular diametric planes in which the optical axes are situated. The incident ought then to pass completely, whether as, ordinary or extraordinary ray, and to pass in a direction parallel to the common bisector of all these axes, and its plane of polarization could not be modified by the crystalline plate. This, however, is never the case completely. It therefore becomes a question, whether the development of the variable colours, which present themselves even when the analyser is rotated, may not be simply a consequence of irregularities in the crystalline structure, rendered sensible by the extreme diminution of the birefractive power in the direction of the bisector, and analogous to those phænomena which have long been known to occur in crystals with one axis, as for instance beryl; and, on the other hand, whether it may not be owing to the reappearance, likewise by reason of the diminution in the double refraction, of the molecular rotatory power proper

to these tartrates. These are points which cannot be determined except by very careful and delicate experiments with crystals of large size and perfect purity. I must therefore postpone their consideration altogether until a future occasion.

The experiments described in this memoir tend to the conclusion, that the mechanical causes which determine the geometrical form are of another order than those which determine the optical characters, because the form remains the same in whole groups of isomorphous substances, while the optical characters experience in their essential elements not only modifications in quantity, but a complete inversion of relative magnitude. One and the same cause could not manifest itself at the same time by similar geometrical effects and opposite optical effects.

The very remarkable relations discovered by Sir David Brewster and the physicists who have followed him, between the optical characters of crystals and geometrical types, may be in reality nothing more than a result of symmetry. It is true, we do not possess any definite ideas as to the reciprocal relations which may exist between luminous agency and ponderable matter; but it is sufficiently evident, that in every molecular network, constituted with the conditions of a regular arrangement in reference to certain directions, the general resultants of the partial forces emanating from each molecule act, whatever may be their nature and cause, in the same directions, and thus become axes of symmetry for all the phænomena dependent upon these forces. Such a character of coordination, expressed moreover in very different characters, in the geometrical form, the optical, thermic and other characters, does not therefore by any means prove the identity of the forces themselves, but merely an identity in the direction of their resultants. To prove the identity of the forces, something more is necessary; their actions in these compulsory directions ought at least to preserve the same order of relative magnitude among each other; but in the case of light the precise contrary obtains; and differences of a similar kind are met with among the phænomena of heat, electricity, magnetism, and in short all the varieties of physical agency, when crystals are submitted to the test of experiments sufficiently varied and delicate.

The manner in which complex media, formed by the crystal-

line association of two isomorphous substances possessing opposite optical characters, behave towards light, indicates the existence of a uniform and continuous mode of aggregation, a homogeneity of structure as perfect as that of their elementary constituents. It appears therefore in the highest degree probable, that this union—in which each of the constituents, by a gradual and mutual concession, merges a part of its individual characters, so as to form a whole capable of acting in its turn in the same manner as a simple system—takes place in the crystalline molecule between elementary particles less gross than those of which the sensible crystal is formed. This line of induction would therefore, as in many other instances, lead us to regard crystals as complex structures consisting of molecular systems of different orders. It would therefore be necessary to ascertain what members of this ascending series produce the determining conditions of the isomorphism and of each physical character; but all speculation on a subject with regard to which the most natural induction has always been a false guide, is at least premature. Consequently I shall more directly arrive at some positive inferences from these experiments, by showing that they furnish a clue to certain hitherto inexplicable optical properties of minerals.

The crystalline associations of several isomorphous compounds of dissimilar or even opposite optical characters, must have their representatives in the mineral kingdom; in fact, few compound minerals present in different specimens optical characters which are absolutely invariable; and there are some instances in which they habitually present considerable discrepancies in this particular.

I do not here speak of irregularities which are obviously consequences of defective homogeneity in the crystalline medium, and manifest themselves as partial alterations of colour, striæ and interior reflexion; without taking into account these accidental phænomena, it is certain that an extreme variability of optical characters appears to be in some sort essential to many natural families of minerals, the sensible characters of which indicate a homogeneity of structure, and which are shown to possess a crystalline continuity by the far more delicate test of polarized light. Such, for instance, are the topazes, in which the angle

of the optical axes varies from 49° (in the yellow Brazilian topaz) to 65° (in the blue crystals from Aberdeenshire), and the micas, in which the deviation commences at 0° and rises as high as 45°, and even more*.

All these anomalies present themselves likewise in the mixed tartrates, for the angle formed by the optical axes may amount to as much as 75°, pass through 0°, and even open to the extent of 60° in a plane normal to the original plane. I have indeed artificially prepared crystals, which are in every respect analogous to the micas, which are shown by their optical characters and chemical relations to result from the crystalline association of several isomorphous compounds. These experiments are in reality nothing more than synthetic demonstrations of the causes which are capable of giving rise to the formation of minerals chemically and optically different; and they justify us in entertaining the opinion, that the family of micas will, when better studied, be found to comprise varieties in which the plane of the optical axes is situated in two diametric rectangular planes†.

Chemical analysis has not as yet thrown so much light upon the composition of the varieties of topaz as it has upon that of

* The form of the micas can never be determined with accuracy, and the results of their chemical analysis scarcely admit of rational interpretation. It is therefore possible to doubt the existence of two species in right and oblique rhombic prisms.

These prisms would both have an angle of 120°, and would both be cleaveable in a direction parallel to their respective base; but in the oblique form there is no crystallographic symmetry which renders it necessary that the optical axes for different colours should have but one bisector, and that this should be normal to the base. On the contrary, such a singular arrangement would be purely fortuitous, and even exceptional, while in the rectangular form it is necessary. But it does not appear that in this respect any essential difference has been observed in the several varieties of mica; and this circumstance is alone sufficient to throw considerable doubt upon the division adopted by many mineralogists.

† M. Biot has always found the plane of the optical axes in the micas to be parallel to the minor diagonal of the rhombic base (*Mém. de l'Institut*, 1816, p. 275); and at the time when this memoir was written, I was not acquainted with any other complete series of optical experiments on these minerals. Since then I have met with a very extended optical investigation by Mr. Silliman, Jun., in which he states that in some varieties, to which Mr. Dana has applied the name of Muscovite, the plane of the optical axes is parallel to the major diagonal. In muscovite the optical axes generally present a very large angle (American Journal of Science, 2nd series, vol. v.).

micas. Physical investigation is with regard to them in advance of chemical. However, there is no doubt that careful examination would lead to the recognition of differences in composition, which are indeed indicated by the differences of colour. It would be very interesting to find modes of investigation so dissimilar mutually confirming each other; and in the same point of view, it would be of great importance to seek in the plumbiferous aragonites for the convergence, and perhaps the dispersion of the optical axes, which the above experiments would appear to point out and lead us to expect.

Article X.

X. *On the Phænomena of closed Electro-Magnets.* *By* J. C. Poggendorff.

[From Poggendorff's *Annalen*, vol. lxxxv. p. 147.]

By closed electro-magnets, the author understands those of the ring form, or of the horse-shoe form with the armature laid on, thus establishing a continuity more or less perfect. Magnets of the first class have been hitherto but little examined; those of the latter, on the contrary, have been often investigated, and more particularly with reference to the amount of the so-called lifting power, as compared with the strength of the galvanic current by which the magnetism is excited, by Fechner, Lenz and Jacobi, Joule and Œrsted. The results of these physicists are however in part divergent and in part contradictory. To the author, therefore, it did not seem superfluous to take up the subject once more, especially as other interesting and hitherto unexhausted questions presented themselves in connexion with it.

The experiments which he has carried out with reference to this object, corroborate, in the first place, a result already suggested by the experiments of Œrsted and some of those of Lenz and Jacobi; namely, *that the lifting power increases more slowly than the strength of the current*, but with the addition, *that this increase with augmented current advances more and more slowly, so that the lifting power approximates asymptotic to a constant value*, a value, the absolute magnitude of which depends, of course, upon the nature of the magnet and its keeper.

In proof of this we will here introduce the results of a series of experiments in which the magnet and keeper were in immediate contact, the strength of the current being measured by the sine-compass.

Relative strength of current.	Relative lifting power.
1	1
1·846	1·757
2·375	2·000

Relative strength of current.	Relative lifting power.
3·181	2·392
4·117	2·595
5·070	2·608
6·025	2·797
7·378	2·851
9·648	2·960
10·350	3·149

If the quotients obtained by dividing the differences of the currents by the corresponding differences of the lifting powers be compared, the above proposition will follow.

Considering the great difficulty presented to the accurate determination of the lifting power, the author does not intend to claim a high degree of exactitude for the above numbers; neither does he believe that the numbers express a general law; for not only is the progression of the lifting power dependent on the character of the magnet and its keeper, but also on the absolute strength of the current, or rather the magnitude of the galvanic moment*. But the convergence of the progression he regards as placed beyond doubt by the above and other results which he has obtained; indeed, if such were not the case, it would be possible to obtain infinitely great lifting powers, which is not only very improbable, but also directly opposed to the results established by J. Müller, as also to the necessary limitation of magnetization which follows from the theory of Ampère †.

The great divergence of the lifting powers from the laws of attraction at a distance, established by Lenz and Jacobi, is strikingly exhibited. If the excitation of magnet and keeper

* Under galvanic moment, the author understands the product of the strength into the length of the current, multiplied by a coefficient which depends upon the form and distance, which, however, like the length of the magnetizing current, remained constant throughout these experiments.

† [This remark appears to us to put the disputed question of magnetic saturation in a new and simple light. The author, if we understand him aright, considers the act of magnetizing to consist in the setting of the molecular currents of Ampère parallel to a common plane. With weak currents, this parallelism is but partially established, and becomes more complete as the strength of the current is augmented. *When complete*, further magnetization is impossible, or, in other words, the point of saturation has been attained. See Weber's investigation, Scien. Mem. Feb. 1853, p. 197. J. T.]

had been proportional to the strength of the current, and the mutual attraction of both proportional to the square of the magnetic force, then by an augmentation of the current from 1 to 10 the lifting power should increase in the ratio of 1 : 100, whereas in reality the increase is only in the ratio of 1 : 3! This extraordinary divergence from the simplest assumption, which certainly in no former experiments exhibited itself so strikingly, is evidently to be referred to the reaction of the keeper upon its magnet, by which the latter is enabled to assume a higher degree of magnetism than could be imparted to it by the action of the galvanic current alone.

A glance at the progression of the lifting powers shows that the addition to the magnetic power is greater in the case of the weaker than of the stronger currents, or that the increase diminishes as the force already possessed by the magnet becomes greater; but we do not learn therefrom in what ratio the total force of the magnet increases with the strength of the current, and just as little, according to what law the magnet and keeper attract each other.

To obtain information on this head, the author resorted to the method of measurement furnished by the induced currents excited by an electro-magnet, which currents may be regarded as proportional to the total amount of the developed magnetism. He surrounded a horse-shoe magnet with two spirals, each of which consisted of two well-insulated wires, and connected these in such a manner that two circuits were formed, in one of which the battery and the sine-compass were included, and in the other the galvanometer used to measure the induced current. To meet all the questions which suggest themselves here, the induced currents were measured in *four* cases, namely, *a* with an open magnet, *b* with a closed magnet and the first closing of the primary circuit, *c* with a closed magnet and a second closing of the circuit, and *d* through the pulling away of the keeper; which latter, in one series of experiments, was wide, and in the other narrow, not being quite sufficient to cover the magnet.

The following table will serve as an example of the results obtained in this manner; the strength of the induced currents is denoted by the letters *a*, *b*, *c*, *d*, the said strength being expressed by the sine of half the angle through which the needle

of the galvanometer is impelled, minus the weak current which the primary spiral excited in the secondary. The current d was, of course, opposite in direction to those which were caused by the closing of the circuit.

Wide keeper.	Narrow keeper.
$i = \sin 4^\circ 25' = 1{\cdot}000.$	
$a = +\ 4{\cdot}36$	$a = +\ 4{\cdot}80$
$b = +32{\cdot}10$	$b = +20{\cdot}77$
$c = +10{\cdot}45$	$c = +\ 9{\cdot}58$
$d = -25{\cdot}01$	$d = -14{\cdot}77$
$i = \sin 5^\circ 31' = 1{\cdot}249.$	
$a' = +10{\cdot}23$	$a' = +10{\cdot}23$
$b' = +49{\cdot}66$	$b' = +28{\cdot}98$
$c' = +17{\cdot}99$	$c' = +17{\cdot}13$
$d' = -34{\cdot}75$	$d' = -17{\cdot}13$
$i = \sin 10^\circ 21' = 2{\cdot}335.$	
$a'' = +16{\cdot}06$	$a'' = +16{\cdot}06$
$b'' = +58{\cdot}86$	$b'' = +37{\cdot}77$
$c'' = +24{\cdot}15$	$c'' = +22{\cdot}04$
$d'' = -39{\cdot}37$	$d'' = -16{\cdot}92$

In consequence of imperfections in the induction-galvanometer, these measurements do not possess very great exactness, but they are sufficient to enable us to form a judgement of the general facts of the case.

Thus we observe that the current b, which is the most important here, is stronger than all the others, especially when the galvanic current is weak, and that its increase with the augmentation of the galvanic current is comparatively slow. It evidently measures the total magnetism assumed by the magnet in consequence of the united action of the keeper and the galvanic current, and they are therefore stronger than the current c, as this is due to the difference between the total and the residual magnetism. The currents d are due essentially to the residual magnetism, and if the permanent magnetism remained unchanged by the separation of the keeper, we must have $b = c + d$.

If it be correct that the currents b measure the total mag-

netism of closed electro-magnets, it is clear that they will stand in a definite relation to the lifting power, and the question now is, in what relation?

Before making the experiments above-mentioned, the author believed that the lifting power, like the *actio in distans*, would be proportional to the square of the strength of the current *b*; a glance however at the numbers obtained, renders this conjecture in the highest degree improbable.

To obtain further information upon this point, new experiments were made, in which, with the same horse-shoe magnet, and with currents of different strength, were measured both the total and the permanent force, as also the current of induction *b*.

Different circumstances prevented these measurements from being made with all the exactitude that might be wished, but they show, nevertheless, in the most decided manner, that the *temporary lifting power*, that is, the total minus the permanent, *is far from being proportional to the square of the inducted current* b, *but on the contrary is nearly proportional to the current itself, or in other words, to the simple force of the magnet.*

Even considering the incompleteness of his measurements, and the great difficulty, notwithstanding all precautions, which the determination of the lifting power will always present, the author would not be disinclined to regard even now the simple proportionality of the lifting power, to the magnetic intensity, as the true expression of the state of the case, did he not consider it more prudent to defer his definite opinion, until by a repetition of the experiments with more perfect instruments, the certainty of the numerical results is more surely established than at present.

The author takes this opportunity of observing, that should future observations confirm his proposition, still the latter would not be quite the same as the results of J. Tyndall*, "that the mutual attraction of an electro-magnet and a sphere of soft iron,

* I take this opportunity of introducing an ingenious and probably well-grounded objection urged by M. Dub against the universality of this result. Where magnet and submagnet are separated by a short interval, the attraction is proportional to the *square* of the strength of the magnet. M. Dub argues thus:—"If the increase in the case of contact be simply proportional to the magnetic strength, and at a distance to the square of the strength, we

in the case where both are in contact, is proportional to the strength of the magnet*." For by Tyndall the strength of the magnet is regarded as proportional to the strength of the galvanic current, and certainly not without reason, as the reaction of a small sphere of iron, which this skilful investigator always makes use of, must be very inconsiderable. The magnetic strength, however, of which the author speaks, is that which is in part due to the reaction of the submagnet, and which is not proportional to the strength of the current, and which depends in a great measure upon the nature and dimensions of the submagnet. But in both cases it would be the *true* magnetic force, which, in apparent opposition to the principle of mechanics, is simply and directly proportional to the attraction.

The author in the same paper treats of the *residual* lifting power of closed electro-magnets, and gives measurements, from which it appears that this is not constant, but increases with the strength of the current, although in a less ratio than the total power. To ascertain how far this residual power depends upon the coercive force of the metal, he determined it with magnets of unhardened steel and with those of soft iron of the same dimensions, both being closed by the same submagnet constructed of the material last-mentioned. The following table will give a view of the results:—

Strength of current.	Lifting power expressed in ounces.					
	Magnet of unhardened steel.			Magnet of soft iron.		
i.	Total.	Residual.	Permanent.	Total.	Residual.	Permanent.
sin 5° 0′ = 1	60½	37½	35½	100½	57½	8
„ 10 2½ = 2	86½	66	35¾	166	74¾	8
„ 15 9½ = 3	129	83	39½	213	93¾	8
„ 21 37½ = 4	162	97⅞	46½	268	100½	8
„ 25 50 = 5	191½	106½	44½	297	100½	8
„ 31 32 = 6	233	127½	45½	326	102¼	8
„ 37 36 = 7	281	125	48½	333	105½	8
„ 44 12 = 8	277	133	48	353	110½	8
„ 51 40 = 9	301	133	48	357	107½	8

should, by increasing the current, soon find the attraction at a distance to be greater than in the case of contact." I record the objection as it appears to be a valid one, reserving its closer examination for another opportunity.—J. T.

* Pogg. *Ann.* vol. lxxxiii. p. 1. Phil. Mag. [iv.] vol. i. p. 265.

Although these results are not free from anomalies, still they plainly show us that the residual lifting power immediately observed is greater in the case of unhardened steel than in that of soft iron, but that the case is reversed where, as seems necessary, as it is included in the observation, the permanent lifting power is subtracted from the residual. This subtraction being made, we see that the residual magnetism in the case of soft iron, notwithstanding its small coercive force, is greater than in the case of unhardened steel. The author would regard this experiment as decisive, with regard to the origin of the residual lifting power, were it not that the objection might be started, that the result observed with steel was a mixed one, produced by the residual lifting power of the steel electro-magnet and the iron submagnet.

He would also observe, as may be seen from the foregoing table, that soft steel is in a high degree capable of assuming temporary magnetism, approaching in this respect very near to soft iron; and that if it is still behind the latter, this is less due to the difference of the direct magnetizability of the particles, than to the difference of what Faraday calls magnetic conductibility, or the propagation of the polarity from particle to particle.

A bar of unhardened steel and a bar of soft iron, both wholly surrounded by equal spirals, exhibit with the same current but slightly different lifting powers; but the difference at once exhibits itself to the disadvantage of the steel, where the ends of the bars protrude even an inch beyond the spirals.

To residual magnetism is to be referred the fact, that a horseshoe magnet closed by its keeper is far from producing the same inductive effect on a second magnetization as it produced with the first, as is sufficiently shown by the above values of the currents *b* and *c*. Each following magnetization in the same direction produces only the weaker current *c*.

If, however, the direction of magnetization be reversed by reversing the galvanic current, the induced current is obtained, not with the primitive, but with nearly two-fold intensity. A second magnetization in this new direction produces again the

weaker current. If, however, the reversion of the magnetism be continued, we always obtain these strengthened currents, which considerably surpass those obtained from the open magnet.

It does not appear that any application of this has been made in the construction of electro-magnetic apparatus. All apparatus of this description seen by the author possess either a straight electro-magnet, or, as that of Neef, a horse-shoe one, which is opened and closed by a moveable keeper, but does not remain permanently closed. From the reversion of the current and the application at the same time of a closed electro-magnet, the most powerful inductive effects are however to be obtained.

The easy reversion of the residual magnetism by a current far feebler than that which produced it, is a proof of its great mobility, and does not appear to corroborate the view of Poisson. In this respect it presents a remarkable contrast with the permanent magnetism, even in soft iron.

If an open horse-shoe magnet of soft iron, which possesses a certain degree of permanent polarity, be magnetized by a galvanic current in the opposite direction, it can be so arranged, that the iron, while the current continues, may possess the reversed polarity, but on interrupting the current will spring back to its former condition. The strength of the current is the only thing of importance here; if this be too great the permanent polarity will also be reversed.

A similar result was obtained some time ago by the author with one of Logeman's magnets. A current being sent round the limits of this magnet, which consist of very hard steel, its polarity during the continuance of the current was reversed; but where the latter ceased, the original polarity, although much weakened, again appeared.

These observations show, that in a magnet two opposite polarizations, or two reversed series of Ampère's currents, may subsist simultaneously.

The foregoing statements with regard to a closed horse-shoe magnet are also applicable to the case when the continuity is perfect, or to the case of an iron ring. This might indeed be surmise, but is nevertheless remarkable.

Such a ring, uniformly magnetized all round, represents a magnet without a pole, which produces no external effect; in the act of magnetizing, nevertheless, it shows the same phænomena of induction as a horse-shoe magnet and its keeper. The author convinced himself of this by coiling two insulated wires round the same ring of iron, one of which was connected with the galvanic battery, and the other with the induction-galvanometer.

Similar rings or tubes of small dimensions might perhaps be made use of for very interesting experiments on frictional electricity.

[J. T.]

Article XI.

On the Nature of those Constituents of the Atmosphere by which the Reflexion of the Light within it is effected. By R. Clausius.

[From Poggendorff's *Annalen*, vol. lxxvi. p. 161.]

IT is a known property of our atmosphere that it is not absolutely transparent, but that it enfeebles the light considerably, and that hence the sun must appear less bright at the earth's surface than to an observer placed beyond the limits of the atmosphere. This diminution is due for the most part to a reflexion which takes place everywhere throughout the atmosphere, which may be inferred from the quantity of reflected light sent from it to the surface of the earth. That the light of the firmament is actually *reflected* light, is pretty manifest of itself, and is rendered certain by the phænomena of polarization which it has lately been discovered to exhibit.

The question as to the real origin of this reflexion within the atmosphere now presents itself to us, and its solution would be of two-fold interest. In the first place, the theoretical investigation regarding the strength and nature of the reflexion must be based upon the knowledge of its cause; secondly, this knowledge would be of service in many other considerations regarding the atmosphere, and serve more particularly to establish a connexion between its optical and meteorological phænomena.

With regard to this subject, which is intimately connected with the blue colour of the firmament and the morning and evening red, various conjectures have been expressed by many investigators, but none of these have been sufficiently established to win general recognition, and the question is still the subject of very divergent and indefinite notions. Probable, however, as it may appear from this that a decision of the question in the present state of science is impossible, and that we must await the result of new observations, I still believe that conclusions may be drawn, even now, from known premises which lead to

a quite definite assumption, and indeed prove the same to be almost the only one possible. These conclusions constitute the object of the following considerations.

In the first place, it is clear that the different strata of air cannot, by reason of their different densities, produce this reflexion. As the limiting surfaces of these strata, in so far as it is permitted to conceive of them separately, are nearly parallel to the earth's surface and to each other, these surfaces could only reflect the sun-light in particular directions, which indeed would never strike the earth's surface at all, so that the firmament must appear dark throughout. From the actual circumstance that at every point of the earth's surface light is received from all directions, we must rather conclude that reflecting surfaces of all possible directions exist in the atmosphere. Hence through the atmosphere we must imagine innnumerable masses to be distributed, the densities of which differ from that of the surrounding matter, and at the limiting surfaces of which the reflexion takes place.

Leaving the question whether these masses are foreign ones which float in the atmosphere, or the constituents of the atmosphere itself, for the present undecided, one question must be stated here and answered; and that is, whether these masses are transparent or opake. With regard to the latter, it might perhaps be affirmed as possible, that small non-transparent bodies which strongly reflect the light at their surfaces may float as dust in the atmosphere, and although imperceptible singly, on account of their minuteness, may still by the number modify the transparency of the atmosphere and effect a sensible reflexion. Against such an assumption, however, so much might be urged, and it is even at first sight so improbable and inexplicable how such masses, which in optical respects at least behave like polished metal, could be so uniformly distributed throughout all regions of the atmosphere, that the supposition may be permitted to fall to the ground. It may perhaps be asserted more definitely, that of *foreign* bodies, only water as the precipitate of vapour can exist in sufficient quantity in the atmosphere to produce so considerable a reflexion.

Assuming, then, the masses which cause the reflexion of light in the atmosphere to be *transparent*, an important starting-

point is hereby gained. Each ray which encounters such a mass will be partially reflected at the first and second surfaces and partly permitted to pass through, and the latter will at its entrance and exit be refracted if it does not by accident strike the surfaces perpendicularly. In general we must assume that the rays which have passed through such a mass do not retain their original direction, but are more or less deflected from it, and thus a dispersion of the total light passing through the atmosphere must take place, which could not escape observation. Now abstracting from certain particular phænomena, such as that of astronomical refraction, the cause of which is sufficiently explained, it is known that the light is propagated in straight lines, as by this alone are we enabled to see objects in their correct position. We have now to examine whether and under what circumstances the refraction which necessarily accompanies the reflexion is to be brought into harmony with this known fact.

Let us here distinguish the assumption before hinted at more clearly, so that we may prove that by it the above harmony is completely established, and hence that it is *admissible*. The further investigation must then be conducted with greater generality; and if it happen that all other assumptions are impossible, that is, not to be reconciled with the propagation of light in straight lines, the *necessity* of the former will follow as a matter of course.

Let us suppose the shape of the masses such that for every ray which strikes them, the forward surface at the point of incidence and the hinder surface at the point of exit are parallel, then each ray refracted by the forward surface will be refracted back again on leaving the mass to its original direction. While then the reflexions sum themselves together, the refractions mutually annul each other, and the transmitted light pursues its primitive course unaltered. That this condition shall be fulfilled, we must assume that all the reflecting bodies possess the form of the thinnest possible plates, whose limiting surfaces are parallel. At first sight such an assumption may appear somewhat forced, but by a closer consideration of the circumstances, we find that such bodies present themselves quite naturally in the atmosphere. By the precipitation of vapour minute bladders of

water are formed which remain floating in the air, and hence the assumption is in no way forced, that even in serene weather some extremely fine bladders are present. Such a vesicle consists of a film of water which is bounded by two spherical surfaces, and fulfils the above condition the more completely the finer it is. Thus by fine vesicles of this kind the direct propagation of light will be almost totally undisturbed, while the reflexion, inasmuch as each ray has to cross four limiting surfaces in passing through each bladder, will be considerable. For the reason last mentioned, a comparatively small number of such vesicles would be sufficient to produce the total reflexion which occurs in the atmosphere. The assumption alluded to is the following:—*that the reflexion in the atmosphere is produced by vesicles of water*, and it has in the first place proved itself satisfactorily by the explanation which it furnishes of the phænomena above mentioned.

We now turn to the more general consideration of masses whose limiting surfaces are not parallel but inclined in any way to each other, in order to determine the dispersion produced by them and to compare the results with what actually exists.

In the case of such masses, we may either assume that all possible inclinations are present without exception, or that definite inclinations obeying any laws whatever are predominant. The latter case would naturally have as a consequence peculiar optical atmospheric phænomena, as for example, those of the halo and mock suns, which may be explained on the assumption of crystals of ice, whose angles of inclination, in the majority of cases, amount to 60°. It would, however, be superfluous to enter more fully upon such particular cases; the impossibility will be evident when the other assumption, that all possible angles of inclination are present, has been completely examined. We shall therefore reserve these, and to exclude every limitation, we will proceed at once from the assumption, *that the shape of the reflecting masses, and hence also the position of their limiting surfaces, are altogether arbitrary and subjected to pure chance.*

With respect to the effect of a great number of bodies arbitrarily formed, we can reduce them to the action of an equal

number of spheres of corresponding size, by the calculus of probabilities. Fixing our attention, in the first place, upon the forward surface where the rays enter, in the case of arbitrary bodies we must, according to the laws of probability, assume that surfaces of all possible directions occur in the same manner. This is the case with each sphere in particular; for it is the essential characteristic of the sphere that its curvature is everywhere the same, and thus the elements of the surface which correspond to the various directions have all the same magnitude. In regard to the hinder surface a difference however exists. In the case of arbitrary bodies, the direction of one surface is independent of that of the other, and purely accidental. But in the sphere, the hinder surface, where it is met by the ray, has the same inclination towards the latter as the forward surface, inasmuch as every cord of the sphere meets the spherical surface under two equal angles. The question now is, what influence will this peculiarity of the sphere have upon the total reflexion and the total dispersion.

In the case of reflexion this influence is not important. If the whole quantity of light reflected at the forward surface in the case of the sphere be equal to that reflected by the arbitrary body, the same may be assumed of the second reflexion at the hinder surface, and in like manner of the third, fourth, &c.; interior reflexions respectively, at least in so far as the diminution which the ray has suffered by the previous reflexions, is not taken into account. As all these quantities of reflected light simply add themselves together, it follows that the total quantity of light which is reflected from a great number of arbitrary bodies, is nearly equal to that reflected from the same number of spheres of corresponding magnitude.

It is otherwise with the refraction. Here the essential circumstance enters, that in the case of a sphere the second refraction takes place in exactly the same direction as the first, while with arbitrary bodies the second plane of refraction can have any inclination whatever towards the first. The difference however can be definitely expressed numerically. Let us suppose the two single deflections small enough to permit of the angle being set for its sine, it is easy to deduce that the average value of the double deflection in the case of spheres is to that in

the case of arbitrary bodies as $2 : \sqrt{2}$, and if the angles of deflection be not sufficiently small, another but similar ratio exists.

If this be taken into account, it is only necessary to confine our attention to the effects of spherical bodies, without thereby limiting the assumption as to the true shape of the bodies which actually exist; it is not even necessary to dwell upon the circumstance, that in many cases where single masses separate freely from each other the spherical form is in reality the most probable.

Let it therefore be assumed in the following, that the reflexion of light in the atmosphere is due to the fact that numerous transparent spherical bodies float in it. The refraction which accompanies the reflexion may then be accurately calculated.

In these considerations it makes an important difference *whether the refractive force of the spheres, as compared with that of the surrounding medium, be regarded as considerable or as very inconsiderable, that is to say, the index of refraction very nearly* $= 1$. In the first case, a superficial calculation suffices to show that phænomena would thereby be produced which are so very different from those which actually exist, that the assumption must be rejected at once; while the second case, on the contrary, needs a somewhat closer examination.

To finish with the first case, we will assume, for example, that in the atmosphere solid spheres of water float so that the refractive index, with which we have to do, amounts to 1·333. With the aid of this value and Fresnel's reflexion formula, we can calculate how much of the light which proceeding, say from the sun, falls upon such a sphere is reflected at its forward and hinder surfaces. We thus find the value 0·12 when the incident is taken as the unit. Now Bouguer and Lambert have made experiments to ascertain how much light is lost in serene weather by its passage through the atmosphere. If the strength of the light of the sun beyond the atmosphere be set $= 1$, and the sun is conceived to occupy the zenith so that his rays pass perpendicularly through the atmosphere, then the intensity of the light on reaching the surface of the earth is,

According to Bouguer	0·81
According to Lambert	0·59

Instead of these considerably divergent numbers let us take the mean of both, the loss endured by the light will therefore be 0·25. Now if this loss were brought about by reflexion on spheres of water, then by comparison with the number above found, 0·12, we must infer that so many spheres are present that the average number crossed by every ray must be two. From this it follows, further, that when the sun is not in the zenith, but in the horizon, where the path of the light through the atmosphere is thirty-five times larger, then the number of spheres crossed by each ray on an average must be 70.

In the estimation of the dispersion caused by these spheres, we will regard the path of the light through the atmosphere so long merely, that each ray cuts one sphere on the average.

To simplify the investigation, let us for the present assume, not that the *average* number crossed by every ray is one, but that every ray cuts one sphere *exactly*. The total light, after passing through this distance, must be dispersed in the same manner as that which has passed through the single sphere. This dispersion is easily determined by means of the ordinary law of refraction, Fresnel's formula of reflexion being made use of and the reflected light being subtracted. The sheaf of light which falls parallel upon the sphere will diverge, after its passage, nearly conically, but so that the intensity of the light in the axis of the cone, which constitutes the production of the incident rays, is greatest, and diminishes the further the light is deflected from its original direction. Denoting the intensity in the axis of the cone by 1, then for the intensities J, which correspond to the various angles of deflection γ, we have the following numbers:—

γ	0	5°	10°	20°	30°	40°	50°	60°
J	1	0·98	0·83	0·54	0·28	0·14	0·06	0·03

Under this assumption, therefore, instead of observing the sharply defined disc of the sun in the zenith on a comparatively dark ground, we should see a large bright space extending with gradually diminishing brightness from the zenith downwards to beyond 60°.

We must now free ourselves from the foregoing limitation, that each ray meets *exactly* one sphere in its passage, and investigate how the result is modified when we simply regard 1

as the average number. It is therefore assumed that the number of spheres present is exactly sufficient, provided the arrangement were made so to block up the passage of the rays that each ray must pass through a sphere; these spheres, however, being now supposed to be arbitrarily scattered through the atmosphere. In this case, of course, many rays would pass through quite free, and others, on the contrary, meet with more than one sphere, and, according to the principles of probability, we can determine the ratios of the various portions of the light which meets *one*, which two, three, four, &c. spheres. Let A be the total quantity of light, and e the basis of the hyperbolic logarithms, that is,

$$e=1+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ \ldots\ =2{\cdot}71828\ \ldots$$

we then obtain the following values:—

$$\left.\begin{array}{lll}
\text{Quantity of light which meets no sphere} & =\frac{A}{e} & =A.0{\cdot}3679 \\
\text{,,} \quad \text{,,} \quad \text{one} \quad \text{,,} & =\frac{A}{e}\frac{1}{1} & =A.0{\cdot}3679 \\
\text{,,} \quad \text{,,} \quad \text{two} \quad \text{,,} & =\frac{A}{e}\frac{1}{2.} & =A.0{\cdot}1839 \\
\text{,,} \quad \text{,,} \quad \text{three} \quad \text{,,} & =\frac{A}{e}\frac{1}{3.} & =A.0{\cdot}0613
\end{array}\right\} (1)$$

According to this, one-third of the light coming from the sun would, it is true, preserve its parallelism, but a second equal quantity would exhibit the dispersion given above, while the remaining portion would suffer a still greater dispersion.

Such dispersion does not at all exist, and still the amount is made much too small, as of the shortest possible path of the light we have only taken the half. It is therefore plain that the assumption of the reflexion being caused by solid spheres of water suspended in the atmosphere must be altogether rejected.

Instead of water, let us suppose the case of masses possessing smaller refractive force, and whose number compensate for their want in this particular, we thus obtain, it must be admitted, a feebler dispersion than before. As long however as the refractive power remains in any degree considerable, we may always convince ourselves by considerations like the fore-

going of the incorrectness of the supposition. The last only of the cases above mentioned remains therefore to be examined, that is, *when the refractive index of the masses compared with that of the surrounding medium* is assumed to be very nearly = 1.

We might here indeed be inclined to seek the cause of the reflexion, not in bodies foreign to the atmosphere, but in the constitution of the air itself, that the latter may be composed of particles separated from each other by empty space, and that the light in its passage from the vacuum to the particle, and *vice versâ*, induces reflexion. Or it might be imagined that the mixture of oxygen and nitrogen, how intimate soever it may appear, is still not perfect enough to form a homogeneous mass, but that, on the contrary, a continual passage from oxygen to nitrogen, and *vice versâ*, takes place. On account of the smallness of the difference between the refractive indices of oxygen and nitrogen, every such passage would only produce a feeble reflexion, but through the innumerable repetitions of the process the total reflexion in the atmosphere might amount to the magnitude observed.

Without entering upon the probability of any such special assumption, we will simply confine ourselves to the condition, that the reflecting masses possess a very small refractive power.

To investigate the dispersion produced by these masses, it is necessary, in the first place, to obtain distinct expressions for the dispersive power of the atmosphere; these expressions will be greatly simplified if we assume that layers of air of the same thickness in all regions of the atmosphere produce equal effects upon the light. This is not the case, in fact, as the air above is less dense than that below; but we may conceive the atmosphere to be so compressed that it possesses everywhere the same density as at the surface of the earth, and if the reflecting and dispersive force be supposed to be the same everywhere, both are easily determined by simple constants.

By the passage of the original parallel sheaf through the atmosphere it is gradually enfeebled, and expands itself conically. Both effects are really produced suddenly at the forward and hinder surfaces of the layers of air. But as each mass, according to hypothesis, exerts a very feeble influence, and the effects need manifold repetition to become considerable, we may re-

gard the progress as *continuous* through the length of the path, by which we secure the advantage of being able to apply the differential calculus.

Let ds be an element of the path, and the intensity of the light at the commencement=J, then the loss of intensity endured by the light in passing over the distance ds, is denoted by

$$J \cdot h\,ds,$$

and h is the constant which expresses the reflective power of the atmosphere.

The determination of the second constant requires a somewhat more tedious development. By means of each ray which descending through the atmosphere meets the eye, a point in the dome of heaven appears to us illuminated; the direction of the ray may be determined by the point in the firmament from which it appears to come, and its intensity by the brightness imparted to this point. To denote the different points of the hemispherical dome of the firmament, let two great circles AX and AY (Plate V. fig. 1) be supposed to pass through any one of them, A. On account of the smallness of the arcs which enter into consideration here, we may regard the portions of the coils as straight lines, and make use of them as rectangular coordinates.

Let us suppose the observer at any point within the atmosphere, regarding from this point the portion of the firmament in the neighbourhood of A. At its different points this may appear of different brightness, but will not exhibit the sudden change exhibited by a star upon the dark blue ground behind it; a gradual increase or decrease of light from point to point will, on the contrary, be observed. For the present purpose, let us assume more definitely, that the brightness v is the same in all lines, such as CD parallel to AY, but that with increasing x, on the contrary, it diminishes according to the equation

$$v=a-x, \quad \ldots\ldots\ldots \quad (2)$$

where a is a constant.

Let us suppose the position of the observer in the atmosphere moved a certain distance backwards, and let him regard the firmament from this new point as before, the light in its passage from his first to his second position having undergone a certain dispersion. By this the rays appear as if they came from other

points, so that the intensity of the light in the firmament appears differently distributed. Now, as in the foregoing case, the spaces left of CD all possessed a greater intensity than the spaces to the right, the dispersion, the tendency of which is to compensate these differences, will cause a portion of the light to be apparently transferred from the left to the right of CD, and moreover an equal quantity in all points of CD, because, as regards y, the brightness is constant. We can therefore express the dispersive power of the atmosphere by stating how much of the luminous intensity passes through an element $=dy$ of the line CD from left to right, while the rays have traversed the element ds through the atmosphere. Denoting this quantity by

$$k\,dy\,ds,$$

then k is the second constant sought.

By means of these constants, h and k, we can form the differential equations by which the changeable distribution of the luminous intensity in the firmament will be determined. For this purpose let us fix our attention on the point M (Plate V. fig. 1) on the firmament, the coordinates of which shall be x and y, and let us take the element $MN\,M'N' = dx\,dy$. Let the brightness of this element, as seen from any point of the atmosphere, be v. Removing the point of observation, the distance ds, backwards in the direction of the rays, the element regarded from this new point appears no more of the brightness v, for this has been changed both by reflexion and dispersion. By the former of the intensities, $v\,dx\,dy$, the portion

$$h\,.\,v\,dx\,dy\,ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I.)$$

is lost upon the way ds. To ascertain the effect of the latter, it must in the first place be remarked, that the above expression, $k\,dy\,ds$, which resulted from the assumption of the definite distribution of the intensity expressed by equation (2), may be easily changed so as to be applicable to every other mode of distribution. According to equation (2), we have $\frac{dv}{dx} = -1$, and it is thus manifest, that, to introduce the generalization just mentioned, it is only necessary to set $\frac{dv}{dx}$ in the place of -1.

The intensity passing through an element dy is therefore

$$-k\frac{dv}{dx}\,dy\,ds.$$

Applying this to the alteration which has taken place in the element of the surface MN M′N′, then while the rays travel the distance ds, the intensity $-k\frac{dv}{dx}\,dy\,ds$ has passed over the boundary MM′ into the element, and, on the other hand, the intensity

$$-k\left(\frac{dv}{dx}+\frac{d^2v}{dx^2}dx\right)dy\,ds$$

has passed out over the boundary NN′, so that the excess which the element has hereby gained is

$$k\frac{d^2v}{dx^2}\,dx\,dy\,ds. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

The two other boundaries, MN and M′N′, must be treated in a similar manner, and thus we find the gain of the element to be

$$k\frac{d^2v}{dy^2}\,dx\,dy\,ds. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

Now as the total increase of intensity which the element has gained by the passage of the rays through the distance ds must be expressed by $\frac{dv}{ds}\,dx\,dy\,ds$, we obtain by combining (I.), (II.) and (III.),

$$\frac{dv}{ds}\,dx\,dy\,ds=-hv\,dx\,dy\,ds+k\frac{d^2v}{dx^2}\,dx\,dy\,ds+k\frac{d^2v}{dy^2}\,dx\,dy\,ds,$$

from which we derive the partial differential equation sought:

$$\frac{dv}{ds}-k\left(\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}\right)+hv=0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By an easy substitution this may be further simplified. Setting

$$v=u\,.\,e^{-hs}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

it becomes

$$\frac{du}{ds}-k\left(\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}\right)=0. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

As this equation quite coincides with the former, with the exception that the last term is absent, we see that u is the brightness obtained for the point M after the rays have passed over

the distance s through the atmosphere, neglecting the reflexion, and taking the dispersion alone into account. To determine from this quantity u the true brightness v, we have only to multiply it by e^{-hs}.

The equation (5) must now be integrated, and moreover the expression which we seek must be so characterized, that it not only satisfies the differential equation (5), but also for $s=0$ it shall exactly represent the appearance which the firmament would present to an observer without the atmosphere. The first condition will be fulfilled by the expression, and by every sum of expressions of the form

$$u=\frac{A}{s}e^{-\frac{(x-\alpha)^2+(y-\beta)^2}{4ks}}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where A, α, and β are arbitrary constants. In regard to the second condition, the expression which fulfils it quite generally shall be only briefly stated here, as for the present purpose the examination of a particular case is sufficient. Let $u=F(x, y)$ be the equation which denotes the original distribution of the luminous intensity on the firmament, then, as Fourier has shown in the theory of heat, we have

$$u=\frac{1}{\pi}\int_{-\infty}^{+\infty}da\int_{-\infty}^{+\infty}d\beta F(\alpha,\beta)\frac{1}{4ks}e^{-\frac{(x-\alpha)^2+(y-\beta)^2}{4ks}}, \quad . \quad (7)$$

an expression which satisfies the differential equation (5), and for $s=0$ passes over into $u=F(x, y)$.

As a particular case, the examination of which we must now enter upon more closely, we will choose the appearance of a single fixed star. Without the atmosphere, a certain portion of the firmament would appear completely dark, and only a small circular space would shine upon it with considerable brightness. Let the radius of the little circle $=\rho$; hence the space taken up by the star $=\rho^2\pi$; and let us further denote the brightness of this space by t; then the total luminous intensity (λ) of the star will be expressed by

$$\lambda=\rho^2\pi t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The space $\rho^2\pi$ is however with all fixed stars so small, that they appear to us as points. Strictly speaking, nevertheless, we cannot set $\rho=0$, for then, in the foregoing equation, in order to

obtain a value of λ different from 0, we must have $t=\infty$; but in the calculation we may neglect ρ in comparison with all distances which can be subjected to observation. We therefore assume that the star observed without the atmosphere is a bright point possessing the intensity λ; and the question now is, How must it appear after the rays have passed through the atmosphere?

Let us return to equation (6),

$$u=\frac{A}{s}.e^{-\frac{(x-\alpha)^2+(y-\beta)^2}{4ks}}.$$

Taking the position of the star as the origin of coordinates, when $s=0$, for all values of x and y, with the exception of zero, u must disappear. As, however, for $x=\alpha$ and $y=\beta$ the above expression does not disappear, then we must have

$$\alpha=\beta=0,$$

and we have therefore

$$u=\frac{A}{s}e^{-\frac{x^2+y^2}{4ks}};$$

or setting $x^2+y^2=r^2$,

$$u=\frac{A}{s}e^{-\frac{r^2}{4ks}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The arbitrary constant A, which still appears in this equation, can be determined in the following manner. The original luminous intensity of the star was λ. Now as by the above u is the expression of the brightness which would be observed if the light suffered no loss in its passage through the atmosphere, but was merely changed in direction, then the light of the star must, as regards quantity, be invariable, and only as regards its distribution in the firmament be dependent on s. If, therefore, by means of the foregoing expression, we determine by integration the intensity for any value whatever of s, we must always obtain the original value λ. We have therefore

$$\lambda=\int_0^\infty u.2r\pi dr=\int_0^\infty \frac{A}{s}.e^{-\frac{r^2}{4ks}}.2r\pi dr=A\pi.4k,$$

from which it follows that $A=\frac{\lambda}{4k\pi}$, so that equation (9) becomes

$$u=\frac{\lambda}{\pi}.\frac{1}{4ks}e^{-\frac{r^2}{4ks}}. \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

Finally, to obtain, instead of u, the true brightness v, we have only, as already stated, to multiply by e^{-hs}; hence we obtain

$$v = \frac{\lambda}{\pi} \cdot \frac{1}{4ks} e^{-hs} \cdot e^{-\frac{r^2}{4ks}}. \quad . \quad . \quad . \quad . \quad (10)$$

Denoting then the brightness at the origin of the coordinates, where $r=0$, by v_0, we have

$$v_0 = \frac{\lambda}{\pi} \frac{1}{4ks} e^{-hs};$$

and introducing this value, the above equation becomes

$$v = v_0 \cdot e^{-\frac{r^2}{4ks}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

From this formula we find, that after the rays have passed through the atmosphere, instead of a very bright point, a circular space must be seen, the brightness of which diminishes from the centre outwards. The nature of this decrease, however, whether it be slow or sudden, depends upon the magnitude of the constant k. The meaning of this constant being already fixed, the next point is to determine its numerical value, which of course will be different according as the refractive index chosen for the reflecting masses is greater or less. The numerical value of the other constant h may be easily determined from the above-mentioned experiments of Bouguer and Lambert; and it is therefore only necessary to determine the ratio $\frac{k}{h}$ in order to find the value of k itself.

It is assumed that in the uniformly dense atmosphere reflecting masses of small refractive power are suspended; and according to our former discussion, we can, without determining anything regarding their actual shape, consider these masses as spherical. Assuming in this atmosphere a length of path (σ) in which each ray strikes an average of one sphere, and expressing the reflexion and dispersion effected in this distance as functions of the index of refraction (n), then will the comparison of these two functions give us the sought relation between h and k.

In the first place, as regards the reflexion, we have only to examine how much of the light which falls parallel upon a *single* sphere will be reflected by the latter, inasmuch as in the case of the spheres of water above, we may conclude that the same re-

lation must very nearly hold good for the entire quantity of light (A) which passes through the said distance σ. For the case that $n-1$ is very small, the investigation may be carried out without much difficulty. Let us imagine a diameter drawn through the sphere in the direction of the rays, and around the end of the diameter let us suppose the entire forward surface divided into narrow zones, and let the attention be fixed on one of those which passes, say the radius i and the width di. The superficial content of the zone, when c denotes the radius of the sphere, is

$$= 2c^2\pi \sin i\, di;$$

and the projection of this surface upon a plane at right angles to the rays is

$$= 2c^2\pi \sin i \cos i\, di,$$

while the projection of the total anterior hemisphere on the same plane is $= c^2\pi$. Assuming the entire quantity of light falling on the sphere as unit, then the quantity which falls upon the elementary zone is

$$= 2 \sin i \cos i\, di.$$

This light meets the spherical surface under the angle i; and if we denote the corresponding angle of refraction by i', then the quantity of light reflected from the zone, according to the formula of Fresnel, is

$$= 2 \sin i \cos i\, di \left[\frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')} + \frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right];$$

and hence the quantity which enters the sphere is

$$= 2 \sin i \cos i\, di \left[1 - \frac{1}{2}\frac{\sin^2(1-i')}{\sin^2(1+i')} - \frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right].$$

This latter suffers at the posterior surface of the sphere a reflexion at the same angle, and is therefore a second time enfeebled in the same proportion, so that the quantity of light which passes through the sphere is

$$= 2 \sin i \cos i\, di \left[1 - \frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')} - \frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2,$$

from which we finally obtain the quantity of light reflected,

$$2 \sin i \cos i\, di \left\{1 - \left[1 - \frac{1}{2}\frac{\sin^2(1-i')}{\sin^2(1+i')} - \frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2\right\}. \quad (11)$$

On account of the assumed smallness of $n-1$, we can simplify this. This causes the angle of reflexion $i-i'$ to be always small, and we can therefore set $\cos^2(i-i')=1$, and neglect $\sin^2(i-i')$ in comparison with 1; but not so with the fraction $\frac{\sin^2(i-i')}{\sin^2(i+i')}$, for this, how small soever $i-i'$ may be, at least for the limiting value $i=90°$, has the value of 1. Applying this, the foregoing formula becomes

$$2\sin i\cos i\,di\left[2\frac{\sin^2(i-i')}{\sin^2(i+i')}-\frac{\sin^4(i-i')}{\sin^4(i+i')}-\sin^2(i-i')\right]. \quad (11a)$$

As this expression merely denotes the portion of the reflected light which corresponds to one elementary zone, it must be integrated between the limits $i=0$ and $i=\frac{\pi}{2}$, which may be effected when the angle i by means of the equation $\sin i'=\frac{\sin i}{n}$ is eliminated. The quantity of light reflected from the entire sphere is thus found to be

$$=(n-1)\frac{5n^3+25n^2+15n+3}{5(n+1)^4}-(n-1)^2\left[\frac{(n^2+1)(n+1)^2}{8n^2}\log\frac{n+1}{n-1}\right.$$
$$\left.-\frac{3n^3+6n^2+7n+2}{12n^2}\right]. \quad \ldots\ldots\ldots \quad (12)$$

Applying this formula, which for the sake of brevity we shall call M, to the total quantity of light A which shines through the distance σ, and remembering that the loss suffered in this distance must, according to the notation already made use of, be expressed by $Ah\sigma$, we obtain

$$h=\frac{M}{\sigma}. \quad \ldots\ldots\ldots \quad (12a)$$

In a similar manner the coefficient of dispersion k must be determined. For this purpose let us return to the considerations by which the meaning of this coefficient was fixed, and determine how much luminous intensity passes through the element $EF=dy$ (Plate V. fig. 1) while the rays are passing over the distance σ through the atmosphere, the original distribution of the luminous intensity in the firmament being expressed by the equation $v=a-x$.

This investigation also will be simplified by the assumption

that each ray encounters on the way σ, not one sphere on the average, but *exactly* one sphere. In the former considerations respecting the spheres of water, attention has been drawn to the difference between these two assumptions. Here, however, where the light does not reach the eye after it has passed over the distance σ, but on the contrary has to travel over many such distances, it is not of importance to inquire in each particular case into the change of the form of the dispersion, but into its magnitude; and this is, as may be readily seen, the same in both cases. The diminution suffered by the ray which encounters no sphere, and therefore remains undispersed, will be compensated by that suffered by others which meet more than a single sphere, and is thus repeatedly dispersed, and, as in the case of reflexion, the smaller the refractive face of the masses the more perfect this compensation will be. We can, therefore, without further hesitation, make the assumption, that on the distance σ each ray encounters one sphere exactly.

According to this, the length σ acts upon the total quantity of light as a single sphere on the light which passes through it. As before, in the case of reflexion, we fixed our attention on a single sphere, so here also let us again suppose the sphere divided into its elementary zones. The portion of light that falls on and penetrates these suffers a definite dispersion, and it must be investigated how much luminous intensity passes through EF when this portion of the light only is taken into account. From this we can afterwards find by integration the *total* intensity which passes through EF.

Assuming on the firmament any element of surface $d\omega$ to the left of CD (Plate V. fig. 2), by the point M, with the coordinates x and y; this element possesses the intensity $(a-x)\,d\omega$, and the light proceeding from it, which falls upon the elementary zone in question, is

$$=(a-x)d\omega \,.\, 2\sin i\cos i\,di. \quad . \quad . \quad . \quad . \quad (13)$$

Of this, according to the foregoing, the quantity

$$(a-x)d\omega \,.\, 2\sin i\cos i\,di\left[1-\frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')}-\frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2$$

passes through the sphere, and then, instead of pursuing its former direction, is by the two-fold refraction deflected from it

through the angle $2(i-i')$. The corresponding intensity on the firmament, therefore, no longer belongs to the element M, but is distributed over an infinitely small ring PHQ, round M, with the radius $MH=2(i-i')$. Now if the distance of the point M from CD is smaller than $2(i-i')$, one portion of the ring lies at the other side of D; and when we conceive that through the gradual enlargement of the ring the latter is pushed over CD, we see at the same time that the bit JK, which lies between the lines of the angle $EMF=\beta$, must have passed through EF. The luminous intensity of this bit is to that distributed over the entire ring as $\beta : 2\pi$; and the magnitude of the angle β, which we may regard as very small, is determined by the angle α which the straight line MG encloses with the normal NG, and the length $MG=g$; that is,

$$\beta=\frac{dy\,.\cos\alpha}{g};$$

and thus we obtain the luminous intensity comprised in the bit JK,

$$=\frac{dy\,.\cos\alpha}{g\,.\,2\pi}.(a-x)d\omega\,2\sin i\cos i\,di\left[1-\frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')}-\frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2. \quad (14)$$

It will be of advantage to combine the consideration of the $d\omega$ by M with that of another situated at an equal distance from CD and exactly opposite, say by the point M' with the coordinates x' and y. The action of this element is opposed to that of the foregoing one, inasmuch as a portion of its light is sent towards the *left* through EF, and the difference of both actions may therefore be regarded as the total action of both. Treating this element exactly as the former, we obtain the same formula as (14), with the exception that x' stands in the place of x. Finding the difference of the two formulæ, and remembering that

$$x'-x=2g\cos\alpha,$$

we obtain

$$\frac{dy\,.\cos^2\alpha}{\pi}\,d\omega\,.\,2\sin i\cos i\,di\left[1-\frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')}-\frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2. \quad (15)$$

The same expression is found for every pair of elements which stand symmetrically on both sides of CD, and whose distances from G are not greater than $2(i-i')$. We must therefore integrate it for the semicircle described round G and over CD. For

this purpose, we will set $g\,dg\,d\alpha$ instead of $d\omega$, and then integrate from $\alpha=-\frac{\pi}{2}$ to $\alpha=\frac{\pi}{2}$, and from $g=0$ to $g=2(i-i')$; we thus obtain

$$dy.2(i-i')^2\sin i\cos i\,di\left[1-\frac{1}{2}\frac{\sin^2(i-i')}{\sin^2(i+i')}-\frac{1}{2}\frac{\tan^2(i-i')}{\tan^2(i+i')}\right]^2. \quad (16)$$

We can here set $\sin^2(i-i')$ for $(i-i')^2$, and also simplify as we did before in formula (11); we then obtain

$$dy.2\sin i\cos i\,di\sin^2(i-i')\left[1-2\frac{\sin^2(i-i')}{\sin^2(i+i')}+\frac{\sin^4(i-i')}{\sin^4(i+i')}\right]. \quad (16a)$$

This expression, which represents the intensity which passes through dy, taking merely the dispersion of one elementary zone into account, must, to find the total intensity which passes through dy, be integrated from $i=0$ to $i=\frac{\pi}{2}$.

The integral thus determined is

$$dy.\frac{(n-1)^2}{n^2}\left[\frac{(n^2+1)(n+1)^2}{8}\log\frac{n+1}{n-1}-\frac{3n^3+6n^2+7n+2}{12}\right.$$
$$\left.-\frac{5n^2+4n+1}{5(n+1)^2}+\frac{21n^4+14n^3+14n^2+6n+1}{42(n+1)^4}\right]; \quad (17)$$

and, for the sake of brevity, let us denote it by $dy.\mathrm{N}$. Now as the same quantity, according to the former notation, was expressed by $k\sigma\,dy$, we obtain

$$k=\frac{\mathrm{N}}{\sigma}. \quad (17a)$$

We have therefore found two separate expressions for h and k, (12a) and (17a). In these, indeed, an unknown quantity σ makes its appearance; forming however the fraction $\frac{k}{h}$, which it was our object to ascertain, the said quantity disappears, and we have remaining simply a function of the index of refraction n, namely,

$$\frac{k}{h}=\frac{\mathrm{N}}{\mathrm{M}}. \quad (18)$$

To obtain definite numerical values from this formula, we must make assumptions as to the possible value of n. Among the indices which can enter our considerations here, the following may be mentioned. From the experiments of Dulong we find,

In the passage from a vacuum into atmospheric air $n=1{\cdot}000294$
In the passage from a vacuum into oxygen . . $n=1{\cdot}000272$
In the passage from a vacuum into nitrogen . . $n=1{\cdot}000300$

and from the two last it follows that

In the passage from oxygen to nitrogen . . . $n=1{\cdot}000028$

In the following table are set down the values of $\frac{k}{h}$ obtained from equation (18) for the first and last of these quantities, or rather, for the sake of simplicity, for $n=1{\cdot}0003$ and $n=1{\cdot}00003$; and to be certain that we embrace all refractive indices conceivable in the case, the same calculation is extended to a series of much smaller indices:—

n	1·0003	1·00003	1·00001	1·000001	1·0000001	1·00000001
$\frac{k}{h}$	0·0034	0·00046	0·00017	0·000021	0·0000025	0·00000029

The value of the coefficient of reflexion h is found as mentioned above, independent of the index of refraction n, from the experiments of Bouguer and Lambert. According to the signification of this coefficient, taking no account of the dispersion of the light in the atmosphere, regarding its weakening by reflexion alone, and denoting the intensity with which a star appears after it has passed over the path s in the atmosphere by v, we have

$$dv=-hv\,ds,$$

and hence

$$v=\mathrm{C}\,.\,e^{-hs}.$$

Now as a star in the zenith sends only 0·75 of its original light to the earth's surface, if we take the perpendicular path through the atmosphere as the unit of the path s, we obtain

$$0{\cdot}75=e^{-h},$$

from which it follows that

$$h=0{\cdot}28768. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It might be objected here, with reference to the use made of the number 0·75, that it is not proved that the entire loss, 0·25, is due to reflexion; a portion of it may perhaps be due to an unknown absorption. At all events, the reflexion must however have much to do with it, inasmuch as the firmament sends us

such a large quantity of reflected light; and the assumption of an absorption which goes on at the same time could therefore only cause a change in the numerical values, which would by no means suffice to compensate the differences which show themselves further on.

By means of this value of h, we obtain from the foregoing table the following values of k:—

n	1·0003	1·00003	1·00001	1·000001	1·0000001	1·00000001
k	0·00099	0·00013	0·000049	0·0000061	0·00000072	0·000000084

These are the values which must be set singly in equation (10a), which latter however we will first simplify by assuming that the star is in the zenith, by which s is rendered $=1$, and (10a) becomes

$$v=ve^{-\frac{r^2}{4k}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The expansion of the luminous circle, which we see in the place of the star, depends now only upon k, and moreover so, that the smaller k is, the more quickly will the brightness diminish as r increases. To furnish a conception of the speed of this diminution in the following table for the different values of n, and hence for those of k, those magnitudes of the radius r are stated which correspond to a brightness equal to $\frac{1}{10}$th of that at the centre:—

n	1·0003	1·00003	1·00001	1·000001	1·0000001	1·00000001
r	5° 29′	2°	1° 13′	25′·7	8′·8	3′·01

Calling to mind that the radius of the sun's disc is about 16′, we see that if the reflexion were caused by the frequent passage from vacuum into the masses of air, or from nitrogen to oxygen, and *vice versâ*, then a star, in case it possessed sufficient intensity to be at all visible, would appear far greater than the sun. In the case of the other smaller refractive indices, the magnitude of this luminous circle certainly diminishes, but in all cases the result remains altogether irreconcileable with the reality; for a fixed star, even observed through a highly magnifying telescope, appears as a point, and not in the zenith alone, but also near the horizon.

It is therefore proved to be impossible that the reflexions in the atmosphere can be due to transparent masses on the surfaces which are not parallel to each other, pushing even the indices of refraction to an extreme limit. Nay, it might be proved that indices which are so nearly $=1$, are, for other reasons, not at all admissible, as they would have peculiar phænomena as consequences which would be quite at variance with the reality.

Summing up the results of the foregoing investigation, we find that if we do not assume that the reflexion takes place on opake bodies floating in the atmosphere, then along with the reflexion refraction must also take place; and that this refraction under all circumstances would produce phænomena completely contradictory to our commonest observations, with the exception of the single case, that the reflecting bodies are thin plates with *parallel* surfaces. We, hence, almost of necessity, are driven to the assumption of fine vesicles of water, which even in serene weather float in the air and produce the reflexion.

[J. T.]

Article XII.

On the Blue Colour of the Sky and the Morning and Evening Red. *By* R. Clausius.

[From Poggendorff's *Annalen*, vol. lxxvi. p. 188.]

It is remarked, in the course of the foregoing paper, that the blue colour of the sky and the morning and evening red stand in close connexion with the reflexion of light in the atmosphere. Among many conjectures on this subject, the assumption that the reflexion takes place on minute bladders of water is often stated to be that which explains these phænomena most completely. For example, Newton* says,—"The blue of the first order, though very faint and little, may possibly be the colour of some substances; and particularly the azure colour of the skies seems to be of this order. For all vapours, when they begin to condense and coalesce into small parcels, become first of that bigness, whereby such an azure must be reflected, before they can constitute clouds of other colours. And so, this being the first colour which vapours begin to reflect, it ought to be the colour of the finest and most transparent skies, in which vapours are not arrived to that grossness requisite to reflect other colours, as we find it is by experience."

Newton does not enter further into the matter, and it appears to me doubtful whether he had a clear view of the subject. When here, and still more plainly in another place†, he says that water particles, when they increase in size, must produce clouds of different colours, in my opinion he is in error, as I shall show further on. It seems also as if by water particles he meant solid spheres instead of hollow vesicles, which supposition, according to the previous discussion, is not admissible.

After Newton, many investigators have given in their adherence to the supposition of vapour vesicles; but I find the theory nowhere mathematically developed, and it is by no means

* Optics, vol. II. p. iii. prop. 7. † *Ibid.*, vol. II. p. iii. prop. 5.

generally accepted. On the contrary, another mode of explanation, that for example of Brandes in Gehler's *Wörterbuch*, according to which the *particles of air* themselves reflect the blue light in a greater proportion than the red, seems to have obtained more general recognition than the other.

After the foregoing considerations, a few words upon this subject will doubtless appear justifiable.

As every vesicle of vapour consists of a filament of water curved spherically, if it be assumed as very thin in comparison with its diameter, it exerts upon a ray of light passing through it twice the influence that would be experienced by a ray in passing through a *plane* layer of water of the same thickness. We may therefore apply in this case the known theory of thin plates.

Let such a plate, of the thickness δ, be given; and upon it, at the angle i, let a sheaf of homogeneous light of the wave length λ, of the intensity a^2, and hence of the amplitude a, be supposed to fall. Further, let the strength of the reflexion which corresponds to the angle i be expressed by the factor r, so that the elongations of the incident and of the once-reflected ray may be to each other in the ratio $1 : r$, then the intensities (b^2 and c^2) of the whole light transmitted and reflected by the plate, taking into account the repeated reflexions within it, will be expressed by the formulæ

$$b^2 = a^2 \cdot \frac{4r^2 \sin^2\left(2\pi\frac{\delta\cos i'}{\lambda'}\right)}{(1-r^2)^2 + 4r^2\sin^2\left(2\pi\frac{\delta\cos i'}{\lambda'}\right)}, \quad \ldots \quad (1)$$

$$c^2 = a^2 \cdot \frac{(1-r^2)^2}{(1-r^2)^2 + 4r^2\sin^2\left(2\pi\frac{\delta\cos i'}{\lambda'}\right)}, \quad \ldots \quad (2)$$

where i' denotes the angle of refraction corresponding to i, and λ' is the length of an undulation within the plate.

From this we see, that, when $\delta\cos i'$ is determined, both intensities depend on λ'. If, therefore, the incident light, instead of being homogeneous, consist of different colours, the action of the plate is not the same on all, and hence the reflected, as well as the transmitted light, will appear coloured; according to

the formulæ, both colours must be complementary, inasmuch as we have

$$b^2 + c^2 = a^2.$$

We will first turn our attention to the *reflected* light, assuming the incident light to be white.

The intensity of the single colours, if the corresponding intensity of the incident light be denoted by a^2, can vary between the limits

$$0 \text{ and } a^2 \cdot \frac{4r^2}{(1+r^2)^2}.$$

Assuming, in the first place, the thickness of the plate as so inconsiderable that it exactly amounts to a quarter of an undulation of the extreme violet, and let the light be supposed to fall perpendicularly, by which we shall have $\cos i' = 1$; then for the violet rays $\sin^2\left(2\pi \frac{\delta \cos i'}{\lambda'}\right)$ becomes $\sin^2\left(\frac{\pi}{2}\right) = 1$, and the reflexion of these rays is thus the greatest possible. The other rays, on the contrary, will be the less reflected the greater their wave-lengths; and the mixture of all gives a whitish-blue. If the light falls obliquely on the same plate, so that $\cos i'$ should be < 1, the violet would no longer undergo its maximum reflexion, while the other circumstance would remain unchanged, that all the remaining rays would be still less reflected, the difference increasing as the wave-lengths diminish. The total light would be therefore weaker, but still blue.

This latter phænomenon must continually take place when the thickness of the plate is still less than that assumed above. The reflected light must appear as a darker and darker blue, until the plate becomes too thin to reflect light at all, a result which is obtained from the simple consideration of the rings of Newton, where blue is the first colour which surrounds the black centre.

Let us, on the contrary, suppose that the thickness of the plate becomes gradually greater than that assumed above, the violet will be more feebly reflected, and instead of it some other colour will undergo its maximum reflexion, and so the tint of the whole light will be changed. As the thickness increases we obtain the series of colours observed in Newton's rings, namely, blue, white, yellowish white, orange, red, violet, blue, &c.

This result must also find its application in the atmosphere, where vesicles of vapour take the place of the thin plates. Those vesicles which float in the air in serene weather must certainly be very thin; and if we assume that their thickness does not exceed the fourth part of an undulation of the violet light, the blue colour of the reflected light of the firmament would follow as a necessary consequence, and the clearer the air, that is, the finer the vesicles, the more deeply blue would it appear.

When, on the other hand, the air becomes more moist and the vesicles no longer possess the requisite fineness, it might be imagined that the firmament, instead of blue, ought to exhibit the other colours in their proper order. This, however, is not the case. When in moist weather the vesicles increase in thickness, new fine vesicles are in the act of forming at the same time, so that we never have a definite thickness for all, but at most a limit value, which does not exclude the smaller ones. When this limit is a little exceeded, then with a large quantity of blue we have a little white. As the thickness increases we have blue, white, yellowish-white, and again blue, white, yellowish-white, orange, &c. Hence new colours perpetually add themselves to those already present, and the mixture of all can only add to the original blue a more or less perfect white, by which the former obtains a milky appearance, and may slowly pass into white altogether.

With this the actual process in the atmosphere coincides most completely. Even in clear weather, the sky towards the horizon generally appears whitish, for the eye thus directed, besides looking through a thicker mass of air, looks along the surface of the earth, where, at least at certain moist places, probably thicker vesicles are suspended than at greater elevations. As the air becomes moister the white expands and the whole appearance of the firmament becomes duller. In fogs and clouds the thickness of the bladders must be assumed to be much greater, but we must not therefore expect, as Newton did, that the clouds can produce, by reflexion, definite colours of a higher order, for it would be quite unnatural to suppose that the cloud is composed of vesicles all of the same thickness. On the contrary, we must expect here the greatest variety, so that

clouds illuminated by white light must appear white, which is indeed the case.

We now pass over to the consideration of the *transmitted* light. As it is complementary to the reflected light, it follows that so far as it is sensible it must appear as orange. With regard to the quantity of the white light mixed along with it, however, an essential difference takes place. In the formula (1) the minimum value is $=0$, and hence, by reflexion from a plate, certain colours might be completely extinguished, so that the entire light may appear highly coloured. In the formula (2), on the contrary, the difference between minimum and maximum is indeed of the same magnitude as in (1), but the minimum is not null. For example, if the plate consists of water, and if the light fall perpendicularly upon it, we have $r^2=0{\cdot}02037$, and from this we obtain as the extreme values of c the quantities

$$0{\cdot}922 \,.\, a^2 \text{ and } a^2,$$

while for b^2 they are

$$0 \text{ and } 0{\cdot}078 \,.\, a^2.$$

In the transmitted light, therefore, the quantity $0{\cdot}922 \,.\, a^2$ is completely white, and only the remaining portion, which at most amounts to $0{\cdot}078 \,.\, a^2$, is coloured, so that the tint of the entire light must be very feeble. When the incidence is oblique the colouring indeed increases, but with great slowness. For example, for 45° we obtain as minimum and maximum for c^2, the values

$$0{\cdot}894 \,.\, a^2 \text{ and } a^2,$$

and the obliquity must be very great when a strong colouring takes place.

Fixing our attention on a vesicle of vapour, all possible angles of incidence are presented to us, but to the greatest and smallest angles, as is readily seen, comparatively small quantities of light belong, so that the medium angles are those to which the principal part belongs. But these produce but a very feeble tinting, and hence the *entire* light which passes through a vesicle must be but feebly coloured. Hence it is that the sun, when it stands high in heaven and the rays pass through a comparatively short length of atmosphere, appears white, particularly as we have no absolutely white beside it with which to

compare it. When, on the contrary, the orb is near the horizon and has to transmit its light through numerous vesicles, the orange colour obtains a decided predominance.

Finally, the circumstance that at sunset not only the sun's disc but also a considerable portion of the horizon, and even clouds which float on high appear coloured orange, is easily explained without assuming that these colours are in the first place due to reflexion. As every object which appears white in white light, in orange light must appear orange, the same takes place on the horizon, which, as remarked above, appears whitish by day.

The explanation of the blue colour of the firmament and the morning and evening red, follow therefore from the assumption of vapour vesicles in the atmosphere, so naturally and simply, that on this account alone the above assumption ought, I think, to be regarded as probable. After having shown however in the foregoing paper that on other grounds the assumption is almost of necessity forced upon us, its easy applicability to the explanation of such grand phænomena furnishes a gratifying corroboration of the result arrived at.

[J. T.]

Article XIII.

On the Theory of Diamagnetism, the Explanation of the transition from the Magnetic to the Diamagnetic deportment, and the Mathematical treatment of the Phænomena observed in Crystals. By M. Plücker, *Professor of Natural Philosophy in the University of Bonn.*

[From Poggendorff's *Annalen*, lxxxvi. 1.]

In December 1849, I handed over to the Society of Sciences at Haarlem, a memoir, the last part of which, treating as it did of the magnetism of crystals with the addition of some further mathematical developments, and the omission of some experiments, was immediately edited in French. These investigations, determined by the official dates and intended for the press, have remained up to the present time unprinted. This circumstance has placed me in a false position with regard to the investigations published on the same subject by MM. Knoblauch and Tyndall. Reluctantly, and with some dissatisfaction, I once more lift my pen to communicate the following literal extracts from the original memoir at present before me.

Part I.

11. It is a known fact that a spiral, through which a galvanic current is suffered to pass, converts a bar of iron placed within it into a magnet. A bar of bismuth under the same circumstances will also become a magnet, but its poles are those of the iron reversed.

To prove this by experiment, I took two equal hollow spirals 120 millims. long, having an interior diameter of 26 millims. and an exterior diameter of 52 millims. (Plate V. fig. 3). The copper used in their construction was 5 millims. in diameter. Both spirals were placed at right angles one above the other, and in the lower one a bar of iron 130 millims. long and 21 millims. thick was placed, while in the upper spiral a bar of bismuth 80 millims. long and 15 millims. thick, was introduced and suspended vertically from the arm of a fine balance. It was so equilibrated by a scale-pan suspended from the opposite arm, and into which fine shot was placed, that the two cy-

linders of iron and bismuth were separated by an interval of from 1 to 2 millims. A current from three of Grove's elements could be sent through each spiral in particular and in any required direction; we will assume the direction to be such that at the upper end of the bar of iron a north pole was excited. In consequence of this the cylinder of bismuth was repelled, but by removing a few grains of shot could be brought into its former position. The current was then conducted through the upper spiral, and first in the same direction as in the lower one; the bismuth cylinder receded again from the iron, but returned to its position as soon as the current was interrupted. When however the direction of the current in the upper spiral was opposed to that below, the cylinder of bismuth was attracted, and did not recede until the current had been intercepted in the upper spiral. The experiment was repeated several times, the action was decided although feeble. The consequences were the same when, through the under spiral, the current was reversed and the upper end of the bar iron thereby converted into a south pole.

12. With the intention of increasing the effect, the same experiment was so modified, that instead of the under spiral the great electro-magnet (Plate V. fig. 4) was made use of. Upon its poles the two heavy semi-armatures were laid, and the round ends brought within a distance of 10 millims. of each other. Then one of the former spirals was so placed upon one of the semi-armatures, that the bismuth cylinders, which moved up and down within it, rested upon the point of the armature when the magnetism was strongest. The magnetism of the great electro-magnet was excited by four of Grove's elements, and then the bismuth cylinder so equilibrated that an interval of 2 millims. existed between it and the armature. When the current from five other elements of Grove was transmitted through the spiral in such a direction that an iron bar, substituted for the bismuth, would have a south pole at its end next the armature, which latter was a north pole, then the cylinder of bismuth receded to a height of 4 millims. above the armature, and returned to its former position when the current was interrupted. All modifications of the original experiment gave the expected result.

In order to be sure that in the last experiment the observed action was not due to any foreign cause, I convinced myself by direct experiment and making use of nine cells, that when the spiral was placed upon a parallelopiped of soft iron and a current sent through it, no sensible magnetism was excited in the iron, and on the other hand, when the bismuth was suspended close to the same mass, there was neither attraction nor repulsion when the mass did not rest upon the electro-magnet.

13. These experiments prove to a certainty that the spiral acts upon the bar of bismuth, and that this action is reversed by the reversion of the current, and hence is a *polar* action. The action of the magnetic poles on the bismuth either coincides with the action of the spiral or is opposed to it; and the whole is completely explained if we assume that the pole of the magnet as well as the spiral excites on the bismuth a polarity which is exactly the reverse of that of iron, so that, for example, in the original experiment, when the direction of the current is the same in both spirals, the polarity of the ends of the iron and bismuth cylinder turned towards each other possess the same polarity.

14. The intensity of the magnetism assumed by a bar of iron within a spiral is so great, that the result of the described experiments, although feeble, cannot be doubted if the assumption, which it is intended to confirm, be the correct one. Let us assume in the original experiment that the end of the iron bar, which has become a magnetic pole, excites in the adjacent end of the bar of bismuth a polarity 10,000 times less than what it could excite in a similar cylinder of iron (I assume this number too small, and at the same time do not take into account that the two iron cylinders, by mutual induction, strengthen each other), then the repulsive force upon the bismuth will be 10,000 times smaller than the attraction of the iron would be, and in this case, with the assistance of a fine balance, we can always observe the action. This repulsion evidently increases or diminishes before a constant magnet, in the same ratio as the polarity of the bismuth increases or diminishes. Assuming, therefore, that the magnetic induction of the bismuth by the upper spiral is to its induction by the cone of iron in the ratio

of $\mu : \nu$, then when the induction in the bismuth excites a pole of the same name, the repulsion will increase in the proportion of $\nu : \mu + \nu$. When the induction is the opposite, the original repulsion will only continue so long as $\mu < \nu$, that is, so long as the induction of the spiral is smaller than the induction of the magnet; when this ceases to be the case the repulsion must pass over into attraction. If the ratio $\mu : \nu$ in the case of the diamagnetic induction of the bismuth does not differ too much from the magnetic induction effected by the iron, then the necessity that the above experiments must succeed becomes manifest.

If in the two spirals we introduce two cylinders of bismuth, then it would follow theoretically that these two cylinders, when the direction of the currents round both is the same, must attract each other, inasmuch as the adjacent ends obtain opposite polarities, exactly as would occur in the case of two cylinders of iron, only the north pole would change with south pole and south pole with north pole. When the directions of the currents in the spirals are opposed to each other, repulsion must take place, for then the adjacent ends of the two cylinders would obtain the same polarity. We can however state beforehand, that in these two cases the attraction or repulsion is so small that we can never hope to observe it by ordinary means. And it is likewise evident that every experiment, no matter how it may be made, to prove the mutual action of diamagnetic bars of bismuth upon each other must be futile. Theoretic considerations warn us not to vainly multiply such experiments to too great an extent, nor to attach a false meaning to the negative results by considering them as proofs against the theory.

The attraction of the bismuth cylinders in the two spirals is manifestly equal to the product of the magnetic excitation of the two adjacent ends, and hence, retaining the numbers already asumed, $\frac{\nu}{\mu} \cdot \frac{\mu + \nu}{\mu}$ 10,000 smaller than the corresponding repulsion between iron and bismuth. As this repulsion is already so small, the attraction of the two induced bars of bismuth must necessarily escape our powers of observation.

If the latter attraction must be regarded as a magnitude of the second order, then the repulsion of diamagnetic bismuth upon

uninduced bismuth must be regarded as a magnitude of the third order, that has a theoretical but no experimental significance.

15. From considerations of which we shall speak afterwards, it appeared to me probable that bismuth not only assumes polarity in the vicinity of a magnetic pole, but that it also retains the polarity for some time after the excitation has taken place; or in other words, that bismuth retains a portion of its magnetism permanently, as steel, unlike soft iron, retains a portion of the magnetism excited in it by induction. My conjecture has been corroborated by experiment.

16. I hung a bar of bismuth 15 millims. long and 5 millims. thick, between the pointed poles of the large electro-magnet; it was suspended horizontally from a double cocoon thread (Plate V. fig. 5). The distance between the points was diminished until the bar could barely swing freely between them. A little bar of glass was brought near to one of the points, so that the bismuth bar, before the magnetism was excited, in consequence of the torsion of the fibre, leaned against the glass and was held by the latter. On exciting the magnet by a current from three of Grove's elements, the bismuth, prevented from assuming the axial position, pressed more forcibly against the glass rod; when the current was interrupted the bar remained still in contact with the glass rod, while its free end vibrated round its position of equilibrium. The current was closed anew, and then reversed by means of a gyrotrope. In consequence of this reversion the bar of bismuth loosing from the glass rod moved towards the axial position, but soon returned and pressed against the glass as before, or in some cases having passed quite through the axial position was driven round with reversed ends into the equatorial.

When the current was interrupted and then slowly closed so as to reverse its direction, two or three seconds being devoted to this purpose, the observed phænomenon did not exhibit itself.

17. This experiment, which was made with some care, was repeated with the same result, when a battery of ten of Grove's elements was made use of. It proves that the bismuth requires time to reverse its polarity. That end of the bar, which, being near the north pole of the magnet, became itself a north pole by induc-

tion and was therefore repelled, retained its north magnetism for a time after the north pole of the electro-magnet, by reversing the current, was converted into a south pole; the consequence of this was the observed attraction of the bismuth. The strong south pole of the magnet however soon changed the magnetism of the end in question of the bismuth bar into south magnetism, and the immediate consequence of this was repulsion.

18. If we combine both the former and the latter of these experiments, we cannot fail to arrive at the conviction, that diamagnetism is identically the same as ordinary magnetism. If we assume iron to be the representant of magnetic bodies, we must of course forget the enormous difference in point of strength. That this difference however may disappear, we should prefer taking some less magnetic metal, say platinum, in its place.

A little bar of bismuth, between the poles of a magnet, becomes then a real magnet, and differs from a bar of platinum only in the circumstance, that its poles are the reverse of those of the latter metal. We need only imagine the bismuth to be reversed longitudinally, nay, we may actually reverse it, and in the moment after reversion we obtain two magnets which are not to be distinguished from each other. If both retained the condition which they assume between the magnetic poles and could be removed in this condition from the magnetic field, then if we did not know the actual positions which they assumed between the poles, we could by no means decide which of them, whether platinum or bismuth, was the magnetic body. The word *diamagnetic* would be therefore superfluous, while regarding the magnetic condition of different bodies.

19. Bismuth and iron both assume their magnetic state under the same circumstances, by *induction*, which can proceed from a magnet just as well as from the wires of a helix. This induction, however, if we regard the polarity excited by it, exhibits itself antithetically in bismuth and iron. We must distinguish a *magnetic* from a *diamagnetic induction*; according as the induced body is *magnetic* or *diamagnetic*, the first or the second takes place. In the case of diamagnetic induction, a north pole excites a north pole, and a south pole a south pole, in the nearer particles of the induced body; while in the case of magnetic induction,

a north pole excites a south pole and a south pole a north pole, in its immediate neighbourhood.

From this way of viewing the subject, the repulsion of the mass of bismuth by the pole of a magnet is explained in exactly the same manner as the attraction of the mass of iron by the same. While in one case the excited pole is always a friendly one, in the other it is hostile.

In like manner, the equatorial position assumed by a bar of bismuth, in opposition to the axial position assumed by a bar of iron, is easily explainable. For each bar we have *two* positions of equilibrium; the axial and the equatorial. Where both bars rotate, the distribution of the magnetism is perpetually changing; in the axial position both bars are longitudinal magnets; in the equatorial position both are transversal magnets. In both cases the bar of iron turns the friendly pole, and the bismuth the hostile pole to the electro-magnet. For the bar of iron the axial position, where its ends are the poles, is the position of *stable* equilibrium, the equatorial position that of *unstable* equilibrium. For the bismuth bar, the contrary holds good; the equatorial position, where the poles are drawn out as it were into straight lines, is that of *stable* equilibrium, and the axial that of *unstable* equilibrium.

The induction now under consideration is that by which, according to Ampère, elementary currents are excited round the ultimate particles of bodies, which as long as the excitation continues remain unchanged, or which, if we assume the currents as already existing, imparts to them a single or at least one predominant direction which they did not possess originally. In magnetic bodies the induced molecular currents have the same direction as the molecular currents of the inducing pole or the current in the inducing spiral: in diamagnetic bodies the direction is the opposite.

[I repress here the statement and discussion of the manifold experiments in which with mixed bodies the magnetic deportment passes over into the diamagnetic. From these I have deduced the empirical, and *as such* incontrovertible law, "*that*

in the amount of the inducing force diamagnetism increases more quickly than magnetism."]

26. I see the cause of this empirical law in the circumstance, that to the excitation of the diamagnetic condition a greater resistance is presented than to the excitation of the magnetic condition. I will in the first place endeavour to illustrate this way of regarding the subject by an analogous experiment, and then endeavour to show how one of the experiments which I have already described presents a new support to our assumption.

27. A different resistance, such as we assume to exist in the case of magnetic and diamagnetic bodies, is a proved fact as regards many kinds of iron and steel, being in this case named coercive force. This in general is deduced from the circumstance, that some of these substances *retain* a greater quantity of magnetism, and for a greater time, than others. But we can also establish experimentally this difference as regards coercive force in the different strength of the magnetism excited by induction, and which afterwards quickly disappears.

For this purpose let two equally long bars of hardened steel and soft iron, the former of which is thicker than the latter, be tied together at the middle, so that they form any angle, say one of 45° with each other, and let them be suspended horizontally (Plate V. fig. 6). If the system be caused gradually to approach the pole of a strong magnet, at a great distance a position will be assumed from which we may infer the predominance of the attraction of the bar of iron, but when we approach the pole the position assumed shows us that the bar of steel is the most strongly attracted. At a great distance the bar of iron sets axial, while near the pole the bar of steel takes up this position. Instead of causing the system of the two bars to approach the magnet, we could also, by choosing a suitable distance, change the strong pole for a weak one, and thus obtain the corresponding action.

In this experiment, the steel, on account of its greater coerceive force, is magnetized with more difficulty than the soft iron. If we assume that the diamagnetic coal assumes the magnetic condition with more difficulty than the iron particles uniformly distributed throughout its mass, then in a mixture of proper proportions a bar of such coal must appear magnetic or diamag-

netic, according to the greater or less distance from the poles, or the greater or less intensity of the magnetic force; it must sometimes be attracted and sometimes repelled. The experiment shows this. Whether our hypothesis has exceptions, must be decided by future experiments.

28. The method of explanation now before us leaves nothing to be desired, if in consequence of it the question might not be started, whether bismuth, when with its strong coercive force it is once excited by a magnet, does not retain its polarity when removed from the magnet. On this point we have heretofore possessed no data, but the experiment in No. 16, which was made with reference to this subject, proves it.

Whether a piece of bismuth or other diamagnetic body, with stronger coercive force, remains a *permanent diamagnetic* for a considerable time, is probably not to be proved on account of the feebleness of the force; for even hardened steel retains manifestly only a small portion of the magnetic excitation which it experiences in the neighbourhood of a strong magnetic pole. It is sufficient here to have proved experimently that bismuth, far from every diamagnetic induction, nay even against the latter, retains a sensible amount of diamagnetism for a short time.

Part II.

[I repress here the enumeration of the known facts regarding the diamagnetic deportment of crystals. In the following I will confine myself strickly to the more mathematical treatment.]

3. These facts are completely determined. The action of a magnet upon crystals is modified by some cause which lies in the crystallization, or even inverted by this cause. From the experiments described, and many other similar ones, the following law is derived.

The optic axis of uniaxal crystals is attracted or repelled by the poles of a magnet according as the crystal is positive or negative.

By this it appears as proved that the same final cause which produces the modification of light within a crystal, also modifies the action of the magnet upon it. The only direction in which no double refraction takes place, which, according as the crystal

is positive or negative, attracts or repels the extraordinary ray; the same direction will, under the same circumstances, be attracted or repelled by the poles of a magnet.

4. The attraction or repulsion of a direction in the interior of a body which exhibits itself as independent of the attraction or repulsion of the mass is an isolated fact, a paradoxical fact which at first sight appears irreconcileable with the universal laws of mechanics.

In order better to appreciate the nature and conditions of the described phænomenon, let us fix our attention upon a prism of tourmaline. This prism will, in the first place, be attracted by the two poles of the magnet, but when the poles are withdrawn to a distance from each other it is repelled; it forsakes the axial position and takes up a position perpendicular to it, like a body whose ultimate particles recede from the magnet. We are here naturally led to the assumption of two hostile forces, one of which acts upon the mass of the tourmaline as on a non-crystalline magnetic body, and tries to set the prism in the axial position, while the other, which depends on the crystallization, forces the prism into the equatorial position*. We are obliged to assume beyond this, that when the distance between the poles increases, the former of these two forces decreases more quickly than the latter. This was the way in which I regarded the subject when, in 1847, I stated the following empirical law as the expression of the phænomena.

The force which produces the repulsion (or attraction) of the optic axis of the crystal decreases more slowly with the distance than the magnetic attraction or diamagnetic repulsion of the mass of the crystal.

5. On the same day I observed first the remarkable deportment of a cylinder of coal, and also the deportment of the tourmaline, between the pointed poles of the large electro-magnet. At first I thought that the behaviour of the coal was determined by its structure, and was thus induced to suspend a plate of tourmaline, whose structure depends upon its crystallization, between the pointed poles. After I had examined all the specimens at my disposal I returned to the coal, and found the explanation in the

* It is by no means asserted that the two forces are independent of each other.

admixture of magnetic substances and the empirical law that the diamagnetic excitation decreased more slowly with the distance from the exciting pole than the magnetic. The manner of reviewing the subject stated in a former paragraph seemed to derive additional confirmation from this new analogy.

6. New experiments caused this analogy completely to disappear. Whatever may be the intensity of the magnetism of the electro-magnet, the prism of tourmaline always turns at the same distance from the one position into the other. Quite otherwise, however, was the deportment of the coal; here the moment when the change of position takes place depends upon the inductive force of the electro-magnet. Here, therefore, the law of distances subordinates itself to another still more general law: as empirical law it remains true, but obtains another signification. The distance as such comes into consideration only so far as the intensity of the inductive action is dependent upon it. Thus I was led to the assumption of the view that magnetism and diamagnetism are forces of the same nature, and that the resistance alone which presents itself to their excitation is different for the various magnetic and diamagnetic bodies.

7. According to this the peculiar action of an electro-magnet upon crystals stands isolated. Could this action, could the repulsion of the prism of tourmaline, to keep the same example in view, be only apparent? Such a prism, after it has taken up the axial position, is really not the less attracted. For when one of the two magnetic poles is removed, the prism approaches the other pole without changing its direction. Hence we find, particularly where the prism is caused to approach very near to the pole, that an *attraction of the mass* and a *repulsion of a direction* take place simultaneously.

8. The idea which first presents itself, is to assume that the magnetic induction suffered by the prism of tourmaline between the poles is different in different directions. Assuming it to be a minimum in the direction of the axis, we can easily give a general explanation of the different positions assumed by a prism of tourmaline between the poles of a magnet.

9. But this hypothesis is contradicted by experiment. From this hypothesis it would follow that a cube cut from such a crystal would not be attracted with equal force when the three

opposite faces are laid successively upon the pole of the electro-magnet. When the vertical faces are perpendicular to the axis the action must be a minimum, and when the vertical faces are parallel to the axis the action must be a maximum. I have proved, by exact measurements, that this does not take place; or rather, that the difference of attraction is so small that it cannot be the cause of the change of position on the part of the crystal.

(I am now convinced that this assertion is incorrect. It was founded on experiments which were not made under altogether favourable circumstances. The substance experimented with was a cube of the protoxide of iron, which stood immediately upon the approximated semi-armatures. Such experiments have been conducted by Tyndall with great exactitude, and he has corrected my assertion.)

10. If we suppose the prism of tourmaline to be divided longitudinally into two equal portions, then at a suitable distance of the poles the centre of gravity of each half recedes from the poles. The same takes place with the entire tourmaline when it is so suspended that it can move freely in a horizontal plane round a point which lies in the production of its axis.

11. These observations, based upon manifold experiments, have caused me to reject the above idea (8), or at least to regard it entirely as an indefinite one, which first needs a closer explanation, and thus finally I have arrived at the conviction that the problem before us is to explain, by the general laws of mechanics, how it is possible, in a freely suspended crystal, a *direction* can be *repelled* by the *magnetic* action of the pole of a magnet, and on the other side a *direction attracted* by the *diamagnetic* action of the same pole; that, in fine, the question is *to show by calculation and to confirm by experiment, that under certain conditions a piece of soft iron is repelled by the poles of a magnet and a piece of uncrystallized bismuth attracted**.

* The following is literally copied from the original memoir:—

"In order to obtain a clear conception of our view that a crystallized body is more ready to assume magnetic polarity in one direction than in another, the familiar thought presents itself of making use of thin bars of iron, which easily assume magnetic polarity at their ends, but with difficulty at their sides, and to unite these bars in a manner conformable with the above view and expose them to the action of a magnet. I will here describe a few such ex-

From this way of viewing the subject, a magnetic needle which is directed by the nearest terrestrial pole, while its centre of gravity sustains no sensible action, presented an analogy. If such a needle be attached at its centre to a lever which is so suspended that the whole can rotate freely in a horizontal plane, the needle moves into the magnetic meridian; and, in order to attain this position, must, according to its original position, either approach or recede from the terrestrial pole.

12. If we represent to ourselves the prism of tourmaline as consisting of an infinite number of small magnets whose mag-

periments, which were made for the purpose of explaining and partially imitating the phænomena presented by crystals.

"I united two bars of soft iron with two bars of brass to a rectangle (Plate V. fig. 7). The bars were 3 millims. thick, and the bars of brass somewhat longer than those of iron. The rectangle was so suspended by silk fibres between the pointed poles that it could swing horizontally, the distance of the poles being so arranged that the whole rectangle did not spring to either. The iron bars were *repelled* with great force by the poles, and set themselves as far as possible from the latter, parallel to the line which united the poles.

"The position assumed by the rectangle remained the same when only one pole was permitted to act upon it, or when one of the iron bars was superseded by a brass one.

"I bored sixteen holes transversely, through a round bar of brass, which was about 50 millims. long and 5 to 6 millims. in thickness, so that the holes were very small, were at equal distances apart, and all lay in the same plane of the axis. Through these I introduced thin iron wires, the protruding ends of which were filed away. Suspended between the poles so that the plane of the wires was horizontal, the bar was decidedly repelled and forced into the equatorial position. When the plane of the perforations was vertical, the bar was attracted and was brought into the axial position.

"When the iron wires passed through the brass wire at a definite angle with the axis of the latter, the bar took up an oblique position, so that the iron wires were parallel to the line forcing the poles of the magnet. When the plane of the wires was vertical the bar set axial.

"A spiral of iron- or steel-wire suspended between the poles (Plate V. fig. 8), assumes the equatorial position with great force; its two halves *were repelled by the poles*, &c.

"I also borrow from the same memoir, without the addition of any further observations, figs. 9 to 12 of Plate V., which are intended to exhibit the deportment of tourmaline, magnetic antimony, calc-spar and bismuth, between the poles of a magnet. The direction of the optic axis is denoted by an arrow, the directions in which the polarity occurs by small lines. In imitation of the two first crystals, these lines represent little bars of iron which are inserted in an indifferent mass; in the two last crystals, bars of a strongly diamagnetic substance."

netic axes are all at right angles to the axis of the crystal, then the pole of the magnet which assumes the place of the earth's pole in the former experiment, will drive the prism of tourmaline into the equatorial position.

13. But, from the assumption just made, it would also follow that the same end of the tourmaline prism must always set itself at the same side of the axial line. In single crystals, indeed, when the force of the magnet is not too great, this deportment is actually observed; with tourmaline and most other crystals, however, no such action is exhibited. Let us therefore imagine the little permanent magnets superseded by such, which, on passing the equator, change their polarity; that is to say, by little bars, which, as in the case of soft iron, become magnetic by induction, without the magnetism thus excited being retained by the coercive force of the bars.

14. We will now submit to calculation those results which we have just derived from general considerations.

Let the plane of the figure be that which passes through the two poles, P and Q, of the electro-magnets (Plate V. fig. 13). Let AB be a little bar of soft iron, attached by its centre to the lever CC′, which can rotate freely round the point O. From the assumption that P is the north and Q the south pole of the magnet, the bar of soft iron obtains a south Pole at A and a north pole at B. We will in the first place calculate the moment of inertia of the point A, in so far as it is produced by the attraction of the pole P.

Let r be the radius of the circle described by the points A and B during the rotation of the lever, c the distance of the pole P from the centre of rotation O, ϕ the angle enclosed between the lever CC′ and the line which unites the poles P and Q, and finally, let a be the angle enclosed between the lever CC′ and the line OA. Then we have

$$OA = OB = r, \qquad OP = c,$$
$$COP = \phi, \qquad COA = COB = a.$$

We will unite the points A and P by a straight line AP; let the length of this line be δ. We then obtain for the attraction of A by P the expression

$$\frac{\mu}{\delta^2},$$

where μ denotes the attraction for the unit of distance. Re-

solving this force now directed along AP into the direction AT, which touches the circle at the point A, and multiplying by the radius r, we find

$$\frac{r\mu}{\delta^2}\cos \mathrm{TAP}$$

for the moment of rotation sought. Remembering that

$$\delta^2 = r^2 + c^2 - 2rc\cos(\phi - a),$$

$$\cos \mathrm{TAP} = \sin \mathrm{PAO} = \frac{c\sin(\phi - a)}{r},$$

this expression changes immediately into the following:

$$\frac{c\mu r \sin(\phi - a)}{\{r^2 + c^2 - 2rc\cos(\phi - a)\}} \quad \ldots\ldots \quad (1)$$

To ascertain the moment of rotation in the same direction due to the repulsion of the same pole P on the other end of the bar of iron B, it is only necessary in the foregoing expression to change the signs of μ and a. We thus obtain

$$\frac{-c\mu r \sin(\phi + a)}{\{r^2 + c^2 - 2rc\cos(\phi + a)\}^{\frac{3}{2}}} \quad \ldots\ldots \quad (2)$$

Adding the two expressions (1) and (2) together, we obtain

$$c\mu r\left\{\frac{\sin(\phi - a)}{\{r^2 + c^2 - 2rc\cos(\phi - a)\}^{\frac{3}{2}}} - \frac{\sin(\phi + a)}{\{r^2 + c^2 - 2rc\cos(\phi + a)\}^{\frac{3}{2}}}\right\} \quad (3)$$

for the total moment of rotation, in so far as it is due to the pole P. The sign of the value of this expression determines the direction of the rotation. The iron bar approaches the pole P when this sign is positive; and it recedes from the pole when the sign is negative.

15. To ascertain the moment of rotation, still in the same direction, produced by the action of the other pole Q, we will assume that this pole lies at the other side of the centre O, and at a distance from it equal to c'; we need then only to set in the last expression (3) c' and $(\pi - \phi)$ in the places of c and ϕ, and to change the sign of μ at the same time. In this way we obtain

$$c'\mu r\left\{\frac{\sin(\phi - a)}{\{r^2 + c'^2 + 2rc'\cos(\phi - a)\}^{\frac{3}{2}}} - \frac{\sin(\phi + a)}{\{r^2 + c'^2 + 2rc'\cos(\phi + a)\}^{\frac{3}{2}}}\right\} \quad (4)$$

16. In the case of a permanently magnetic bar, which is subjected to the action of the earth, and whose south pole is A, we must assume c and c' infinitely great in comparison to r. According to this the two expressions (3) and (4) become

$$\frac{\mu r}{c^2}\Big\{\sin(\phi-a)-\sin(\phi+a)\Big\}=-\frac{2\mu r}{c^2}\sin a\cos\phi,$$

$$\frac{\mu r}{c'^2}\Big\{\sin(\phi-a)-\sin(\phi+a)\Big\}=-\frac{2\mu r}{c'^2}\sin a\cos\phi.$$

The magnetic forces which proceed from the two poles of the earth, unite in driving the needle into the magnetic meridian with a relative force which is inversely proportional to the distance from the poles. The magnetic bar, therefore, in the case of the figure, recedes from the (by reason of its nearness) stronger pole P. The total moment of rotation, if we denote the length of the needle $2r\sin a$ by l, is

$$-\mu\left(\frac{1}{c^2}+\frac{1}{c'^2}\right)l\cos\phi.$$

It decreases, therefore, as $\cos\phi$, disappears when the needle has attained the magnetic meridian, and for all positions is proportional to the length of the needle l*.

17. For the case of a bar of soft iron which becomes magnetic under the influence of the two poles P and Q, our formulæ are not applicable when one of the two ends A and B is situated at the other side of the axial line PQ. If both ends pass over to the other side they are again applicable; but in this case we must change the sign of μ. In the following we will confine ourselves to the case where the length of the bar, in comparison with the length of the lever and with the distance of the poles from each other, may be neglected. Here our formulæ are of general applicability, provided only that we change the sign of μ when the bar of iron crosses over the axial line PQ.

18. Inasmuch as the expressions (3) and (4) only change their signs when the sign of a is changed, from the development of

* The above formula can be applied to the earth only when we assume that the magnetism of the globe is uniformly distributed, and that, besides this, we are situated at the equator. μ then denotes the force proceeding from each pole projected upon the plane of the horizon. The result deduced from the last form remains applicable when the needle is suspended at any latitude whatever. The coefficient of $l\cos\phi$ alone changes.

them the even powers of a must disappear. Retaining, therefore, the first power only of a, we neglect only the third, and those higher than the third. We thus obtain

$$\sin(\phi \mp a) = \sin\phi \pm a\cos\phi,$$
$$\cos(\phi \mp a) = \cos\phi \pm a\sin\phi;$$

and if, for the sake of brevity, we set

$$r^2 + c^2 - 2rc\cos\phi = \overline{CP}^2 = m^2 \quad . \quad . \quad . \quad . \quad (5).$$

and develope, we have

$$\{r^2 + c^2 - 2rc\cos(\phi \mp a)\}^{-\frac{3}{2}} = (m^2 \mp 2acr\sin\phi)^{-\frac{3}{2}}$$
$$= m^{-3} \pm 3acr\sin\phi \,.\, m^{-5}.$$

Substituting these values, the expression (3) passes into the following:—

$$\frac{2ac\mu r}{m^3}\left\{\frac{3cr}{m^2}\sin^2\phi - \cos\phi\right\},$$

and may also be written in the following form:

$$-\frac{2a\mu c^2 r^2}{m^5}\left\{\cos^2\phi + \left(\frac{r}{c} + \frac{c}{r}\right)\cos\phi - 3\right\} \quad . \quad . \quad . \quad (6)$$

The expression (3) therefore gives the moment of rotation produced by the action of the pole P upon the little needle of soft iron. The corresponding moment of the pole Q is

$$\frac{2a\mu c'^2 r^2}{m'^5}\left\{\cos^2\phi - \left(\frac{r}{c'^2} + \frac{c'^2}{r}\right)\cos\phi - 3\right\}, \quad . \quad . \quad . \quad (7)$$

where, for the sake of shortness, we have set

$$r^2 + c^2 + 2rc\cos\phi = m'^2.$$

This expression is obtained immediately from the foregoing, when, as in paragraph 15, c is exchanged for c', and the sign of μ and $\cos\phi$ is at the same time altered.

19. If the lever CC′ be caused to rotate in the horizontal plane around the point O, then the momenta (6) and (7) change with the angle ϕ. These also change with r, that is, when the little iron bar, while remaining at right angles to the lever, is placed at different distances from the centre of rotation O. If we regard r and ϕ as variable quantities, by which they will obtain the significance of polar coordinates, we can calculate the moments of an inertia for all points of a straight line, consisting of infinitely small magnets, which are all at right angles to the

length, and rotate with the line around the fixed point O. To determine the points in the horizontal plane for which the moment of inertia due to each of the single poles P and Q disappears, we obtain immediately the two following equations in polar coordinates:

$$\cos^2\phi+\left(\frac{r}{c}+\frac{c}{r}\right)\cos\phi-3=0, \quad \ldots\ldots \quad (8)$$

$$\cos^2\phi-\left(\frac{r}{c'^2}+\frac{c'^2}{r}\right)\cos\phi-3=0. \quad \ldots\ldots \quad (9)$$

The two curves represented by these equations are similar, but extend from the point O in different directions; the first towards the pole P, the second towards the pole Q. If the point O lies in the middle between P and Q, the two curves are equal.

20. As the equations (8) and (9) are independent of μ, the two curves in question, geometrical loci of the indifferent points, remain the same, no matter how small or how great the magnetic intensity of the poles P and Q may be, or how the polarity induced by these poles in the bar of iron during its rotation round the point O may change in point of intensity.

21. Instead of the polar coordinates r and ϕ, we will introduce rectangular ones, x and y, by assuming O as the origin and permitting the x axis to pass through the poles P and Q. Then we have

$$\cos\phi=\frac{x}{r},\ r^2=x^2+y^2,$$

by which the first part of the equation (8) becomes the following:

$$\frac{1}{cr^2}\left\{(x-3c)y^2+x(x-c)^2\right\}. \quad \ldots\ldots \quad (10)$$

The curve of indifferent points corresponding to the pole P is therefore represented by the equation

$$y^2=-\frac{x(x-c)^2}{x-3c}. \quad \ldots\ldots \quad (11)$$

It is only necessary to change the value of c and the sign of x, to obtain from the last equation the equation of the curve corresponding to the pole Q.

The equation (11) shows us immediately, that the curve represented by it (Plate V. fig. 14), which is the geometric locus of those indifferent points obtained by regarding the action of the pole P alone, extends from the centre O laterally, that it is symmetrical in respect to the straight line which unites the two poles P and Q, that it has the pole P for double points, and for asymptote the straight line AA, which is perpendicular to PQ, and is three times as far from the centre O as the pole P. The figure exhibits the course of the curve TPCPU.

If we give the little iron bars different positions, either by turning the lever or permitting it to remain fixed and moving the bar along it parallel with itself, then as often as the curve is crossed the moment of rotation changes its sign; if the iron bar were previously attracted it will afterwards be repelled, and *vica versâ*. It is easy to convince ourselves, that within the oval of the curve the moment of inertia is negative, and that hence the pole here repels the iron bar.

Let us suppose the case of a lever KN (Plate V. fig. 15) of indefinite length and composed of an infinite number of infinitely small needles, which are all at right angles to the length, it is clear that all these needles which lie between O and L, and which lie beyond M, will be *repelled* by the pole P; while those which lie between L and M and beyond the point O in the opposite direction, will be *attracted* by the same pole. The manner of action is indicated in the figure by little arrows.

The way in which the second pole acts on the little needle, will be determined by the second curve in exactly the same manner; this curve, under the assumption that $c' = c$, that is, that the centre of rotation O is midway between P and Q, is also shown on the figure.

22. We will first consider only one half of the lever ON, in any one of its positions. We have then two forces which proceed from the pole P, and act upon the segments OE and MN, as well as a third force which proceeds from Q and acts upon the whole length ON; all three forces tend to force the lever into the equatorial position. Only one force, which proceeds from P and acts upon the segment LM, tends to force the lever into the axial position.

Regarding the other half of the lever, we find forces which

produce a resultant moment of rotation, equal to the moment of rotation of the first arm and acting in the same direction.

Each of the two halves of the lever, when submitted to the action of the two poles, must therefore move exactly in the same manner as the whole lever.

When the pole P recedes from the centre of suspension, the dimensions of the oval OLP increase, Plate V. fig. 14. The portion of the lever which lies within this oval and whose points are repelled becomes greater, while the portion attracted by this pole recedes further from the centre.

To fix our attention upon a definite case, we will assume that M is the end of the lever. Two conflicting forces are then active, one of which seeks to place the lever in the axial position, while the other forces it to the equator. (In this instance, we do not take the more distant pole Q, which acts upon the entire length of the arms of the lever, and seeks to place it in the equatorial position, at all into account.) When the pole P recedes from the point O in the proportion of OL to OM, the original oval passes into the one shown by a dotted line in the figure, which passes through the end M of the lever and through the point P′, which denotes the new position of the pole, and this embraces the entire arm of the lever. No point of the lever arm then exists which is not forced towards the equatorial position.

23. The action of the pole P on the different points of the lever increases, when these points recede from those which lie upon the curve (11). For every position of the lever, we must have *four* maxima of action, namely, a maximum of repulsion between O and L, and another beyond M; further, a maximum of attraction between L and M, and a second beyond O towards K. By turning the lever, we obtain as the geometric locus of these maxima of attraction and repulsion an easily determined curve*. For this purpose it is only necessary to differentiate the expression

$$-\frac{2a\mu c^2 r^2}{m^5}\left\{\cos^2\phi+\left(\frac{r}{c}+\frac{c}{r}\right)\cos\phi-3\right\} \quad . \quad . \quad (6)$$

with regard to r, and to set the differential coefficients $=0$. In

* It is of course understood that these maxima are to be referred to *moments of rotation* produced by attraction and repulsion.

this way we find, when we neglect the factor $\frac{2a\mu c^2}{m^7}$ and assume c as the unit of length,

$$-5\{\cos\phi \,.\, r^3+(\cos^2\phi-3)r^2+\cos\phi \,.\, r\}(r-\cos\phi)$$
$$+\{3\cos\phi \,.\, r^2+2(\cos^2\phi-3)r+\cos\phi\}(r^2-2r\cos\phi+1)=0,$$

or when we develope

$$2\cos\phi \,.\, r^4+(4\cos^2\phi-9)r^3-\cos\phi(\cos^2\phi-4)r^2$$
$$-(5\cos^2\phi-6)r-\cos\phi=0. \quad . \quad . \quad . \quad . \quad (12)$$

This is the polar equation of the curve sought. To express the curve by means of rectangular co-ordinates, we will multiply by r, $r\cos\phi$ by x, and for r^2 substitute (x^2+y^2). We then find

$$(2x-9)(y^2+x^2)^2+(4x^2+4x+6)(y^2+x^2)$$
$$-x(x^2+5x+1)=0. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The figure shows the course of the curve, which in every position of the lever gives the point, whose moment of rotation is a maximum. This, less strongly pronounced, curve is of the fifth degree, and consists of two separate portions. One of these forms an oval which passes through the point P and cuts the axial line a second time in the point S, which is half the distance of the pole P from the centre O. The point of this oval corresponds to an attraction. The other portion of the curve, like the geometric locus of the indifferent points, has the point P as double point, and an asymptote which is at right angles to the axial line, but which stands further by one-half from the centre of rotation, and an oval which lies within the oval of the geometric locus just mentioned. The points of this second portion of the curve correspond to a repulsion on the part of the pole P.

To obtain the total action of both poles on a lever of given magnitude, which consists of an infinite number of infinitely small iron bars, all perpendicular to the direction of the length, the expressions (6) and (7) must be multiplied by dr and then integrated. Let the length of the lever be 2R. For the sake of brevity, let

$$-\int_0^R \frac{\mu dr}{(r^2-2r\cos\phi+1)^{\frac{5}{2}}}\left\{r^2(\cos^2\phi-3)+(r^3+r)\cos\phi\right\}=\Phi,$$
$$\int_0^R \frac{\mu dr}{(r^2+2r\cos\phi+1)^{\frac{5}{2}}}\left\{r^2(\cos^2\phi-3)-(r^3+r)\cos\phi\right\}=\Psi.$$

Assuming the centre of rotation of the lever to be central between the two poles, we find the moment of rotation due to the action of both poles on the entire length of the lever to be

$$\{4a\Phi+\Psi\ .$$

This expression must be reduced to one-half, if we take only one-half of the lever, or regard one of the poles only as active.

25. The last expression is a function of R and ϕ. For a given position of the lever, which is determined by the value of ϕ, the value and sign of this expression depend upon the length of the lever R. When the sign is positive the lever approaches to the axial position, when it is negative it recedes from this position. If we regard R and ϕ as polar coordinates and then set the last expression equal null, then the resulting equation

$$\Phi+\Psi=0$$

represents a curve which immediately determines the motion of a lever of given length in all its positions. The development of this equation is easy but somewhat tedious.

26. In 23 we have regarded the coefficient μ as constant. This would really be the case if the little needles at right angles to the length were permanent magnets, whose intensity is not changed by the induction of the poles P and Q. If however these needles are of soft iron, then the intensity of the magnetism excited in them by the poles, depends upon their distance from these poles. From the analogy of central forces, must we not assume that for the case of magnetic saturation—a case perhaps which does not exist—the intensity of the excited magnetism varies inversely as the square of the distance?

Let κ be the magnetic intensity of the pole P, and λ that of two ends of the needle; then, according to the first assumption, we have

$$\mu=\kappa\lambda.$$

According to the last assumption, we should have

$$\mu=\frac{\kappa\lambda}{m^2},$$

if we denote the intensity of the magnetism excited in the ends of the iron needle by the pole P, at the unit of distance by λ and permit m to retain its former meaning. The natural case

lies between both. The curve of the indifferent points will be in no way affected, whether we assume μ as constant or as variable; we can always compare the results of calculation with those of experiment. In the assumption that μ is variable, certain modifications of the developments in 23 are necessary, and the values of the limited integrals Φ and Ψ do not remain the same.

27. According to the foregoing, the deportment of a prism of tourmaline suspended between the poles is no longer a mechanical paradox. It is only necessary to assume that its ultimate particles become magnetic under the action of the magnet, but so that the straight line which unites the two poles of every particle is perpendicular to the optic axis. To imitate the tourmaline, it is only necessary to construct a prism of thick brass wire, or any other feebly magnetic or non-magnetic substance, and through the prism, at right angles to its axis, to introduce bits of iron wire in all possible directions. Thin iron plates, perpendicular to the axis, and whose centres are in the axis, might be substituted for the wires.

28. To explain the phænomena exhibited by crystals of oxide of tin, described above, it is only necessary to assume that in this case the permanent polarity, or that excited by induction, always takes place in the direction of the optic axis. To imitate these phænomena, we can introduce through a mass, which is but feebly acted upon by the magnet, a number of iron wires having a common direction; this direction will represent the optic axis of the crystal.

29. Finally, if instead of such crystals as we have regarded in the two last paragraphs, we take those whose substance is *diamagnetic*, the phænomena are similarly explained, when we assume that *the diamagnetic induction is opposed to the magnetic*. We thus make the assumption, that in calc-spar the directions in which the polarity takes place are all at right angles to the optic axis, while in the case of ice these directions all coincide with the optic axis.

30. According to the theory of Ampère, the hypothesis that we have made in 27 to 29, reduces itself to the assumption, that in those crystals which do not belong to the regular system

the molecular currents do not take place in all directions indifferently.

31. This is not the place to enter into the detail of the experiments which I have undertaken with the intention of imitating the different phænomena which are exhibited by crystals (particularly by biaxial ones) between the magnetic poles. One difficulty still remains, which springs from the experiment alluded to in 9.

32. Faraday was the first who discovered that in certain crystals a definite direction was attracted by the poles of a magnet; as I was the first to observe the repulsion of such a direction, nothing was more natural than to ascribe both classes of phænomena to the same cause. Of the well-established fact, that in crystals certain directions, dependent on their structure, occur, which are either attracted or repelled by the poles of a magnet, I have, I conceive, in the foregoing given a sufficient mechanical explanation. The great question, however, still remains to be decided as to what cause we are to attribute these phænomena, which in fact would lead us to pronounce upon the nature of magnetism itself. Is it the form or position of the ultimate particles of the crystal which modifies the action of the magnet; or is it rather that magnetism, like light, is due to the motion of an æther, the elasticity of which is subjected to modifications within the crystal?

33. Adopting the great English experimenter's view, that the attracted line is perpendicular to the plane of most eminent cleavage, and assuming, further, that in other crystals the same direction is repelled, we arrive of necessity to the conclusion—whether it be that one surface of cleavage is present or several such surfaces, and we must regard the resultant action,—that in uniaxial crystals a single direction exists, which is either attracted or repelled by the magnetic poles, and that this axis must coincide with the optic axis round which the crystal is symmetrical. In the uniaxial negative crystals which I have examined the optic axis is repelled, and in the positive attracted. I have thought myself justified, in accordance with this view, in extending the classification of crystals under positive and negative to the opake ones, such as bismuth and antimony. But even should subsequent observations compel us to give up

the analogy between light and magnetism, as regards the latter, the classification must still continue to exist. The magnetic axes and the optic axes must both be regarded; the former as well as the latter are either positive or negative. In a similar manner different axes have been observed in crystals, for the different colours which either do or do not lie in the same plane. The uniaxial crystals do not enable us to decide here, and I have been compelled to turn to the biaxial crystals in order to seek the universal response. As this investigation is not yet complete, for the present I refrain from drawing any inferences from it.

34. The experimental investigations are rendered complicated in a high degree by the circumstance that foreign substances are generally mixed up with the crystals. Thus, for instance, Faraday's crystals of arsenic were diamagnetic, mine, evidently because they contained iron, magnetic at all distances from the poles. But the crystals of both set so that the direction perpendicular to the principal plane of cleavage coincided with the line which united the poles. If the iron entered as chemical constituent into the ultimate particles of the crystal, then we must expect the axis of the crystal in one case to be attracted, and in the other case repelled. Inasmuch, however, as the opposite takes place, I believe I am justified in concluding that the iron was a foreign body in the crystals which I examined.

35. From our way of viewing the subject, we ought to expect that in organic substances phænomena must exhibit themselves analogous to those observed in the case of crystals. Thus, for instance, we should imagine that an organic web, saturated with a magnetic substance, would not in magnetic respects prove the same in all directions, but that, on the contrary, in the direction of the length the capillary canals filled with the magnetic substance would cause the magnetism to exhibit itself more forcibly in this direction than in any other.

36. In certain crystals it might, indeed, be doubted whether the observed phænomena were due to the crystalline form and its possible action upon the æther, and not, on the contrary, to an admixture of foreign magnetic substances uniformly distributed between the ultimate particles of the crystals. I reckon here many crystals, which in their state of purity contain no iron,

but which are often found to contain an admixture of iron. I will here refer only to tinstone and wernerite, in which the phænomena spoken of only exhibit themselves when the substance is magnetic, but then in such a high degree that the first-mentioned crystal is directed by the magnetism of the earth.

37. When the foreign magnetic substances are irregularly distributed through the mass of a crystallized body, for example, through a crystal of tourmaline where the unequal distribution is indicated by a difference of colour, the action of the magnet will necessarily be distributed and a direction may be given to the crystal, which, without it, would show no direction whatever*.

Finally, a trace of iron, which appears under circumstances which deserve mentioning, may give rise to error. To cite a distinct case, let a plate or prism of rock crystal be taken, both of which, suspended between the poles, turn in consequence of the ordinary diamagnetic action into the equatorial position. If the length be shortened so as to annul the effect of form, the shortening being effected by a hammer, we generally find a trace of iron attached to the crystal through contact with the hammer. This iron will not be at all detected, if we bruise the crystal close to a powerful magnetic pole, but when placed at a sufficient distance, the magnetic attraction of the iron overcomes the diamagnetic repulsion of the mass. This follows from the law deduced in 5†. Without this law, we must of necessity conclude from the motion of the crystal between the poles, that in the case of the prism the optic axis is attracted, while in the case of a plate cut perpendicular to the optic axis, repulsion of the latter takes place. At different times I have fallen into this two-fold error; the first time, considering our knowledge of the subject, I was certainly not to blame. The above phæno-

* There is the strongest reason for supposing these remarks applicable to the crystals of calc-spar examined by M. Plücker, on which he bases his conclusions that there are *diamagnetic* crystals of this class whose optic axes are *attracted*, and *magnetic* specimens whose optic axes are *repelled*. See Phil. Mag.—J. T.

† The law in No. 5 is at variance with the above fact, but the contradiction is doubtless due to an error in printing.—J. T.

mena do not exhibit themselves if the crystal be first placed in acid which dissolves the iron. I must now express my adherence to the result of Faraday, that the axis of rock crystal is neither attracted sensibly nor repelled by the poles of an electromagnet.

Note.

M. Plücker complains of the false position in which the non-publication of the foregoing memoir has placed him. It gives me pleasure to do him the justice of placing it before the English public. Granting, however, that the memoir had been published with all possible despatch, it would, I imagine, but slightly modify the tone and purport of the papers which have since appeared upon the same subject. We have, for example, the law regarding the attraction of the axes of positive crystals and the repulsion of the axes of negative crystals restated in the foregoing memoir. This, I believe, M. Plücker himself now admits to be an error. The turning of a crystal from the axial to the equatorial position is also dwelt upon, a fact which formed the basis of a most important general law; but there is not the slightest indication of the veritable cause of this phænomenon, that it is due to the action of the pointed poles made use of in the experiments; nor can I gather even from the latest writings of M. Plücker, that he is as yet aware of the extent to which this circumstance has vitiated his otherwise beautiful investigations. In a note at the bottom of page 341, it is stated that it is not meant to be asserted that the optic axis force is independent of the magnetism or diamagnetism of the mass. Similar statements are made in other publications of M. Plücker. As long, however, as his results remained uncontroverted, the independence spoken of is constantly affirmed. The optic axis force is called "a *new force* not hitherto indicated by any phænomenon," and an individuality is ascribed to it of the same order as magnetism itself. No astronomer, I imagine, would think of dividing the forces of the solar system into—

1st. The force of gravitation;

2nd. The force which produces the precession of the equinoxes.

And yet, when we grant that M. Plücker knew the relation

between magnetism and magnecrystallic deportment, there is a strict analogy between his classification and the above. I have no desire to impugn the accuracy of the statement, that the independence contended against was never meant to be asserted; I only urge that the fair grammatical interpretation of the language used would lead any body to suppose that it *was* meant, and this, I trust, will justify me in the course which I have taken, even recently, in connexion with this subject. Few have given more attention to the question before us than M. de la Rive, and few I think understand it better. Let us inquire how he has interpreted the words of Prof. Plücker. In the English translation of his valuable treatise on electricity, he states the view of this philosopher to be, "*that the axis, in its quality of axis, and independently of the very nature of the substance of the crystal, enjoys peculiar properties more frequently in apposition* (opposition?) *to those possessed by the substance itself, or which at least are altogether independent of it.*"

There is undoubtedly an approach to the true solution of the question in the foregoing memoir. We obtain from the perusal of it the conviction, that had M. Plücker been left to the undisturbed prosecution of the subject, he would finally have completed his valuable discovery by the development of its true laws. Still, although the tendency is in the right direction, the end is far from being attained. The fundamental cause of all magnecrystallic phænomena is, that the mass is attracted or repelled with peculiar force in one or more directions, a fact first established in the case of bismuth by Faraday. In the memoir before us this cause was not only not assigned, but a series of experiments were cited for the express purpose of disproving its existence. On this cardinal point M. Plücker has the candour to admit himself in error. It might be readily inferred from this and other papers that M. Plücker had been misinterpreted; sufficient, I trust, has been said to show, that, if such has been the case, the mistake was not wilful, and also to prove, that however quickly the foregoing memoir might have been printed, it would by no means have rendered the researches which have been since made unnecessary.—J. T.

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